Examination of High School Students' Understanding of Geometry

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EXAMINATION OF HIGH SCHOOL STUDENTS’ UNDERSTANDING OF GEOMETRY

by

Brantley Grant Pierce

A Thesis
Submitted to the Graduate School
of The University of Southern Mississippi
in Partial Fulfillment of the Requirements
for the Degree of Master of Science

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August 2014
ABSTRACT

EXAMINATION OF HIGH SCHOOL STUDENTS’ UNDERSTANDING OF GEOMETRY

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Not every student learns geometry instruction the same. Inside today’s classroom, one will find a diverse collection of students with different learning styles, background knowledge, and cognitive abilities. Students with high cognitive skills may sit next to those who struggle to maintain the material of a single subject. It is the job of an educator to accept the students as they are and guide them through a successful academic journey. This process is called Differentiated Instruction. Gregory and Chapman, authors of Differentiated Instructional Strategies: One Size Doesn’t Fit All, state that the term differentiation is a philosophy that allows instructors the ability to plan their classes in a strategic manner in order to meet the needs of each diverse learner in the classroom. Tomlinson states that teachers can differentiate instruction in four main areas: content, process, products, and learning environment. In order to test the effectiveness of differentiated instruction, the researcher gathered and analyzed data from a 2014 spring geometry class. This study attempted to draw comparisons between differentiated lessons versus traditional lecture based lessons.
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CHAPTER I
INTRODUCTION

Geometry has long been a secondary subject heavily based on paper and pencil learning. Teachers too often rely excessively on textbooks; as a result, many students are unable to form a deep mathematical connection to the material being taught. It becomes a mere memorization game in which many students do not excel. Educators now need to format their classroom agendas to adapt to their diverse classrooms in order for their students to understand the material at length.

The National Council of Teachers of Mathematics stresses that teachers should relate mathematical material to real world problems (NTCM, 2000). However, the outdated textbooks offered at many rural southeastern Mississippi school districts use geometric examples that involve Reel-to-Reel tape recorders, newspaper routes, and phone books; they use day-to-day technology that is now obsolete and foreign to the students. While it might not be true that up-to-date textbooks would provide better real world examples, it can be said that students are not inspired by reading about the real world problems presented in these outdated books.

It is from the researcher’s standpoint that students are influenced by hands-on experiences where they have the opportunity to see math in action. Unfortunately, most students go through their entire secondary careers without seeing math at work in the real world. That is why teachers repeatedly hear the question, “When will we ever use this in real life?” For these reasons, the researcher attempted to incorporate a collection of differentiated lesson plans where the students could see math at work. It can be asserted
that by using these differentiated lesson plans, the students were more attentive and able to absorb information more readily.

Differentiated Instruction

Differentiated instruction is a philosophy that aims at providing students alternative learning routes for obtaining content. Gregory and Chapman, authors of *Differentiated Instructional Strategies: One Size Doesn’t Fit All*, state that the term differentiation is a philosophy that allows instructors the ability to plan their classes in a strategic manner in order to meet the needs of each diverse learner in the classroom. They continue by stating that teachers can no longer teach a standard lesson in the hopes that every student is going to understand. Instead, they stress that teachers have to put consideration into each individual student’s needs, readiness, preferences, and interest (Gregory & Chapman 2007). Kellough says that students prefer different methods according to their personal interests. While some students prefer learning via a visual aid, others prefer talking in groups, listening to the instructor, or being physically involved with the lesson (Kellough 1999).

Teachers can differentiate instruction in four main areas: content, process, products, and learning environment (Tomlinson 2000). Content is described as what a student needs in order to successfully access the information clearly. Tomlinson continues by stating that in order to achieve differentiation, educators need to use classroom resources effectively as well as create various materials that can more adequately address the students’ needs. Ways in which this can be achieved include showing the students how they can examine a problem from different perspectives to more fully understand the problem,
utilizing different resources that can induce interest in the subject, and allowing students to view a wide range of topics in the classroom (Tomlinson 2000).

Process offers guided activities that can be used during the instruction. Such activities include whole-class introductory discussions and student-led big idea discussions. These help in keeping the students engaged during the lesson. Products are activities used to review acquired knowledge. For example, benchmark assessments, evaluation projects, and tests. Such activities help in assessing how much of the material the students have retained by reaffirming the material (Tomlinson 2000). Lastly, learning environments describe the way the classrooms look, feel, and operate. Tomlinson states that the best way to create a learning conducive environment is to allow time and space for hands-on activities and presentations as well as small group and whole class learning (Tomlinson 2000). Making use of these different teaching methods will allow for students of varying ranges of abilities to get the most out of the lesson and allow for their individual learning style to flourish.

In the researcher’s differentiated lessons, all four areas were differentiated. The lesson plans presented the content through auditory, kinesthetic, and visual means by explaining the content through hands-on activities and visual aids. The learning environment was differentiated by delivering lessons both inside and outside of the classroom, giving the overall class atmosphere a positive environment conducive to positive student-led learning. In order to differentiate the process, whole-class discussions were held students lead discussion during outside activities. Also, before each unit test, the researcher held review activities in order to differentiate the area of product.
The first differentiated lesson of the spring semester was integrating multiple outside-of-the-classroom lessons with the first unit. These lessons included angle measurements, parallel lines, as well as perpendicular lines. The students were separated into think-pair-share groups in order to facilitate group discussions, and they were provided with a protractor and blue painter’s tape. They were asked to construct various angles and line segments on the sidewalk with the tape. For example, they constructed vertical angles, parallel lines intersected by a transversal, and other line segments intersected by transversals on the sidewalk with the tape. The goal with this unit was for students to discover for themselves the geometric relationships between the existing angles, lines, and measurements.

Conversely, for the traditional based lecture, the researcher began the class with a warm-up based on the previous day’s lesson on angles and line segments. After the students completed this exercise, they were then provided with vocabulary that would be used throughout the day’s lesson as well as a few properties and some brief examples. The student were then given five problems to complete on their own that were similar to the examples completed for them on the board. After they completed these problems, the researcher continued the lecture by completing more complex problems for the students. The researcher attempted to relate these problems to the problems and vocabulary that were introduced at the beginning of the lecture. Lastly, the students were provided with an in-class assignment to complete that addressed all of the lecture points.

Later in Unit II, another differentiated lesson which concentrated on triangle congruence was introduced. The students performed more hands on activities in order to discover the geometric relationships and properties of triangles. For example, in section
two of Unit II, the students were divided into groups, provided with various sizes of paper, and instructed to create any type of triangle that they desired. The students were then instructed to label each vertex, tear the labeled vertices off of their triangles, and align the vertices on the floor vertex to vertex. Aligning the vertices on the floor was an integral part of this activity as it allowed the vertices to line up against the straight lines on the floor. This activity allowed the students to witness that in spite of the type and size of a triangle, all interior angles added up to equal 180 degrees.

The following day, the researcher performed another traditional lesson with the class. The class began with a quick review of the previous day’s highpoints. The students were asked questions to evaluate how much information they had retained from the previous day. Afterwards, the researcher began introducing the new vocabulary and properties and followed up with some basic problems. The students then completed a few problems on their own like with the previous traditional lesson. After completing the problems, the researcher continued adding on to the material. After completing a couple of the more complex problems, the students also completed a few problems that were similar to the completed ones on the board. To complete the lesson, the researcher gave a comprehensive overview of all lecture points and assigned the students a few problems to turn in the following day.

As the class approached the middle of the semester, the researcher began collecting data on Unit III: Properties of Polygons and Quadrilaterals. For the differentiated portion of this unit, the researcher divided the students into small groups of two or three. Students who were struggling in the course were paired with upper classmen who seemed to have a better understanding of the material. This was done to ensure that none of the
students felt lost during the activity. The purpose of this activity was for the students to discover by themselves the polygon angle sum theorem, which is used to obtain the sum of interior angles of different convex polygons. The researcher gave the groups multiple convex polygons with the interior angle measure already stated. The students then had to construct a formula that would calculate all interior sum measures regardless of the size of the polygon. It took approximately 30 minutes for all groups to obtain the correct formula: \((n-2)\times180\) where \(n\) represents the number of edges. Following this, the researcher provided the students with other convex polygons and the sum of the exterior angle measurements. The researcher then asked the students to find the polygon exterior angle sum theorem. The students quickly realized that the angle measurement was 360 degrees for all convex polygons.

The following day, the researcher continued instruction on Unit III but with a traditional approach. The majority of the class period was spent going over properties of convex polygons, parallelograms, and trapezoids. The researcher wrote properties, theorems, and vocabulary words on the board and asked the students to work problems using their newly acquired knowledge of quadrilaterals. The researcher concluded the lesson by assigning the students their take home assignments. The next day was a continuation of Unit III, followed by two more days of application and review. Unit III concluded with a comprehensive unit test that was comprised of both traditional and differentiated questions.

Problem Statement

It is from the researcher’s viewpoint that students who can analytically apply what they know are comfortable making assumptions and approximations to simplify compli-
cated scenarios that often arise in real life. The researcher’s concern is that students are not building on their analytical and mathematical foundations needed to be successful academically and in real life. Therefore, the researcher gathered and analyzed data in order to determine if the differentiated lesson plans resulted in student analytical growth and understanding.

van Hiele

Van De Walle states that geometry is essential within human life, and it can be seen in both science and art even from the earliest of times (Van De Walle, 2001). Geometry has the capabilities within a mathematics curriculum to allow students to develop problem solving skills, learn to create comparison, and effectively make generalizations and summarizations. It allows for students to develop deduction and reasoning skills while contained within a natural environment by examining different shapes and forming relations between them (Napitupulu, 2001). The renewed standards of the National Council of Teachers of Mathematics in America has placed more importance on geometry being incorporated in primary school mathematics curricula (Lehrer & Chazan 1998). The NCTM points out that geometry at this level plays a proactive role in the students’ mathematical thinking ability and interactions with math (NCTM 2000).

In 1957 Pierre van Hiele and his wife Dina van Hiele-Geldof examined how students learn geometry. They noticed that students have difficulty with the higher order cognitive processes required for success in geometry and that students tended to level out at certain points in their understanding of geometry. The van Hieles identified these benchmarks as levels.
Each level indicates how individuals think over the concepts in geometry. In order to be at a level, the previous levels must be passed. Therefore, the levels are hierarchical. The transitions from one level to the other are dependent upon several factors. Some of these factors include the subject that is being taught, the quality of education as well as the experiences of the teachers and students. The different levels of the Van Hiele geometric thinking are as follows (Crowley, 1987; Usiskin, 1982; van Hiele, 1959; Van de Walle, 2001):

- **Level 1 Visualization**: At this level, the focus of a child’s thinking is on individual shapes, which the child is learning to classify by judging their holistic appearance.

- **Level 2 Analysis**: At this level, the shapes become bearers of their properties. The objects of thought are classes of shapes, which the child has learned to analyze as having properties.

- **Level 3 Abstraction**: At this level, properties are ordered. The objects of thought are geometric properties, which the student has learned to connect deductively.

- **Level 4 Deduction**: Students at this level understand the meaning of deduction. The object of thought is deductive reasoning (simple proofs), which the student learns to combine to form a system of formal proofs.

- **Level 5 Rigor**: At this level, geometry is understood at the level of a mathematician. Students understand that definitions are arbitrary and need not actually refer to any concrete realization.
Research Questions

The researcher investigated two different questions in the secondary geometry class:

1. Was there an increase in Van Hiele Levels over the course of the semester?

2. Did the students’ scores differ from differentiated lessons versus traditional lecture based lessons?

Null Hypotheses

1. There was not an increase in Van Hiele Levels over the course of the semester.

2. There was not a significant difference between scores collected from the differentiated lesson and the traditional lecture based lessons.

In the differentiated lessons, the researcher’s goal was to provide students with the skills needed to become comfortable in making conjectures about given geometric relationships and plan a solution pathway rather than asking for formulas and precise ways to complete given tasks. By using this approach, the researcher anticipated that the students would develop the mathematical discipline needed to engage with the subject matter as they continuously built upon their mathematical foundation.

Review of Terms

- *The National Council of Teachers of Mathematics*: The public voice of mathematics education, supporting teachers to ensure equitable mathematics learning of the highest quality for all students through vision, leader-
ship, professional development, and research.

(http://www.nctm.org/about/default.aspx?id=166)

- **Differentiated Lesson Plans:** A way to reach students with different learning styles, different abilities to absorb information and different ways of expressing what they have learned.

(http://www.scholastic.com/teachers/article/what-differentiated-instruction)

- **Block Schedule:** Type of secondary scheduling in which each student has fewer, but longer classes each day and classes last for one semester as opposed to a full year.

- **Common Core State Standards:** A state-led effort that established a single set of clear educational standards for kindergarten through 12th grade in English language arts and mathematics that states voluntarily adopt.

(http://www.corestandards.org/)

- **Think-Pair-Share:** A strategy designed to differentiate instruction by providing students time and structure for thinking on a given topic, enabling them to formulate individual ideas and share these ideas with a peer.

(http://www.readwritethink.org/professional-development/strategy-guides/using-think-pair-share-30626.html)

**Limitations**

For this study, only one geometry class was available. Also, geometry must be taken after Algebra I but before Algebra II. Therefore, the student population was not randomly selected. The researcher will be the only instructor for this course; thus, the
study may be at risk for biased judgments and assessments. Also, a majority of the re-
search was studying correlations. Therefore, one limitation may be that the researcher
cannot make causal conclusions from the findings because one cannot rule out all other
explanations for the discoveries made. For example, the researcher may or may not be
able to claim that the differentiated approach to geometry was the sole factor for the stu-
dents’ Van Hiele significance. Also, one cannot control how serious a student takes the
exam. Occasionally, some students will answer test questions at random. By doing so,
this could have an impact on the research findings. Additionally, research is often mold-
ed to fit the needs of a selected population. Therefore, it is often difficult to make a claim
about a population from the findings of a qualitative study. For example, the researcher
conducted a case study which identified students’ geometrical growth in a southeastern
Mississippi school district. Although certain growths and significances were found with-
in the population, it is impossible to derive wider conclusions from a single case study
that all Mississippi geometry students will show growth.

Longitudinal effects would also be a limitation to this study due to the time allot-
ted to complete the research. Many researchers have years to study a single problem in
order to conduct ongoing research, but the researcher in this study had only 5 months.
The time available to investigate the research questions and to measure the sample was
constrained by the due dates.

The measures the researcher used to collect student data could be a limitation as
well. Given that this school district had limited resources, the researcher sometimes did
not have enough material to implement into the lesson. For example, there were not
enough textbooks provided for students to take home and practice the in-class material.
CHAPTER II
RELEVANT LITERATURE AND STUDIES

Geometry is a stringent secondary subject that is often presented differently than other mathematics classes. The students are introduced to abstract ideas (postulates, theorems, definitions, and proofs) and asked to think and learn in an unfamiliar way. It is from the researcher’s point-of-view that this system can often lead to student-teacher miscommunications as well as confusion.

Van Hiele Theory

In 1957, Dina van Hiele-Geldof and Pierre Marie van Hiele, two Dutch mathematics educators, recognized this complication and constructed an approach to explain why many students have difficulty learning geometry. Their method was titled the Van Hiele Level Theory (Van Hiele, 1959). This theory has been applied to explain why many students have difficulty with the higher order cognitive processes required for success in secondary geometry. To begin with, according to this theory, there are five levels of understanding that must be consecutively completed for maximum achievement (Crowley, 1987; Usiskin, 1982; van Hiele, 1959; Van de Walle, 2001).

- **Level 1: Visualization/Recognition:** At this level, an individual is capable of distinguishing the different features of shapes and classifying them according to appearance. Squares and triangles are different from each other. “A square is a square for the individual and he or she is unable to comprehend neither the definition nor the features attributed to a square. Depending on the definition, the individual can just say the name according to the appearance. For example, he or she is not capable of noticing that the rectangle or square is spe-
cial. The suitable activities that can be done with an individual at this level include letting them play with items that contain geometric shapes, letting them tell their observations and experiences about these items, and providing opportunities for the individuals to draw these items.

- **Level 2: Analysis**: An individual at this level is capable of explaining the features of each shape in a class, but the individual cannot establish the relationship between these shapes. The individual at this level are able to derive some generalizations about the shapes. For example, the individual can say that all the edges of a square are equal and perpendicular to each other or that the opposite sides of a parallelogram are equal and parallel to each other. They can classify the shapes according to their characteristics such as an angle’s edges. Appropriate activities for individuals at this level include measuring objects, identifying and transforming a shape, and classifying an object.

- **Level 3: Informal Deduction /Order**: Individuals at this level are able to sort the shapes and relationships logically but may not be able to understand the shape’s mathematical properties. They can make simple, informal inferences but are not capable of understanding the proofs involved. They can distinguish other relations from the relations they know using informal expressions. For example, when one says that the perpendicular edge going down from the top point of a triangle is both the angle bisector and median, a student at this level can notice that this triangle is an isosceles triangle or an equilateral triangle.
• **Level 4: Deduction:** This level corresponds to a high school course. Individuals at this level can compare and discuss the features of shapes. Additionally, the individual can explain the relationships between axioms and theorems, postulates and definitions, and can comprehend the processes of reasoning by induction.

• **Level 5: Rigor:** Individuals at this level can understand various axiomatic systems and comprehend the relationships between them. They can understand the non-Euclidean geometries that are not included in a standard geometry course.

Initially, these levels were placed on a scale of 0-4 (Carroll, 1998; Usiskin, 1992; Van de Walle, 2001; van Hiele, 1959). They were later placed on a scale of 1-5 in order to make use of level “0” for those individuals who could not be assigned to the first level (Bulut & Bulut, 2012; Senk, 1989). Level “0” is said to be the level in which the individual can only distinguish between cornered and uncornered geometric shapes (Clements & Battista, 1990).

Without having first built a strong foundation of geometric relationships and ideas, students cannot be expected to construct and prove geometric theorems and definitions. This foundation cannot be learned by memorization or repetition, but must be refined through experiencing various examples, properties, and property order. Educators call this the fixed sequence property of the levels. The five levels were postulated by the van Hieles, and they describe how students advance through this understanding.

• **Property 1:** (fixed sequence) a student cannot be at Van Hiele level n without having first completed level (n-1).
- **Property 2**: (adjacency) at each level of thought, what was intrinsic in the preceding level becomes extrinsic in the current level.

- **Property 3**: (distinction) each level has its own linguistic symbol and its own network of relationships connecting those symbols.

- **Property 4**: (separation) two people who reason at different levels cannot understand each other.

- **Property 5**: (attainment) the learning process leading to complete understanding at the next level has five phases: inquiry, directed orientation, explanation, free orientation, and integration.

1. **Inquiry** - students become acquainted with the material and begin to discover its structure.

2. **Orientation** - students do tasks that enable them to explore implicit relationships.

3. **Explanation** - students express what they have discovered and vocabulary is introduced.

4. **Free Orientation** - students do more complex tasks enabling them to master the network of relationships in the material.

5. **Integration** - students summarize what they have learned and commit it to memory.
In 1979, Zalman Usiskin, an educator at the University of Chicago, developed the CDASSG project (The Cognitive Development and Achievement in Secondary School Geometry) in an attempt to establish the validity of van Hiele’s claims. The CDASSG was a study that tested approximately 2,500 geometry students and aimed to address a collection of questions relating to the Van Hiele Theory and achievement. The overall design was given in the form of a standard pre-test and post test design. Four tests were given to all students, and one of three forms of a proof test given to some students in accordance with the following schedule:

First week of school:  
- Entering Geometry Test (EG)  
- Van Hiele Level Test (VHF)

Three to five weeks before end of school:  
- Van Hiele Level Test (VHF)  
- Comprehensive Assessment Program Geometry Test  
- Proof Test (PrF)

Over the course of three years, Usiskin addressed the following questions:

- How are entering geometry students distributed with respect to the levels in the Van Hiele scheme?
- What changes in Van Hiele levels take place after a year’s study of geometry?
- To what extent are Van Hiele levels related to concurrent geometry achievement?
- To what extent do Van Hiele levels predict geometry achievement after a year’s study?
• What generalizations can be made concerning the entering Van Hiele level and geometry knowledge of students who are later found to be unsuccessful in their study of geometry?

• To what extent is the geometry being taught to students appropriate to their Van Hiele levels?

• To what extent do geometry classes in different schools and socio-economic settings differ in the appropriateness of the content to the Van Hiele level of the student?

Usiskin found that students’ Van Hiele levels are an adequate classification of the student’s current foundation in geometry and excellent predictors of later achievements. The weaker performances of many students are strongly associated with being at a lower Van Hiele level. Thus, Usiskin’s study confirms the use of the Van Hiele level theory to explain why many students have trouble learning and performing in geometry classes.

Furthermore, the geometry course was not working for large numbers of students. At the end of the courses, many students did not possess even trivial information regarding geometry terminology and measurement. Half of the students who enrolled in proof oriented courses experienced very little to no success with proofs. The major cause appeared to be lack of knowledge at the beginning of the year. This shows a need for systematic geometry instruction before high school. In order for the students to obtain greater geometry knowledge and proof writing success, students need to be educated sufficiently throughout their educational careers.
Not every student learns geometry instruction the same. Inside today’s classroom, one will find a diverse collection of students with different learning styles, background knowledge, and cognitive abilities. Students with high cognitive skills may sit next to those who struggle to maintain the material of a single subject. It is the job of an educator to accept the students as they are and guide them through a successful academic journey. This process is called Differentiated Instruction.

Ward

In 1961, Dr. Virgil Ward first coined the term differential education. Virgil Ward is considered one of the pioneers of differentiated instruction in that he realized that standard teaching methods were insufficient. He first brought forth the idea of differentiated teaching to further the current teaching curriculum of the time for the gifted and talented students. He believed the curriculum to be inadequate for producing the best results from these select students. Ward’s idea was to base the curriculum on who the students were as individuals and how they best learned material (Bravmann, 2004).

In Ward’s “Lifetime Education-Propositions toward General Theory of Education,” he stated that the curriculum of the time focused only on factual information and, thus, bore dependent learners incapable of furthering knowledge on their own. However, he believed that by enhancing the curriculum, independent learners could be produced. Ward believed the way to unlock these gifted students’ potential was to instruct them in a manner that catered to each individual student’s talents and interests (Ward, 1967). There are still several schools that seem to cater to traditional teaching methods and are devoid of relevant information which could be seen as beneficial to the students’ skill development.
Ward was one of the first to recognize the need for change within schools and act upon that realization. Ward’s article titled *Systematic Intensification and Extensification of the School Curriculum* states that the “gifted” curriculum at the time focused only on one subject. He proposed the programs should be expanded to encompass all subjects and have students relate different subjects to each other (Tomlinson, 2004). This can be seen as an excellent method for showing students how all of their knowledge correlates together. Instructors far too often become pigeonholed in one area and forget to demonstrate the broader picture of what the students are learning and the reason behind why they are learning. Ward states that the ideal state of the classroom would be to create a comprehensive and balanced sequence of experiences that adhere to the intellectual as well as the behavioral potential of the students.

Ward did receive some criticism for his theories because of the fact that his work focused only on those students deemed to be gifted. What he deemed as gifted could also be seen as a rather narrow scale as he exclusively used test scores to assess intellectual ability. Nevertheless, his research and teaching methods became the foundation for other researchers to expand upon and discover more methods on how to best tap into each student’s potential (Tomlinson & Reis, 2004).

Tomlinson

In 2000, Dr. Carol Ann Tomlinson explained the differentiated teaching philosophy to be a form of teaching that provides all students with different pathways to effectively learning material. In an interview with Echo Wu in 2013, Tomlinson says she decided to adopt differentiating teaching methods during her third year as an educator when she noticed a large gap between her students’ knowledge levels. She had some who
could barely read and others that already knew the material she taught. She turned to her other colleagues to begin researching how to transform standard teaching methods in order to allow all of her students to learn from the curriculum (Wu, 2013).

Tomlinson states that the purpose of differentiation is to accommodate each student’s needs in a diverse classroom. She continues by asserting that the idea behind differentiation is that each student is an important member of the classroom. In an article written in 2000, Tomlinson says that practical application of the material is how students learn best because it makes the learning process feel more natural and important (Tomlinson, 2000). Real-world application is imperative to the learning process. When students feel that what they are learning can be applied pragmatically, it gives them a greater drive to learn the material by giving the material a feeling of importance.

Tomlinson also writes about her views and concerns regarding formative assessment in her article titled, Between Today and Tomorrow’s Lesson. She writes how it is great that more people in the education field are taking notice of formative assessment and its benefits. Formative assessment is defined as the continuous exchange between educators and students and is meant to help teachers contribute to the growth of their students in a positive way. However, Tomlinson fears that many teachers may be using formative assessment in the wrong way. She states that many are using formative assessment merely as way to raise test scores instead of providing students with long-term learning goals. (Tomlinson 2014) Tomlinson makes a valid point by stating that educators are overly concerned with raising test scores instead of focusing on whether or not the students are learning from the class. Having a focus on the students’ long term growth is essential to having a successful classroom.
There are several differentiation strategies Tomlinson suggests that teachers can incorporate into their classrooms. One of the most important strategies is a teacher working with small groups within the classroom. Tomlinson argues that within a normal classroom, teachers are unable to know whether each student has fully grasped the material or not. However, within small groups the teacher is able to recognize almost immediately what the student understands or does not understand by asking each individual student questions pertaining to the material being discussed (Wu, 2013).

A few other significant strategies Tomlinson suggests are learning stations and learning contracts. By using learning stations, the students go to each station to work on a certain set of skills they may need help with. Instructions are provided at each station that state how the student is to finish the work correctly as well as how they can receive more help and where to place the finished assignment. In this way, each student can work on the skills he or she needs more help with while still using classroom time effectively. Learning contracts allow the teacher to design tasks for certain students that are either readiness-based or student interest-based. This allows for more flexibility for the teacher and allows the student to learn more efficiently. (Wu, 2013)

Tomlinson suggests that differentiated instruction takes time to incorporate fully, and teachers should approach these methods slowly (Wu, 2013). By the educators fully understanding the methods and knowing the end goal they want to achieve in their classroom, the classroom will naturally become a more learning-conducive environment. The world of education has widely adopted Dr. Carol Ann Tomlinson’s version of differentiated instruction. If educators aspire to grow academically as a whole, they must meet the needs of all learners in the classroom.
CHAPTER III

METHODS

Participants

This study was conducted at a rural high school located in southeastern Mississippi. This experiment was executed on a block schedule with classes that change each semester. The geometry students of the spring semester were not the same students the researcher taught in the fall semester. Therefore, the spring semester began with a Van Hiele pre-test. The Van Hiele test allowed the researcher to verify the students’ current learning levels in geometry.

There are a total of 322 students enrolled in grades 9-12 during the 2013-2014 school year. The gender makeup of the school is approximately 48% female and 52% male. The racial makeup of the school is about 33% African American, 66% White, and 1% Other. Sixty-five percent of the school’s students are eligible to receive free lunch. The students’ average MCT2 score, which is a state test that all 8th grade students are required to take, was 148. Students are scored as minimal, basic, proficient, or advanced. A score of 148 falls into the basic classification. Their Algebra I state test score was 648, which also falls under the basic category. Taking into account the students’ state test scores as well as in-class observations, the students’ math skills could be said to be slightly below average.

The population for this study consisted of all students enrolled in the spring semester geometry course. Given that this high school operates on a block schedule with classes that change each semester, the students of the spring semester were new to the researcher’s class. In general, the population of this course is taken by 45% tenth grad-
ers, 30% 11th graders, and 25% seniors. Of these students, 45% of the population was male, and 55% was female. Since there is only one geometry course offered at this high school, the sample and population were the same.

Research Locale

The location for this study was not chosen at random. It was chosen because it is the researcher’s current place of employment. Also, the geometry students are not randomly selected. The students are pre-assigned to the researcher’s roster by the administration. The researcher had no control over the class selection process, which is a limitation to this study. A suggestion for future studies would be to have a second geometry class and instructor to use as a control group.

Procedures

To address research question one and see whether or not the Van Hiele levels showed an increase, the students were administered a standard pre-test/post test. The schedule was as follows:

- First week of school: Van Hiele Level Test (pre-test)
- 14 weeks of integrated lectures: 14 traditional lectures and 10 differentiated lessons throughout the 14 weeks.
- One to three weeks before the end of the semester: Van Hiele Level Test (post test).

The Van Hiele test is a 25-question multiple-choice assessment that was created by Zalman Usiskin in 1982. It measures each student’s Van Hiele Level. The test items were written to correspond directly to the van Hiele characteristics for each level to determine what the student should be able to comprehend and perform (Usiskin, 1982). The stu-
dents’ pre-test Van Hiele scores as well as their post test Van Hiele scores were input into SPSS. The researcher then ran a related-samples Wilcoxon signed rank test, which is a non-parametric statistical hypothesis test used when comparing two related samples, to determine if there is a difference in the population mean rank.

Next, in order to determine if there was a difference in the students’ scores with the differentiated lessons versus the traditional lesson, the researcher collected student data (homework, tests, and warm-up/exit-ticket assignments) from both the traditional and differentiated lessons. Both sets of data were then compared to each other. The researcher was able to determine from these data sets whether or not the differentiated lessons were effective. The researcher analyzed this data by first collecting students’ assigned homework from a differentiated lesson. The researcher then collected homework from a preceding traditional lecture based lesson. Both homework grades were then input into SPSS. Once the grades had been input into the program, the researcher performed a related-samples Friedman’s two-way analysis of variance by ranks test. This test is used to detect differences in treatments across multiple test attempts and ranks each row while taking the values of each rank into account. The researcher used this test to determine if there was a growth correlation that would indicate that one teaching strategy was more effective over the other.
Table 1

*Research Methods*

<table>
<thead>
<tr>
<th>Research Questions</th>
<th>How the data will be analyzed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Will there be an increase in Van Hiele Levels over</td>
<td>a) Van Hiele post test (graded by scantron)</td>
</tr>
<tr>
<td>the course of the semester?</td>
<td>b) Van Hiele pre-test (graded by scantron)</td>
</tr>
<tr>
<td></td>
<td>c) Use a related-samples Wilcoxon signed rank test in SPSS to determine if there exists a correlation.</td>
</tr>
<tr>
<td>2. Will the students’ scores differ from</td>
<td>a) Collect student homework data over the course of 14 weeks.</td>
</tr>
<tr>
<td>differentiated lessons versus traditional lecture based</td>
<td>b) Use a related-samples Friedman’s two-analysis of variance by rank test to determine if there exists a correlation.</td>
</tr>
</tbody>
</table>
CHAPTER IV
DATA ANALYSIS

Van Hiele Data Analysis

After compiling all the data and inputting it into the SPSS program, the results from the related-samples Wilcoxon signed rank test were used to address the research question concerning changes in students’ Van Hiele scores. The null hypothesis states that there will not be an increase in Van Hiele levels. There were 21 students who took the Van Hiele pre-test. The mean of those scores was 1.38. Twenty-three students took the post test, and produced a mean of 1.91. There was one student not present during the pre-test and one student added to the class in the time between the pre-test and post test, which resulted in a different number of students taking each test. From this data, the researcher received a significance level of .04, which is less than a .05 significance level; therefore, the researcher rejected the null hypothesis.

Table 2

<table>
<thead>
<tr>
<th>Van Hiele Significance</th>
<th>N</th>
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<td>Van Hiele Post</td>
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<td>Valid N</td>
<td>13</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Unit 1 Data Analysis

Next, the researcher addressed the second research question, which questioned whether there would be a difference in the students’ test scores from a differentiated lesson as opposed to a traditional lecture based lesson. The null hypothesis for this section states that there will not be a difference in students’ scores with differentiated lessons versus traditional lessons. The researcher did this by analyzing the students’ data from three different units of instruction. The first unit was on angle measurements during week 2 of school.

For this first unit, the researcher collected data from a differentiated warm-up, traditional warm-up, differentiated homework I, traditional homework I, and the cumulative unit test. The first data set used for Unit I was the warm-up following the outside differentiated lesson plan. The warm-up included both differentiated as well as traditional lecture format questions, and 21 students took the warm-up. The lowest score received was 0%, while the highest was 100%. The overall average for the warm-up was 68.3%. The average score on the differentiated questions for the warm-up was 66.7%, and the average for the traditional questions was 76%. This data was used to run a related-samples Friedman’s two-analysis of variance by rank test. The researcher received a significance level of .103, which is greater than .05; therefore, the researcher retained the null hypothesis. There was no significance in the data collected.

Before leaving for the day, the researcher assigned the students a series of homework problems which was closely aligned with differentiated lesson I. The data collected from the homework problems served as the second data set. Like the warm-up, the homework involved both differentiated and traditional lesson format questions. Eighteen
students turned in the homework. The highest grade received was 100%, and the lowest was 80%. The overall average of the homework was 90.67%. On the differentiated questions, the average was 89%, while the average of the traditional questions was 91.89%. This data set received a significance level of .813, revealing that this data was not significant. Thus, the researcher retained the null hypothesis.

The following day, the researcher taught a traditional lesson plan, which was a continuation of the previous day’s angle segments lesson. The class began with another warm-up, much like the previous day. The data collected from this warm-up was used as the 3rd data set of Unit I. Nineteen students did the warm-up. The highest score was 100%, and the lowest was 0%. The overall mean was 60.5%. The mean for the differentiated questions was 63%, and the mean for the traditional questions was 58%. This data set received a significance level of .895. Therefore, this data was not significant, and the researcher retained the null hypothesis.

At the end of the traditional-based lesson, the researcher issued another homework that addressed the main points of the day’s lecture and used that data as the fourth data set of Unit I. Nineteen students turned in the homework. The highest grade was 100%, and the lowest was 68%. The overall mean for this homework was 82.11%. The differentiated questions’ mean was 86.16%, and the traditional questions’ mean was 71.95%. After inputting the data into the SPSS program, the data received a significance level of .257. The data showed no significance; thus, the researcher retained the null hypothesis.

After one week of concept introductions and vocabulary, differentiated lessons, traditional lessons, and review days, the researcher then issued a unit test. The unit test
was a collection of both differentiated questions and traditional questions from Unit I, and this data served as the fifth data set for Unit I. Twenty-three students took the unit test. The highest score from the test was 92%, while the lowest score was 8%. The overall mean produced from these scores was 56.43%. The mean for the differentiated questions was 61.78%, and the mean for the traditional questions were 49.1%. This data set received a significance level of .119, showing no significance to the data, and thus, resulting in retaining the null hypothesis.

Table 3

*Unit 1 Data Analysis*

<table>
<thead>
<tr>
<th></th>
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Table 4

*Unit I Data Significance*

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<tr>
<td>Unit Test 1</td>
<td>.119</td>
<td>Retain the null hypothesis</td>
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Unit II Data Analysis

For the second unit of instruction, the researcher conducted a lesson on triangle congruence and collected data from one warm-up, one homework assignment, and a cumulative unit test. As in the previous unit, the warm-up, homework, and the unit test in-
cluded both differentiated and traditional questions. This unit was slightly different from the first one in that the researcher conducted a differentiated lesson, a traditional lesson, and then assigned a warm-up and homework on the following day. The first data set of Unit II was compiled from the warm-up. Twenty-two students took the warm-up. The lowest score was 33%, and the highest score was 100%. The overall mean from this warm-up was 65.14%. The mean for the differentiated questions was 71.23%, while the traditional questions’ mean was 59.05%. After running Friedman’s two-way test, this data set received a significance level of .135, showing the data was not significant. Thus, the researcher retained the null hypothesis.

The second data set was comprised of data collected from a homework assignment that followed a review of both the differentiated and traditional lesson. Twenty-two students turned in the homework. The highest score received was 100%, and the lowest was 33%. The overall mean was 68.05% with a differentiated mean of 76.64% and a traditional mean of 59.86%. The significance level received from this data set was .002, showing that this data was significant; thus, the researcher rejected the null hypothesis.

Table 5

**Unit II Data Analysis**

<table>
<thead>
<tr>
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<td>.2</td>
<td>.83</td>
<td>.6264</td>
<td>.19180</td>
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Table 6

Unit II Data Significance

<table>
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<td>Retain the null hypothesis</td>
</tr>
<tr>
<td>Diff/Trad H.W.</td>
<td>.002</td>
<td>Reject the null hypothesis</td>
</tr>
<tr>
<td>Unit Test 2</td>
<td>.304</td>
<td>Retain the null hypothesis</td>
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</tbody>
</table>

Unit III Data Analysis

Lastly, the third data set from this unit used the scores from the Unit II cumulative test, which was completed following the 5th day of instruction. The researcher conducted
another traditional and differentiated lesson on day 4 and day 5; however, the researcher did not assign any warm-ups or homework to be completed. Twenty-two students took Unit Test II. The highest score was 86%, and the lowest score was 43%. A mean of 67.27% was produced from these scores. The differentiated question mean was 71.27%, and the traditional question mean was 62.64%. This data set received a significance level of .304. The data showed no significance; thus, the researcher retained the null hypothesis.

The third unit from which the researcher collected data was on polygons and quadrilaterals. The researcher collected data from two warm-ups, a homework assignment, and a comprehensive unit test. This unit began with a differentiated lesson as well. Preceding the actual lesson, the researcher gave the students a warm-up, which was used as the first data set of Unit III. Nineteen students did this warm-up, resulting in a highest score of 100% and a lowest score of 50%. The overall mean was 73.7%, and the differentiated mean and traditional mean were both 74%. The significance level received was 1.0, showing no significance. Therefore, the researcher retained the null hypothesis.

The next day, the researcher conducted a traditional lesson. Before the actual lesson, the students were given another warm-up. This was the second data set for this unit. Twenty-one students took this warm-up. The highest score was 100%, and the lowest was 50%. The overall mean of the scores was 76.2% with a differentiated mean of 81% and a traditional mean of 71%. The significance level of this data set was .670. The results from this data set show that the data was not significant. Thus, the researcher retained the null hypothesis.
Following the traditional lesson, the researcher assigned the students a homework assignment. This served as the third data set for Unit III. Twenty-three students turned in the homework. The highest score received was 100%, and the lowest score received was 52%. The overall mean for the homework was 82.61%. The differentiated mean was 78.35%, while the traditional mean was 85.22%. The significance level received was .002, showing this data was significant. Therefore, the researcher rejected the null hypothesis.

After two more days of instruction and a review day, the students were issued a unit test, which served as the fourth and final data set of Unit III. Twenty-three students took the test. The highest score received was 90%, while the lowest score received was 50%. The overall mean of the scores was 71.09%, the differentiated mean was 66.26%, and the traditional mean was 75.13%. The significance level received for this data set was .676. This data was not significant; thus, the researcher retained the null hypothesis.

Table 7

*Unit III Data Analysis*

<table>
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Table 8

*Unit III Data Significance*

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CHAPTER V

CONCLUSIONS

Overview

Differentiated instruction was first created in the late 1950s, but it has only been in recent years that it has received attention as a way to truly make a difference in the classroom. Coleman stated that keeping students actively engaged in the lesson helps them to see learning as a cumulative whole (Coleman, 2001). Differentiated instruction was created for the diverse learner and is meant to be used as a way of thinking about each individual student’s learning needs while at the same time maximizing each student’s learning potential (Tomlinson, 2000).

The use of a traditional lecture, a one-size-fits-all approach, does not meet the needs of the majority of learners in the classroom. Traditional lectures do not take into account different learning styles and interests of the students. The researcher’s belief is that by taking into account each individual student’s differences and interests, students will be more motivated to learn and enhance their overall learning experience in the classroom. Every student deserves to have an engaging learning experience and have the opportunity to reach his or her potential. Unfortunately, many educational curricula do not address the needs of the students. Therefore, the purpose of this paper is that the findings from the research prove to be substantial enough to provoke the interest of other educators, so that they may observe how beneficial incorporating differentiated instruction into current curricula can be.

One concern expressed by the researcher before writing this paper was that secondary students were not making a deep connection to the mathematical material they
were learning in the classroom. From the researcher’s observations, students are influenced by hands-on experiences, interactive course work, and relevant real world examples. However, many students in southeastern Mississippi seem to go through their entire secondary career without even seeing math at work in the real world or in the classroom. In order to address these concerns and present geometry more effectively in the classroom, the researcher investigated differentiated instruction. The modified lesson plans created for the sample classroom consisted of differentiated instruction and think-pair-share mathematical learning and engagement.

The class in which the researcher conducted the research on was a spring semester geometry class at a high school in southeastern Mississippi. The reasons behind using geometry for the research were mainly in order to incorporate the use of the Van Hiele test. Also, it can be said that geometry is a truly integral part of any school curriculum. Geometry is a stringent secondary subject that is often presented differently than other mathematics classes. The students are introduced to abstract ideas (postulates, theorems, definitions, and proofs) and asked to think and learn in an unfamiliar way. This system often leads to student-to-teacher miscommunications as well as confusion.

In geometry class, students learn characteristic features and the relations among them with geometric shapes and structures. The most important part of geometry is spatial visualization, which is thinking of two or three dimensions of a geometric shape in space and looking at various aspects (NTCM, 2000). Students need to be allowed to hypothesize and explore theorems and relations. The researcher decided that using a differentiated instructional approach would be the best way to allow for this and unlock each student’s potential.
Students who can analytically apply what they know are comfortable making assumptions and approximations to simplify complicated scenarios that often arise in real life. One concern addressed by this paper is that students are not building on the analytical and mathematical foundations needed to be successful academically and in real life. Students need to either have already reached or reach a deductive reasoning level on the Van Hiele scale in geometry. The data obtained from this group of students revealed the highest level that any of them had obtained was level 3. They had not yet reached a level in which they could fully understand the concepts presented in geometry, which resulted in many of the students struggling throughout the course.

After completing all research, the researcher organized and analyzed data to determine if there was any student analytical growth and understanding from using a differentiated approach.

The researcher drafted two research questions to use as guides for the research:

1. Will there be an increase in Van Hiele Levels over the course of the semester?

2. Will the students’ scores differ from differentiated lessons versus traditional lecture based lessons?

Null hypotheses:

1. There was not an increase in Van Hiele Levels over the course of the semester.

2. There was not a significant difference between scores collected from the differentiated lesson and the traditional lecture based lessons.
Many educators have attempted differentiated instruction; however, the researcher found Dr. Carol Tomlinson’s approach to be the most effective. Tomlinson’s approach shows that teachers can differentiate learning in four main areas: content, process, products, and learning environments (Tomlinson, 2014). The researcher attempted to differentiate all four areas Tomlinson suggests. The researcher wanted to take geometry and not only deliver it to the students through auditory, kinesthetic, and visual means, but also have each individual student play an important role in the geometry learning process.

In order to differentiate the learning environment, lessons were conducted both inside and outside of the classroom. For example, in the first differentiated lesson, the researcher integrated multiple classroom lessons outdoors. The researcher observed that conducting lessons both indoors and outdoors helped in diversifying the learning process and also helped the students retain information more readily. The researcher was also able to incorporate different types of activities by making use of both the classroom and the outdoors.

To address differentiation in terms of content, the researcher chose to use several problems that were very hands-on in nature. The units of instruction were on angle measurements, parallel lines, as well as perpendicular lines. Each student participated in the lesson outside by creating the angles on the sidewalk with blue painter’s tape. By doing so, each student was allowed to participate and voice their thought processes with each geometric assignment. The researcher observed that these types of activities kept the students very engaged with the lesson and allowed them to think about the material in a different way.
The researcher also tried to differentiate the process by holding whole-class discussions at the beginning of lessons as well as allowing for student-led discussions during the activities held outside. Before each lesson, the researcher engaged the whole class in discussion in order to prepare them for the lesson. The students were also separated into think-pair-share groups during outside activities to facilitate student-led discussions. Lastly, in order to differentiate product, the researcher held review activities before each unit test. Many of the students said they found the review activities before each test helped in reinforcing the material and showing them in which areas they needed to review more.

One of the methods used to gauge whether or not the new teaching strategy was effective was the Van Hiele test. The reasons for using this test were to find whether there was any growth from the students throughout the semester. This test was administered at the beginning of the semester and again at the end of the semester to establish if there would be any difference in the test scores. The Van Hiele test was first created in 1982 by Usiskin to apply the Van Hiele theory. The Van Hiele theory has been applied to explain why many students have difficulty with the higher order cognitive processes required for success in secondary geometry. The benchmarks they found for the students’ understanding of geometry were turned into hierarchical levels; each level must be passed before reaching the subsequent one.

The results obtained after running the Wilcoxon signed rank test with the data from the Van Hiele tests showed that the data was significant, and there was positive growth in the test scores. The mean of the pre-test was 1.38, while the mean of the post-test was 1.91. These results suggest that the differentiated instruction approach was ben-
eficial to the students’ scores. Although the research did show positive results, the researcher must address the possible limitations of the study. One possible reason for the positive growth in the scores could be due to the fact that the students finished a course of geometry and not just because of the difference in instruction.

The researcher also collected student data on the differentiated lessons and their traditional lesson counterparts in order to further determine whether or not the differentiated lessons were truly making a difference. The researcher collected data from class warm-ups, homework, as well as tests from three different units. With each data set, the questions were separated into differentiated lecture questions and traditional lecture questions. The researcher later input this data into SPSS to determine specific correlations.

Results and Discussion

The three units of instruction the researcher collected data on throughout the semester made up 12 different data sets from which to draw conclusions. These three units were chosen because they had the most potential for hands-on activities. The researcher believed that this data would reveal large significance in several data sets from the researcher’s differentiated instruction efforts. However, out of the 12 data sets, only two of them showed significance. In the majority of the other data sets, the data showed no difference between the differentiated lessons and the traditional lessons. Therefore, the data shows there is no definitive answer as to whether or not the differentiated lessons were more beneficial than the traditional lessons. In order to find the reasons behind why only two of the data sets showed significance, the researcher would have to delve into the lessons that were taught on those days in order to find any differences in instruction.
A majority of the research was on studying correlations. Therefore, one limitation may be that the researcher cannot make causal conclusions from this paper’s findings because one cannot rule out all other explanations for the researcher’s discoveries. For example, the researcher may or may not be able to claim that the differentiated approach to geometry was the sole factor for a student’s increase in Van Hiele levels. Also, the researcher cannot control how seriously a student takes the exam. Occasionally, there will be some students who will answer test questions at random. By not taking the tests in earnest, these students could have an impact on the researcher’s findings.

Conclusions and Recommendations

Research is often molded to fit the needs of a selected population. Therefore, it is often difficult to make a claim about a population from the findings of a qualitative study. For example, the researcher conducted a case study which identified students’ geometrical growth in a southeastern Mississippi school district. Although the researcher found certain growths and significances within the sample, it is impossible to derive a wider conclusion from a single case study that all Mississippi geometry students will show growth. In order to address this limitation, the research should be conducted on a larger scale in future studies. It is the researcher’s belief that an increase in the population studied would, in fact, show more positive growth as well as more significant data. Having a different instructor conducting lessons as well could result in less bias in results for future studies.

Longitudinal effects would also be a limitation to this study due to the time allotted to complete the research. Professors sometimes have years to study a single problem and conduct ongoing research, but the researcher only had five months. The time availa-
ble to investigate the research questions and to measure the sample was constrained by the due date of this thesis. A suggestion for the future would be to conduct this study on a seven period schedule. In doing so, the instructor will have access to the students for two semesters. Furthermore, the researcher suggests conducting this research free from ongoing deadlines in order to gather a substantial amount of data from which to draw conclusions.

The measures in which the researcher used to collect student data could be a limitation as well. Given that the school district had limited resources, the researcher sometimes did not have enough material to implement into the lesson. For example, there were not enough textbooks provided for students to take home and practice the in-class material. Suggestions for future studies would be to have access to more in-class resources.

To conclude, it can be said that Tomlinson’s method of differentiated instruction, which was incorporated into this classroom, is very beneficial. However, in this particular study, the researcher was unable to find much positive growth with this method. The results concluded with finding significance in only 2 data sets. Initially, the researcher believed that the research would demonstrate a vast difference between the traditional based and the differentiated based instruction. Although the research did not provide conclusive evidence of this, further examination of the researcher’s exact teaching methods could bring about further findings. The researcher could take these results to further the research in differentiated instruction methods and find better ways to incorporate differentiated instruction methods into the classroom.
NOTICE OF COMMITTEE ACTION

The project has been reviewed by The University of Southern Mississippi Institutional Review Board in accordance with Federal Drug Administration regulations (21 CFR 26, 111), Department of Health and Human Services (45 CFR Part 46), and university guidelines to ensure adherence to the following criteria:

- The risks to subjects are minimized.
- The risks to subjects are reasonable in relation to the anticipated benefits.
- The selection of subjects is equitable.
- Informed consent is adequate and appropriately documented.
- Where appropriate, the research plan makes adequate provisions for monitoring the data collected to ensure the safety of the subjects.
- Where appropriate, there are adequate provisions to protect the privacy of subjects and to maintain the confidentiality of all data.
- Appropriate additional safeguards have been included to protect vulnerable subjects.
- Any unanticipated, serious, or continuing problems encountered regarding risks to subjects must be reported immediately, but not later than 10 days following the event. This should be reported to the IRB Office via the “Adverse Effect Report Form”.
- If approved, the maximum period of approval is limited to twelve months. Projects that exceed this period must submit an application for renewal or continuation.

PROTOCOL NUMBER: 14072107
PROJECT TITLE: Examination of High School Student's Understanding of Geometry
PROJECT TYPE: New Project
RESEARCHER(S): Brantley Pierce
COLLEGE/DIVISION: College of Science and Technology
DEPARTMENT: Mathematics
FUNDING AGENCY/SPONSOR: N/A
IRB COMMITTEE ACTION: Exempt Review Approval
PERIOD OF APPROVAL: 07/25/2014 to 07/24/2015

Lawrence A. Hosman, Ph.D.
Institutional Review Board
APPENDIX B

PERMISSION LETTER

THE UNIVERSITY OF SOUTHERN MISSISSIPPI
AUTHORIZATION TO PARTICIPATE IN RESEARCH PROJECT
Consent is hereby given to participate in the study titled:
Examination of High School Students' Understanding of Geometry

Mr. Pierce will be conducting a research project in his high school geometry class to measure the students’ understanding of geometry concepts. The data collected in this project will be used to write his master’s thesis as part of the requirement for completion of his master’s degree in mathematics.

Data for the research project will be collected from two different sources. Students will complete a mathematical survey that will collect information about study habits, technology access and use, beliefs about mathematics, and mathematics background. Students will also complete a test that will determine their level of understanding of geometry concepts. The test will NOT count toward the grade for the course. This test will be given at the beginning of the course and again at the end of the course. Other than the approximately 30 minute time period that it will take for the students to take the test at the beginning and at the end of the course, the research project will have no other impact on the course. The instructor, Mr. Pierce, will cover the required content for the course in a manner consistent with state and national recommendations. There will be no research activities that will in any way affect students’ achievement in the course. Therefore, there are no risks or benefits for students to participate in the research project.

The data collected by the survey and the test will be kept strictly confidential and will not impact students’ grade in the course. Mr. Pierce and his thesis advisor, Dr. Susan Ross, will be the only ones that will have access to the data from the survey and the test. All of the data will be combined and reported as a class. No individual data will be used and no individual student will be identified.

If a student chooses not to participate in the study, his or her information will be removed from the class data pool. Since the test will be given to the entire class, all students will complete the test. However, individual test results can be removed. Participation in this project is completely voluntary, and students may withdraw from this study at any time without penalty or prejudice.

Questions concerning the research project should be directed to Mr. Brantley Pierce at 601.964.3235 or to Dr. Susan Ross at 601.266.6257.

By signing below, you are giving permission for the student to participate in the research study.

[Signature of Student]  8/9/2013  [Date]

[Signature of Parent/Guardian]  8/9/13  [Date]

[Signature of Mr. Brantley Pierce]  8/9/2013  [Date]
APPENDIX C

VAN HIELE TEST

Directions:

This test contains 25 questions. It is not expected that you know everything on this test. Please answer each question to the best of your ability.

When you are told to begin:

1. Read each question carefully.
2. Decide upon the answer you think is correct. There is only one correct answer to each question. Carefully circle in the correct answer on the answer sheet.
3. If you want to change an answer, please make sure you completely erase the first answer.
4. Use the scrap paper for finding the solution to a problem. DO NOT mark in the test booklet.
5. Use only a pencil to mark your answer on the answer sheet. If you need a new pencil, please raise your hand.

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Van Hiele Geometry Test

1. Which of these are squares?

(A) K only
(B) L only
(C) M only
(D) L and M only
(E) All are squares

2. Which of these are triangles?

(A) None of these are triangles.
(B) V only
(C) W only
(D) W and X only
(E) V and W only
3. Which of these are rectangles?

(A) S only  
(B) T only  
(C) S and T only  
(D) S and U only  
(E) All are rectangles

4. Which of these are squares?

(A) None of these are squares.  
(B) G only  
(C) F and G only  
(D) G and I only  
(E) All are squares
5. Which of these are parallelograms?

(A) J only
(B) L only
(C) J and M only
(D) None of these are parallelograms
(E) All are parallelograms

6. PQRS is a square

Which relationship is true in all squares?

(A) $\overline{PR}$ and $\overline{RS}$ have the same length.
(B) $\overline{QS}$ and $\overline{PR}$ are perpendicular.
(C) $\overline{PS}$ and $\overline{QR}$ are perpendicular.
(D) $\overline{PS}$ and $\overline{QS}$ have the same length.
(E) Angle $Q$ is larger than angle $R$. 
7. In the rectangle GHJK, \( \overline{GJ} \) and \( \overline{HK} \) are the diagonals.

Which of the (A) – (D) is not true in every rectangle?

(A) There are four right angles.
(B) There are four sides
(C) The diagonals have the same length.
(D) The opposite sides have the same length.
(E) All of (A) – (D) are true in every rectangle.

8. A rhombus is a four sided figure with all sides of the same length. Here are some examples.

Which of (A) – (D) is not true in every rhombus?

(A) The two diagonals have the same length.
(B) Each diagonal bisects two angles of the rhombus.
(C) The two diagonals are perpendicular.
(D) The opposite angles have the same measure.
(E) All of (A) – (D) are true in every rhombus.
9. An isosceles triangle is a triangle with two sides of equal length. Here are three examples.

Which of (A) – (D) is true in every isosceles triangle?

(A) The three sides must have the same length.
(B) One side must have twice the length of another side.
(C) There must be at least two angles with the same measure.
(D) The three angles must have the same measure.
(E) None of (A) – (D) is true in every isosceles triangle.

10. Two circles with centers P and Q intersect at R and S to form a 4-sided figure PRQS.

Here are two examples.

Which of (A) – (D) is not always true?

(A) PRQS will always have two pair of sides of equal length.
(B) PRQS will have at least two angles of equal measure.
(C) \( \overline{PQ} \) and \( \overline{RS} \) will be perpendicular.
(D) Angles P and Q will have the same measure.
(E) All of (A) – (D) are true.
11. Here are two statements.

   Statement 1: Figure F is a rectangle.

   Statement 2: Figure F is a triangle.

Which is correct?

   (A) If 1 is true, then 2 is true.
   (B) If 1 is false, then 2 is true.
   (C) 1 and 2 cannot both be true.
   (D) 1 and 2 cannot both be false.
   (E) None of (A) - (D) is correct.

12. Here are two statements.

   Statement S: \( \triangle \text{ABC} \) has three sides of the same length.

   Statement T: In \( \triangle \text{ABC} \), angle B and angle C have the same measure.

Which is correct?

   (A) Statements S and T cannot both be true.
   (B) If S is true, then T is true.
   (C) If T is true, then S is true.
   (D) If S is false, then T is false.
   (E) None of (A) - (D) is correct.
13. Which of these can be called rectangles?

Which is correct?

(A) All can.
(B) Q only
(C) R only
(D) P and Q only
(E) Q and R only

14. Which is true?

(A) All properties of rectangles are properties of all squares.
(B) All properties of squares are properties of all rectangles.
(C) All properties of rectangles are properties of all parallelograms.
(D) All properties of squares are properties of all parallelograms.
(E) None of (A) - (D) is true.
15. What do all rectangles have that some parallelograms do not have?

(A) Opposite sides equal.
(B) Diagonals equal.
(C) Opposite sides parallel.
(D) Opposite angles equal.
(E) None of (A) - (D).

16. Shown below is right triangle ABC. Equilateral triangles ACE, ABF, and BCD have been constructed on the sides of ABC.

From this information one can prove that $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common. What would this proof tell you?

(A) Only in this triangle drawn can we be sure that $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common.
(B) In some but not all right triangles, $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common.
(C) In any right triangle, $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common.
(D) In any triangle, $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common.
(E) In any equilateral triangle, $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common.
17. Here are three properties of a figure.

Property D: It has diagonals of equal length.

Property S: It is a square.

Property R: It is a rectangle.

Which is true?

(A) D implies S which implies R.
(B) D implies R which implies S.
(C) S implies R which implies D.
(D) R implies D which implies S.
(E) R implies S which implies D.

18. Here are two statements.

I: If a figure is a rectangle, then its diagonals bisect each other.

II: If the diagonals of a figure bisect each other, then the figure is a rectangle.

Which is correct?

(A) To prove I is true, it is enough to prove II is true.
(B) To prove II is true, it is enough to prove I is true.
(C) To prove II is true, it is enough to find one rectangle whose diagonals bisect each other.
(D) To prove II is false, it is enough to find one non-rectangle whose diagonals bisect each other.
(E) None of (A) - (D) is correct.

19. In geometry which of the following statements is correct.

(A) Every term can be defined and every true statement can be proved true.
(B) Every term can be defined but it is necessary to assume that certain statements are true.
(C) Some terms must be left undefined but every true statement can be proved true.
(D) Some terms must be left undefined and it is necessary to have some statements which are assumed true.
(E) None of (A) - (D) is correct.
20. Examine these three sentences.

(1) Two lines perpendicular to the same line are parallel.
(2) A line that is perpendicular to one of two parallel lines is perpendicular to each other.
(3) If two lines are equidistant, then they are parallel.

In the figure below, it is given that lines \( m \) and \( p \) are perpendicular and lines \( n \) and \( p \) are perpendicular. Which of the above sentences could be the reason that line \( m \) is parallel to line \( n \)?

![Diagram showing lines \( m \), \( n \), and \( p \) with perpendicular relationships.]

(A) (1) only
(B) (2) only
(C) (3) only
(D) Either (1) or (2)
(E) Either (2) or (3)

21. In F-geometry, one that is different from the one you are used to, there are exactly four points and six lines. Every line contains exactly two points. If the points are \( P, Q, R, \) and \( S \), the lines are \( \{P,Q\}, \{P,R\}, \{P,S\}, \{Q,R\}, \{Q,S\}, \) and \( \{R,S\} \).

Here are how the words “intersect” and “parallel” are used in F-geometry. The lines \( \{P,Q\} \) and \( \{P,R\} \) intersect at \( P \) because \( \{P,Q\} \) and \( \{P,R\} \) have point \( P \) in common. The lines \( \{P,Q\} \) and \( \{R,S\} \) are parallel because they have no points in common.

From this information, which is correct?

(A) \( \{P,R\} \) and \( \{Q,S\} \) intersect.
(B) \( \{P,R\} \) and \( \{Q,S\} \) are parallel.
(C) \( \{Q,R\} \) and \( \{R,S\} \) are parallel.
(D) \( \{P,S\} \) and \( \{Q,R\} \) intersect.
(E) None of (A) - (D) are correct.
22. To trisect an angle means to divide it into three parts of equal measure. In 1847, P.L. Wantzel proved that, in general, it is impossible to trisect angles using only a compass and an unmarked ruler. From this proof, what can you conclude?

(A) In general, it is impossible to bisect angles using only a compass and an unmarked ruler.
(B) In general, it is impossible to trisect angles using only a compass and a marked ruler.
(C) In general, it is impossible to trisect angles using any drawing instruments.
(D) It is still possible that in the future someone may find a general way to trisect angles using only a compass and an unmarked ruler.
(E) No one will ever be able to find a general method for trisecting angles using only a compass and an unmarked ruler.

23. There is a geometry invented by a mathematician J in which the following is true:

The sum of the measures of the angles of a triangle is less than 180°.

Which is correct?

(A) J made a mistake in measuring the angles of the triangle.
(B) J made a mistake in logical reasoning.
(C) J has a wrong idea of what is meant by “true”.
(D) J started with different assumptions than those in the usual geometry.
(E) None of (A) - (D) is correct.

24. Two geometry books define the word rectangle in different ways.

Which is true?

(A) One of the books has an error.
(B) One of the definitions is wrong. There cannot be two different definitions for a rectangle.
(C) The rectangles in one of the books must have different properties from those in the other book.
(D) The rectangles in one of the books must have the same properties as those in the other book.
(E) The properties of rectangles in the two books might be different.
25. Suppose you proved statements I and II.

I. If p, then q.

II. If s, then not q.

Which statement follows from statements I and II?

(A) If p, then s.
(B) If not p, then not q.
(C) If p or q, then s.
(D) If s, then not p.
(E) If not s, then p.
APPENDIX D

DATA DESCRIPTIVES

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<td>Valid N (listwise)</td>
<td>18</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Descriptives

<table>
<thead>
<tr>
<th>Descriptive Statistics</th>
<th>N</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit Test 1:</td>
<td>23</td>
<td>.08</td>
<td>.92</td>
<td>.5643</td>
<td>.22962</td>
</tr>
<tr>
<td>UT1 Diff. Inst. UT1: Diff. Inst. ?</td>
<td>23</td>
<td>.14</td>
<td>1.00</td>
<td>.6178</td>
<td>.23924</td>
</tr>
<tr>
<td>UT1 Trad. L. UT1: Trad. L. ?</td>
<td>23</td>
<td>.0</td>
<td>1.0</td>
<td>.491</td>
<td>.2999</td>
</tr>
<tr>
<td>Trad. Less. WU Trad. Less. W-U</td>
<td>19</td>
<td>.0</td>
<td>1.0</td>
<td>.605</td>
<td>.3566</td>
</tr>
<tr>
<td>T.L. WU Diff. T.L. W-U Diff. ?</td>
<td>19</td>
<td>0</td>
<td>1</td>
<td>.63</td>
<td>.496</td>
</tr>
<tr>
<td>T.L. WU Trad. T.L. W-U Trad. ?</td>
<td>19</td>
<td>0</td>
<td>1</td>
<td>.58</td>
<td>.507</td>
</tr>
<tr>
<td>Diff. Inst. WU Diff. Inst. W-U</td>
<td>21</td>
<td>.00</td>
<td>1.00</td>
<td>.6833</td>
<td>.32497</td>
</tr>
<tr>
<td>H.W.1 Diff. Inst. L. H.W. 1 Diff. Inst. L.</td>
<td>18</td>
<td>.80</td>
<td>1.00</td>
<td>.9067</td>
<td>.06287</td>
</tr>
<tr>
<td>H.W.1 Diff. H.W. 1 Diff. ?</td>
<td>18</td>
<td>.67</td>
<td>1.00</td>
<td>.8900</td>
<td>.11931</td>
</tr>
<tr>
<td>H.W.1 Trad. H.W. 1 Trad. ?</td>
<td>18</td>
<td>.75</td>
<td>1.00</td>
<td>.9189</td>
<td>.06747</td>
</tr>
<tr>
<td>H.W.2 Trad. L. H.W. 2 Trad. L.</td>
<td>19</td>
<td>.68</td>
<td>1.00</td>
<td>.8211</td>
<td>.10944</td>
</tr>
<tr>
<td>H.W.2 Diff. H.W. 2 Diff. ?</td>
<td>19</td>
<td>.65</td>
<td>1.00</td>
<td>.8816</td>
<td>.09477</td>
</tr>
<tr>
<td>H.W.2 Trad. H.W. 2 Trad. ?</td>
<td>19</td>
<td>.25</td>
<td>1.00</td>
<td>.7195</td>
<td>.27658</td>
</tr>
<tr>
<td>V.H. PreT V.H. Pre-T</td>
<td>21</td>
<td>0</td>
<td>3</td>
<td>1.38</td>
<td>.921</td>
</tr>
<tr>
<td>V.H. Pos. T</td>
<td>23</td>
<td>0</td>
<td>3</td>
<td>1.91</td>
<td>1.125</td>
</tr>
<tr>
<td>Valid N (listwise)</td>
<td>13</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Hypothesis Test Summary

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Test</th>
<th>Sig.</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>The distributions of Unit Test 1; UT1: Diff. Inst ? and UT1: Trad. L. ? are the same.</td>
<td>Related-Samples Friedman's Two-Way Analysis of Variance by Ranks</td>
<td>.119</td>
<td>Retain the null hypothesis.</td>
</tr>
</tbody>
</table>

Asymptotic significances are displayed. The significance level is .05.

---

#### Related-Samples Friedman's Two-Way Analysis of Variance by Ranks

<table>
<thead>
<tr>
<th>Unit Test 1:</th>
<th>UT1: Diff. Inst ?</th>
<th>UT1: Trad. L. ?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Rank = 2.00</td>
<td>Mean Rank = 2.30</td>
<td>Mean Rank = 1.70</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rank</th>
<th>Frequency</th>
<th>Rank</th>
<th>Frequency</th>
<th>Rank</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td></td>
<td>3</td>
<td></td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total N</th>
<th>Test Statistic</th>
<th>Degrees of Freedom</th>
<th>Asymptotic Sig. (2-sided test)</th>
</tr>
</thead>
<tbody>
<tr>
<td>23</td>
<td>4.261</td>
<td>2</td>
<td>.119</td>
</tr>
</tbody>
</table>

1. Multiple comparisons are not performed because the overall test retained the null hypothesis of no differences.
Hypothesis Test Summary

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Test</th>
<th>Sig.</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>The distributions of Trad. Less. W-U, T.L.W-U Diff?</td>
<td>Related-Samples Friedman's Two-Way Analysis of Variance by Ranks</td>
<td>.895</td>
<td>Retain the null hypothesis.</td>
</tr>
<tr>
<td>Trad ? are the same.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Asymptotic significances are displayed. The significance level is .05.

**Related-Samples Friedman's Two-Way Analysis of Variance by Ranks**

<table>
<thead>
<tr>
<th></th>
<th>Frequency</th>
<th></th>
<th></th>
<th>Frequency</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Trad. Less. W-U</td>
<td>0.0</td>
<td>5.0</td>
<td>10.0</td>
<td>15.0</td>
<td>20.0</td>
<td></td>
</tr>
<tr>
<td>T.L.W-U Diff ?</td>
<td>0.0</td>
<td>5.0</td>
<td>10.0</td>
<td>15.0</td>
<td>20.0</td>
<td></td>
</tr>
<tr>
<td>T.L.W-U Trad ?</td>
<td>0.0</td>
<td>5.0</td>
<td>10.0</td>
<td>15.0</td>
<td>20.0</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rank</th>
<th>Frequency</th>
<th></th>
<th></th>
<th>Frequency</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10.0</td>
<td>0.0</td>
<td>0.0</td>
<td>5.0</td>
<td>10.0</td>
<td>15.0</td>
</tr>
<tr>
<td>2</td>
<td>5.0</td>
<td>0.0</td>
<td>0.0</td>
<td>10.0</td>
<td>15.0</td>
<td>20.0</td>
</tr>
<tr>
<td>3</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>4</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

| Total N               | 19        |                      |                      |           |                      |                      |
| Test Statistic        | .222      |                      |                      |           |                      |                      |
| Degrees of Freedom    | 2         |                      |                      |           |                      |                      |
| Asymptotic Sig. (2-sided test) | .895     |                      |                      |           |                      |                      |

1. Multiple comparisons are not performed because the overall test retained the null hypothesis of no differences.
Hypothesis Test Summary

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Test</th>
<th>Sig.</th>
<th>Decision</th>
</tr>
</thead>
</table>

Asymptotic significances are displayed. The significance level is .05.

Related-Samples Friedman's Two-Way Analysis of Variance by Ranks

```
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Rank = 2.00</td>
<td>Mean Rank = 1.76</td>
<td>Mean Rank = 2.24</td>
</tr>
</tbody>
</table>
```

<table>
<thead>
<tr>
<th>Total N</th>
<th>21</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test Statistic</td>
<td>4.545</td>
</tr>
<tr>
<td>Degrees of Freedom</td>
<td>2</td>
</tr>
<tr>
<td>Asymptotic Sig. (2-sided test)</td>
<td>.103</td>
</tr>
</tbody>
</table>

1. Multiple comparisons are not performed because the overall test retained the null hypothesis of no differences.
### Hypothesis Test Summary

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Test</th>
<th>Sig.</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>The distributions of H.W. 1 Diff. Inst. L., H.W. 1 Diff.? and H.W. 1 Trad.? are the same.</td>
<td>Related-Samples Friedman's Two-Way Analysis of Variance by Ranks</td>
<td>.813</td>
<td>Retain the null hypothesis.</td>
</tr>
</tbody>
</table>

Asymptotic significances are displayed. The significance level is .05.

### Related-Samples Friedman's Two-Way Analysis of Variance by Ranks

<table>
<thead>
<tr>
<th>H.W. 1 Diff. Inst. L.</th>
<th>H.W. 1 Diff.?</th>
<th>H.W. 1 Trad.?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Rank = 1.94</td>
<td>Mean Rank = 1.94</td>
<td>Mean Rank = 2.11</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rank</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total N</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test Statistic</td>
<td>.414</td>
</tr>
<tr>
<td>Degrees of Freedom</td>
<td>2</td>
</tr>
<tr>
<td>Asymptotic Sig. (2-sided test)</td>
<td>.813</td>
</tr>
</tbody>
</table>

1. Multiple comparisons are not performed because the overall test retained the null hypothesis of no differences.
Hypothesis Test Summary

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Test</th>
<th>Sig.</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>The distributions of H.W. 2 Trad. L., H.W. 2 Diff.? and H.W. 2 Trad ? are the same.</td>
<td>Related-Samples Friedman's Two-Way Analysis of Variance by Ranks</td>
<td>.257</td>
<td>Retain the null hypothesis.</td>
</tr>
</tbody>
</table>

Asymptotic significances are displayed. The significance level is .05.

Related-Samples Friedman's Two-Way Analysis of Variance by Ranks

<table>
<thead>
<tr>
<th>H.W. 2 Trad. L.</th>
<th>H.W. 2 Diff.?</th>
<th>H.W. 2 Trad ?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Rank = 2.03</td>
<td>Mean Rank = 2.24</td>
<td>Mean Rank = 1.74</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Frequency</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>5.0</td>
<td>10.0</td>
</tr>
<tr>
<td>5.0</td>
<td>10.0</td>
<td>15.0</td>
</tr>
<tr>
<td>10.0</td>
<td>15.0</td>
<td>20.0</td>
</tr>
<tr>
<td>15.0</td>
<td>20.0</td>
<td>25.0</td>
</tr>
<tr>
<td>20.0</td>
<td>25.0</td>
<td>30.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total N</th>
<th>19</th>
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</thead>
<tbody>
<tr>
<td>Test Statistic</td>
<td>2.716</td>
</tr>
<tr>
<td>Degrees of Freedom</td>
<td>2</td>
</tr>
<tr>
<td>Asymptotic Sig. (2-sided test)</td>
<td>.257</td>
</tr>
</tbody>
</table>

1. Multiple comparisons are not performed because the overall test retained the null hypothesis of no differences.
## Crosstabs

V.H.PreT V.H.Pre-T * V.H.Pos.T Crosstabulation

<table>
<thead>
<tr>
<th></th>
<th>V.H.Pos.T</th>
<th></th>
<th></th>
<th></th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>V.H.PreT V.H.Pre-T</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td>2</td>
<td>6</td>
<td>4</td>
<td>9</td>
<td>21</td>
</tr>
</tbody>
</table>

### Directional Measures

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Asymp. Std. Error&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Approx. t&lt;sup&gt;b&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ordinal by Ordinal</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Somers' d</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Symmetric</td>
<td>.288</td>
<td>.184</td>
<td>1.552</td>
</tr>
<tr>
<td>V.H.PreT V.H.Pre-T</td>
<td>.289</td>
<td>.182</td>
<td>1.552</td>
</tr>
<tr>
<td>Dependent</td>
<td>.286</td>
<td>.186</td>
<td>1.552</td>
</tr>
<tr>
<td>V.H.Pos.T Dependent</td>
<td>.286</td>
<td>.186</td>
<td>1.552</td>
</tr>
</tbody>
</table>

### Nonparametric Tests

- a. Not assuming the null hypothesis.
- b. Using the asymptotic standard error assuming the null hypothesis.
### Hypothesis Test Summary

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Test</th>
<th>Sig.</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>The median of differences between V.H.Pre-T and V.H.Pos.T equals 0.</td>
<td>Related-Samples Wilcoxon Signed Rank Test</td>
<td>.040</td>
<td>Reject the null hypothesis.</td>
</tr>
</tbody>
</table>

Asymptotic significances are displayed. The significance level is .05.

### Related-Samples Wilcoxon Signed Rank Test

![Graph showing frequency distribution of differences between V.H.Pos.T and V.H.Pre-T.]

- **Total N**: 21
- **Test Statistic**: 94.000
- **Standard Error**: 16.557
- **Standardized Test Statistic**: 2.054
- **Asymptotic Sig. (2-sided test)**: .040
Hypothesis Test Summary

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Test</th>
<th>Sig.</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>The distributions of Unit Test 2, UT2: Diff. Inst ? and UT2: Trad. L ? are the same.</td>
<td>Related-Samples Friedman's Two-Way Analysis of Variance by Ranks</td>
<td>.304</td>
<td>Retain the null hypothesis.</td>
</tr>
</tbody>
</table>

Asymptotic significances are displayed. The significance level is .05.

Related-Samples Friedman's Two-Way Analysis of Variance by Ranks

Unit Test 2

- Mean Rank = 2.00

UT2: Diff. Inst ?

- Mean Rank = 2.23

UT2: Trad. L ?

- Mean Rank = 1.77

<table>
<thead>
<tr>
<th>Total N</th>
<th>22</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test Statistic</td>
<td>2.381</td>
</tr>
<tr>
<td>Degrees of Freedom</td>
<td>2</td>
</tr>
<tr>
<td>Asymptotic Sig. (2-sided test)</td>
<td>.304</td>
</tr>
</tbody>
</table>

1. Multiple comparisons are not performed because the overall test retained the null hypothesis of no differences.
Hypothesis Test Summary

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Test</th>
<th>Sig.</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>The distributions of Diff/Trad. W-U, D/T. W-U Diff ? and D/T. W-U Trad. ? are the same.</td>
<td>Related-Samples Friedman's Two-Way Analysis of Variance by Ranks</td>
<td>.135</td>
<td>Retain the null hypothesis.</td>
</tr>
</tbody>
</table>

Asymptotic significances are displayed. The significance level is .05.

Related-Samples Friedman's Two-Way Analysis of Variance by Ranks

<table>
<thead>
<tr>
<th>Diff/Trad. W-U</th>
<th>D/T. W-U Diff ?</th>
<th>D/T. W-U Trad. ?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Rank = 2.00</td>
<td>Mean Rank = 2.18</td>
<td>Mean Rank = 1.62</td>
</tr>
</tbody>
</table>

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total N</td>
<td>22</td>
</tr>
<tr>
<td>Test Statistic</td>
<td>4.000</td>
</tr>
<tr>
<td>Degrees of Freedom</td>
<td>2</td>
</tr>
<tr>
<td>Asymptotic Sig. (2-sided test)</td>
<td>.136</td>
</tr>
</tbody>
</table>

1. Multiple comparisons are not performed because the overall test retained the null hypothesis of no differences.
Hypothesis Test Summary

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Test</th>
<th>Sig.</th>
<th>Decision</th>
</tr>
</thead>
</table>

Asymptotic significances are displayed. The significance level is .05.

Related-Samples Friedman’s Two-Way Analysis of Variance by Ranks

<table>
<thead>
<tr>
<th>Rank</th>
<th>Frequency</th>
<th>Mean Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>22</td>
<td>2.00</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>2.45</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>1.55</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total N</th>
<th>Test Statistic</th>
<th>Degrees of Freedom</th>
<th>Asymptotic Sig. (2-sided test)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>12.500</td>
<td>2</td>
<td>.002</td>
</tr>
</tbody>
</table>
### Pairwise Comparisons

Each node shows the sample average rank.

<table>
<thead>
<tr>
<th>Sample 1-Sample 2</th>
<th>Test Statistic</th>
<th>Std. Error</th>
<th>Std. Test Statistic</th>
<th>Sig.</th>
<th>Adj.Sig.</th>
</tr>
</thead>
</table>

Each row tests the null hypothesis that the Sample 1 and Sample 2 distributions are the same. Asymptotic significances (2-sided tests) are displayed. The significance level is .05.
Hypothesis Test Summary

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Test</th>
<th>Sig.</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>The distributions of Unit Test 3, UT3: Diff. Inst ? and UT3: Trad. L. ? are the same.</td>
<td>Related-Samples Friedman's Two-Way Analysis of Variance by Ranks</td>
<td>.676</td>
<td>Retain the null hypothesis.</td>
</tr>
</tbody>
</table>

Asymptotic significances are displayed. The significance level is .05.

Related-Samples Friedman's Two-Way Analysis of Variance by Ranks

Unit Test 3
Mean Rank = 2.00

UT3: Diff. Inst ?
Mean Rank = 1.87

UT3: Trad. L. ?
Mean Rank = 2.13

Total N 23
Test Statistic .783
Degrees of Freedom 2
Asymptotic Sig. (2-sided test) .676

1. Multiple comparisons are not performed because the overall test retained the null hypothesis of no differences.
Hypothesis Test Summary

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Test</th>
<th>Sig.</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>The distributions of Trad. Less. W-U, T.L.W-U Diff ? and T.L.W-U Trad ? are the same.</td>
<td>Related-Samples Friedman's Two-Way Analysis of Variance by Ranks</td>
<td>.670</td>
<td>Retain the null hypothesis.</td>
</tr>
</tbody>
</table>

Asymptotic significances are displayed. The significance level is .05.

Related-Samples Friedman's Two-Way Analysis of Variance by Ranks

<table>
<thead>
<tr>
<th>Trad. Less. W-U</th>
<th>T.L.W-U Diff ?</th>
<th>T.L.W-U Trad ?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Rank = 2.00</td>
<td>Mean Rank = 2.10</td>
<td>Mean Rank = 1.90</td>
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</table>

<table>
<thead>
<tr>
<th>Total N</th>
<th>Test Statistic</th>
<th>Degrees of Freedom</th>
<th>Asymptotic Sig. (2-sided test)</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>.800</td>
<td>2</td>
<td>.670</td>
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1. Multiple comparisons are not performed because the overall test retained the null hypothesis of no differences.
Hypothesis Test Summary

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Test</th>
<th>Sig.</th>
<th>Decision</th>
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</thead>
</table>

Asymptotic significances are displayed. The significance level is .05.

Related-Samples Friedman's Two-Way Analysis of Variance by Ranks

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>Mean Rank = 2.00</td>
<td>Mean Rank = 2.00</td>
<td>Mean Rank = 2.00</td>
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</table>

<table>
<thead>
<tr>
<th>Total N</th>
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<tbody>
<tr>
<td>Test Statistic</td>
<td>.000</td>
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<tr>
<td>Degrees of Freedom</td>
<td>2</td>
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<tr>
<td>Asymptotic Sig. (2-sided test)</td>
<td>1.000</td>
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</tbody>
</table>

1. Multiple comparisons are not performed because the overall test retained the null hypothesis of no differences.
Hypothesis Test Summary

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Test</th>
<th>Sig.</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>The distributions of Diff/Trad. H.W., Diff/Trad. H.W. Diff ? and Diff/Trad. H.W. Trad ? are the same.</td>
<td>Related-Samples Friedman's Two-Way Analysis of Variance by Ranks</td>
<td>.005</td>
<td>Reject the null hypothesis.</td>
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Asymptotic significances are displayed. The significance level is .05.

Related-Samples Friedman's Two-Way Analysis of Variance by Ranks

<table>
<thead>
<tr>
<th>Rank</th>
<th>Frequency</th>
<th>Rank</th>
<th>Frequency</th>
<th>Rank</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>2</td>
<td></td>
<td>3</td>
<td></td>
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</tbody>
</table>

Mean Rank = 2.04
Mean Rank = 1.52
Mean Rank = 2.43

Total N = 23
Test Statistic = 10.571
Degrees of Freedom = 2
Asymptotic Sig. (2-sided test) = .005
Pairwise Comparisons

Each node shows the sample average rank.

<table>
<thead>
<tr>
<th>Sample1-Sample2</th>
<th>Test Statistic</th>
<th>Std. Error</th>
<th>Std. Test Statistic</th>
<th>Sig.</th>
<th>Adj.Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diff/Trad. H.W. - Diff/Trad. H.W. Trad?</td>
<td>-.391</td>
<td>.295</td>
<td>-1.327</td>
<td>.185</td>
<td>.554</td>
</tr>
</tbody>
</table>

Each row tests the null hypothesis that the Sample 1 and Sample 2 distributions are the same. Asymptotic significances (2-sided tests) are displayed. The significance level is .05.
APPENDIX E

EXAMPLES OF HOMEWORK, TESTS, AND WARM-UPS

6-1 Exercises

GUIDED PRACTICE

1. Vocabulary Explain why an equilateral polygon is not necessarily a regular polygon.

Tell whether each outlined shape is a polygon. If it is a polygon, name it by the number of its sides.

2. Tell whether each polygon is regular or irregular. Tell whether it is concave or convex.

9. Find the measure of each interior angle of pentagon ABCDE.

10. Find the measure of each interior angle of a regular dodecagon.

11. Find the sum of the interior angle measures of a convex 20-gon.

12. Find the value of \( y \) in polygon JKLM.

13. Find the measure of each exterior angle of a regular pentagon.

Safety Use the photograph of the traffic sign for Exercises 14 and 15.

14. Name the polygon by the number of its sides.

15. In the polygon, \( \angle P, \angle R, \) and \( \angle T \) are right angles, and \( \angle Q \equiv \angle S \). What are \( m\angle Q \) and \( m\angle S \)?

PRACTICE AND PROBLEM SOLVING

Tell whether each figure is a polygon. If it is a polygon, name it by the number of its sides.

16. Tell whether each polygon is regular or irregular. Tell whether it is concave or convex.
22. Find the measure of each interior angle of quadrilateral RSTV.

23. Find the measure of each interior angle of a regular 18-gon.

24. Find the sum of the interior angle measures of a convex heptagon.

25. Find the measure of each exterior angle of a regular nonagon.

26. A pentagon has exterior angle measures of $5a^\circ$, $4a^\circ$, $10a^\circ$, $3a^\circ$, and $8a^\circ$. Find the value of $a$.

Crafts  The folds on the lid of the gift box form a regular hexagon. Find each measure.

27. $\text{m} \angle JKM$

28. $\text{m} \angle MKL$

Algebra  Find the value of $x$ in each figure.

29. \[ \begin{align*}
   \text{(x - 3)}^\circ & \quad 130^\circ \\
   110^\circ & \quad x^\circ
\end{align*} \]

30. \[ \begin{align*}
   (x + 22)^\circ & \quad (x + 23)^\circ
\end{align*} \]

31. \[ \begin{align*}
   (x + 22)^\circ & \quad (x + 23)^\circ
\end{align*} \]

Find the number of sides a regular polygon must have to meet each condition.

32. Each interior angle measure equals each exterior angle measure.

33. Each interior angle measure is four times the measure of each exterior angle.

34. Each exterior angle measure is one eighth the measure of each interior angle.

Name the convex polygon whose interior angle measures have each given sum.

35. $540^\circ$  36. $900^\circ$  37. $1800^\circ$  38. $2520^\circ$

Multi-Step  An exterior angle measure of a regular polygon is given. Find the number of its sides and the measure of each interior angle.

39. $120^\circ$  40. $72^\circ$  41. $36^\circ$  42. $24^\circ$

43. /ERROR ANALYSIS/ Which conclusion is incorrect?

Explain the error.

A. The figure is a polygon.

B. The figure is not a polygon.

44. Estimation  Graph the polygon formed by the points $A(-2, -6)$, $B(-4, -1)$, $C(-1, 2)$, $D(4, 0)$, and $E(3, -5)$. Estimate the measure of each interior angle. Make a conjecture about whether the polygon is equiangular. Now measure each interior angle with a protractor. Was your conjecture correct?

45. This problem will prepare you for the Multi-Step Test Prep on page 406.

In this quartz crystal, $\text{m} \angle A = 95^\circ$, $\text{m} \angle B = 125^\circ$, $\text{m} \angle E = \text{m} \angle D = 130^\circ$, and $\angle C \equiv \angle F \equiv \angle G$.

a. Name polygon $ABCD\overline{EFG}$ by the number of sides.

b. What is the sum of the interior angle measures of $ABCD\overline{EFG}$?

c. Find $\text{m} \angle F$.
GUIDED PRACTICE

Vocabulary  Apply the vocabulary from this lesson to answer each question.

1. Explain why the figure at right is NOT a \textit{parallelogram}.

2. Draw $\square PQRS$. Name the opposite sides and opposite angles.

3. $BD \quad 4. \quad CD$

4. $BE \quad 5. \quad m\angle ABC$

5. $\angle ADC \quad 6. \quad m\angle DAB$

\textbf{SEE EXAMPLE} \hspace{1cm} p. 392

\textbf{Safety}  The handrail is made from congruent parallelograms. In $\square ABCD$, $AB = 17.5$, $DE = 18$, and $m\angle BCD = 110^\circ$. Find each measure.

\begin{align*}
3. \quad BD & \quad 4. \quad CD \\
5. \quad BE & \quad 6. \quad m\angle ABC \\
7. \quad \angle ADC & \quad 8. \quad m\angle DAB \\
\end{align*}

\textbf{SEE EXAMPLE} \hspace{1cm} p. 393

\textbf{JKLM} is a parallelogram. Find each measure.

\begin{align*}
9. \quad JK & \quad 10. \quad LM \\
11. \quad m\angle L & \quad 12. \quad m\angle M \\
\end{align*}

\textbf{SEE EXAMPLE} \hspace{1cm} p. 393

\textbf{Multi-Step}  Three vertices of $\square DFGH$ are $D(-9, 4)$, $F(-1, 5)$, and $G(2, 0)$. Find the coordinates of vertex $H$.

\textbf{SEE EXAMPLE} \hspace{1cm} p. 394

\textbf{Write a two-column proof.}

\textbf{Given:} $\square PSTV$ is a parallelogram. $PQ \cong RQ$  

\textbf{Prove:} $\angle STV \cong \angle R$

\textbf{PRACTICE AND PROBLEM SOLVING}

\textbf{Shipping}  Cranes can be used to load cargo onto ships. In $\square JKLM$, $JL = 165.8$, $JK = 110$, and $m\angle JML = 50^\circ$. Find the measure of each part of the crane.

\begin{align*}
15. \quad JN & \quad 16. \quad LM \\
17. \quad LN & \quad 18. \quad m\angle JKL \\
19. \quad m\angle KLM & \quad 20. \quad m\angle MJK \\
\end{align*}

\textbf{WXYZ} is a parallelogram. Find each measure.

\begin{align*}
21. \quad WZ & \quad 22. \quad YW \\
23. \quad XZ & \quad 24. \quad ZV \\
\end{align*}

\textbf{Multi-Step}  Three vertices of $\square PRTV$ are $P(-4, -4)$, $R(-10, 0)$, and $V(5, -1)$. Find the coordinates of vertex $T$.

\textbf{Write a two-column proof.}

\textbf{Given:} $ABCD$ and $AFGH$ are parallelograms. 

\textbf{Prove:} $\angle C \cong \angle G$

\textbf{6-2 Properties of Parallelograms} 395
Algebra The perimeter of \( \square PQRS \) is 84. Find the length of each side of \( \square PQRS \) under the given conditions.

27. \( PQ = QR \)  
28. \( QR = 3(RS) \)  
29. \( RS = SP - 7 \)  
30. \( SP = RS^2 \)

31. Cars To repair a large truck, a mechanic might use a parallelogram lift. In the lift, \( FG \parallel GH \parallel IL \parallel JK \), and \( PL \parallel GK \parallel HF \).
   a. Which angles are congruent to \( \angle 1 \)? Justify your answer.
   b. What is the relationship between \( \angle 1 \) and each of the remaining labeled angles? Justify your answer.

Complete each statement about \( \square KMPR \). Justify your answer.

32. \( \angle MPR \equiv ? \)
33. \( \angle PRK \equiv ? \)
34. \( MT \equiv ? \)
35. \( FR \equiv ? \)
36. \( MP \parallel ? \)
37. \( MK \parallel ? \)
38. \( \angle MPK \equiv ? \)
39. \( \angle MTK \equiv ? \)
40. \( m \angle MKR + m \angle PRK = ? \)

Find the values of \( x, y, \) and \( z \) in each parallelogram.

41. \[ \begin{array}{c}
\angle x \\
\angle y \\
\angle z \end{array} \]
42. \[ \begin{array}{c}
\angle x \\
\angle y \\
\angle z \end{array} \]
43. \[ \begin{array}{c}
\angle x \\
\angle y \\
\angle z \end{array} \]

44. Complete the paragraph proof of Theorem 6-2-4 by filling in the blanks.
   Given: \( ABCD \) is a parallelogram.
   Prove: \( AC \) and \( BD \) bisect each other at \( E \).
   Proof: It is given that \( ABCD \) is a parallelogram. By the definition of a parallelogram, \( AB \parallel a \). By the Alternate Interior Angles Theorem, \( \angle 1 \equiv b \). \( \angle 3 \equiv c \). \( AB \equiv CD \) because \( d \). This means that \( \triangle ABE \equiv \triangle CDE \) by \( e \). So by \( f \), \( AB \equiv CE \), and \( BE \equiv DE \). Therefore \( AC \) and \( BD \) bisect each other at \( E \) by the definition of \( g \).

45. Write a two-column proof of Theorem 6-2-3: If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

Algebra Find the values of \( x \) and \( y \) in each parallelogram.

46. \[ \begin{array}{c}
x \\
2y - 9 \\
x \end{array} \]
47. \[ \begin{array}{c}
x \\
y + 7 \\
x + 3 \\
y \end{array} \]

48. This problem will prepare you for the Multi-Step Test Prep page 496.
   In this calcite crystal, the face \( ABCD \) is a parallelogram.
   a. In \( \square ABCD \), \( m \angle B = (6x + 12)^\circ \), and \( m \angle D = (9x - 33)^\circ \). Find \( m \angle B \).
   b. Find \( m \angle A \) and \( m \angle C \). Which theorem or theorems did you use to find these angle measures?
Warm-up

1) Given the following . . . solve for $x$.

\[ 110^\circ \quad 130^\circ \quad (x-3)^\circ \quad x^\circ \]

2) Given . . .

\[ x^\circ \]

solve for $x$. 
Geometry Pre-class Warm-up

1) Solve the following proportion: \( \frac{50}{2t + 4} = \frac{2t + 4}{2} \)

2) Given that \( 34 = \frac{51}{y} \). Find the ratio of \( y : x \). Then find the ratio of \( x : y \).

3) Given \( \triangle ABC \sim \triangle DEF \). The similarity ratio of \( \triangle ABC \) to \( \triangle DEF \) is \( 5/2 \). What is the length of \( AC, BC, AB \). Where:

   \[ \text{Diagram of \( \triangle ABC \) and \( \triangle DEF \) with points labeled A, B, C, E, D, F.} \]

   \( \text{Hint: BC corresponds to EF.} \)
Geometry Unit 6 Test:

1) If a parallelogram is a rhombus, then its diagonals are:

2) If a parallelogram is a rectangle, then its diagonals are:

3) If a quadrilateral is a rhombus, then it is also a:

4) Every rectangle is a:

5) Every square is a:

6) A trapezoid always equals an isosceles trapezoid: True or False

7) Draw a detailed picture of the quadrilateral family tree:

8) Given: \[ \text{find } X \]

9) In polygon ABCD, angle A = 49 degrees, angle B = 109 degrees, angle C = angle D. What is the measure of angle C?
10) State the precise definition of a regular polygon:

11) Name the convex polygon whose interior angle measure has the given sum: 2520

12) Given 3 vertices in the following parallelogram A (1,-2) B (-2, 3) D (5,-1). Find the coordinates of C:

13) Find the measure of each interior angle of a regular 24-gon:

14) Given the following rectangle

Where angle 2 = 57 degrees. Find angle 3: angle 4: angle 5: angle 1:

15) Given the following parallelogram where angle EBA = y degrees, angle BEC = 125 degrees, angle BAE = 75 degrees, angle DAE = 31 degrees, angle ADE = X degrees, and angle EDC = Z degrees. Find the value of X, Y, and Z.

16) Given the following Rhombus: Find the measure of angle 2, angle 3, angle 4, angle 5, and angle 1.
17) The vertices of a square JKLM are J (-2, 4) K (-3,-1) L (2,-2) M (3, 3). Show that this square fits the definition of Rhombus, Rectangle, and Parallelogram.
### APPENDIX F

#### EXAMPLE OF LESSON PLANS

<table>
<thead>
<tr>
<th>Essential Questions</th>
<th>Academic Vocabulary/Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>- To identify parallel and perpendicular lines.</td>
<td>- Parallel Lines</td>
</tr>
<tr>
<td>- To identify the angles formed by two lines and a transversal.</td>
<td>- Perpendicular Lines</td>
</tr>
</tbody>
</table>

**Monday**

- Modeling (10-15 minutes)

This section will be incorporated into my Guided Practice section.
Teacher: Brantley Grant Pierce  
Dates: 1/13/14  
School: PCHS  
Grade: 10th-12th

### Guided Practice (10-15 minutes)

After a brief reintroduction of parallel and perpendicular lines (from Transitions Algebra) I will explain the outside activity to the students. During this time I will provide them with a protractor and blue painter's tape. I will then write on the board a very brief set of directions. The directions will be as follows:

Construct vertical angles, parallel lines intersected by a transversal, and other line segments intersected by a transversal, on the sidewalk with the tape. You will then discover the geometric relationships between existing angles, lines, and measurements.

### Independent Practice (10-30 minutes)

Today's independent practice will be the outside activity mentioned above. After the activity is complete the students and I will come back inside to discuss our geometric findings and hypothesis's.

### Accommodations/Modifications

Given that I have no prior knowledge of the students mathematical background; extra time, assessment, and differentiated instruction will be given if needed.
Teacher: Brantley Grant Pierce  
Dates: 1/13/14  
School: PCHS  
Grade: 10th-12th

<table>
<thead>
<tr>
<th>Assessment (Embedded)</th>
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</thead>
<tbody>
<tr>
<td>Beginning Class warm up:</td>
<td></td>
</tr>
<tr>
<td>Taking up signed Syllabi from the previous Friday. Perform other administrative duties (if needed).</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Closure</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Exit ticket: A brief collection of H.W. problems to be copied down into student’s notes.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Homework</th>
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</thead>
<tbody>
<tr>
<td>H.W. Pg: 148: Holt Geometry: 1-32 even (Due Wednesday)</td>
<td></td>
</tr>
</tbody>
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REFERENCES


