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A Comparison of van Hiele Levels and Final Exam Grades of Students at The University of Southern Mississippi

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A COMPARISON OF VAN HIELE LEVELS AND FINAL EXAM GRADES OF STUDENTS AT THE UNIVERSITY OF SOUTHERN MISSISSIPPI

by

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A Thesis

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A Research Study

A Comparison of van Hiele Levels, Final Grades, and Aided Curriculum of Students at the University of Southern Mississippi

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Abstract

This research analyzed students final exam scores in a college mathematics class with geometric components and their van Hiele levels upon entering the class. After the class was completed, each student's final exam grade was calculated. The researcher used a Spearman correlation to compare the two; the result was a correlation coefficient of 0.742. The researcher then reported that the results of the van Hiele test are a major component in predicting a student's success in such a class.

Introduction

One of the key components in mathematics is geometry. It is imperative for a student to learn, not only the basics of geometry, but also its applications to the real world. Geometry extends beyond the classroom; it encompasses nearly all aspects of life. Even though most students only see it as the study of shapes, studying geometry also teaches students deductive thinking and logic. Students are taught the minimal geometrical concepts while in elementary school which includes different types of shapes. However, geometry is usually not revisited until later in high school, causing students to miss out on the necessary deductive and logical thinking skills. Therefore, once students enter a high school geometry class, they are not mentally prepared for the topic at hand. After minimal geometry knowledge in high school, geometry is not as prevalent in the college setting. As a result, students may not proceed to a higher order of geometrical thinking (Jones, Critical Issues in the Design of the School Geometry Curriculum, 2000).

One way of measuring what level of geometrical thinking a student is on in a common geometry class is to administer a van Hiele test. Dina van Hiele-Geldof and Pierre Marie van Hiele developed a theory which places individuals on a level of 1 to 5 based on their knowledge of geometry. Level 1, Visualization, is where students can merely recognize a shape. At Level 2, Analysis, an individual is able to analyze a shape because they know the properties of the shapes in Level 1. After attaining the first two levels, Level 3, Abstraction, is the level at which students have learned geometric properties. At level 4, Deduction, the student is able to construct proofs of geometric properties. Finally, at level 5, Rigor, the student is able to understand the implications of non-Euclidian geometry. Dr. Zalman Usiskin, from the University of Chicago, developed an exam that will determine the van Hiele level at which a student is working (Usiskin, Van Hiele Levels and Achievement in Secondary School Geometry, 1982).

The purpose of this study is to determine if there is a positive correlation between a student's van Hiele level at the beginning of the class and his/her final exam grade in the class. From this, the researcher will determine if the van Hiele test, when used as a pretest, can adequately predict a student's performance in a class heavily based on geometry. The researcher will do this by using the van Hiele test on two select mathematics classes at the University of Southern Mississippi. According to the van Hiele levels and the NCTM geometry curriculum, each student is expected to leave a high school geometry class on a level 4 of geometrical thinking (Fuys, Geddes, & Tischler, 1988). The researcher will test whether students are performing on this level. Using the test developed by Usiskin, we will determine the van Hiele level of the student and gather final exam grades of the students. We will compare the final exams of each student to their van Hiele level upon completion of the course to determine if there is a correlation between grades and van Hiele levels. This study will ultimately answer the question of whether the van Hiele test is an adequate tool to predict a student's success in a class where geometry is a key component. The results of our study will then be analyzed and reported.

Literature Review

An Understanding of Geometry

Merriam Webster defines geometry to be "a branch of mathematics that deals with the measurement, properties, and relationships of points, lines, angles, surfaces, and solids." More broadly, however, it is "the study of properties of given elements that remain invariant under specified transformations" (Merriam-Webster Geometry, 2011). From this definition alone, geometry encompasses a variety of aspects. According to Amsco's Geometry 2008, geometry is "the branch of mathematics that defines and relates the basic properties and measurements of line segments and angles." Euclid, often known as the "Father of Geometry", is credited with most of today's geometrical concepts. His collection of books, "Euclid's Elements", written in 300 B.C., was a compilation of known facts of geometry at that time. Some very famous and useful facts came from these 13 books. Euclid developed five postulates which cannot be proved. They are

Postulate 1: It is possible to draw a straight line from any point to any point.

Postulate 2: It is possible to produce a finite straight line continuously in a straight line.

Postulate 3: It is possible to describe a circle with any center and radius.

Postulate 4: All right angles congruent to one another.

Postulate 5: That, if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

Postulate five is of extreme importance. Many geometers tried to prove this but constantly failed. Eventually, it was proved that it cannot be proved. Today, it is known as the infamous parallel postulate. It is impressive that Euclid's Elements have been used to the extent that they have. Its printed copies are second only to the Bible (Musser, Timpe, & Maurer, 2007). It should be noted that Euclidian geometry is focused around straight lines, parallel lines, and right triangles.

It is important to understand that Euclidian geometry is not the only form of geometry; this is what makes the study of geometry so diverse. Other types of geometry are simply classified as non-Euclidian Geometry. Elliptic Geometry is an example. In this form, there is no "straight" line. Lines are part of a circle, or a sphere. Another form of non Euclidian geometry is Hyperbolic Geometry. One main point from this type of geometry is in a two dimensional plane, if there is a line and one single point not on the line, then there are an infinite amount of lines through A that do not intersect the original line; this is vastly different from Euclidian geometry (Musser, Timpe, & Maurer, 2007).

Geometry Curriculum in Secondary Education

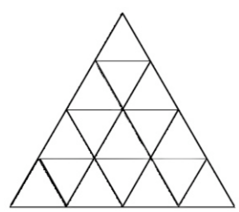
Many in the field of education agree that a problem exists in children learning geometry. Throughout elementary school, children learn the basics, and sometimes more, of arithmetic. They know how to add, subtract, multiply, divide, and solve word problems by the end of the sixth grade. By the end of elementary school, students' knowledge of geometry is a scattered array of shapes and formulas (Usiskin, Resolving the Continuing Dilemmas in School Geometry, 1987).

After elementary school mathematics is completed, the main chunk of mathematical learning is to understand algebra and its components. Geometry is usually not revisited until the 10th grade (on average). The concept of different subjects of mathematics is similar to a language; if its foundations are not set in stone at an early age, such as elementary school, it is a very arduous task for it to be fully understood. Therefore, when most students enter a high school level geometry class, their knowledge of geometry is still at an elementary level. Nonetheless, this is not the only problem of the low success rates in geometry; the curriculum for the teachers to follow is not uniform.

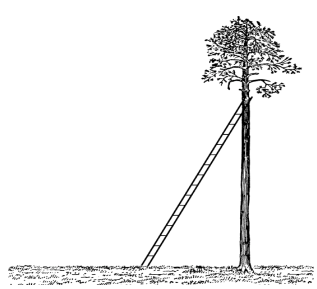
There is no set curriculum for geometry regarding both the topics to be covered as well as what order they should be covered, especially in comparison to arithmetic and algebra. In Mississippi, there is a state Algebra Test. Teachers have an established list of objectives to teach the students aiding them in performing to the best of their ability on the test. However, this is not the case for geometry. To help with the curriculum problem, the National Council of Teachers of Mathematics (NCTM) has helped develop a curriculum for geometry, but it is not mandatory for schools to follow it.

One reason that having a set curriculum for geometry is such a hard task to complete is because professionals in the field cannot come to a consensus on the actual subject of geometry. Some basic ways that this can be fixed is to first explain the actual meaning of geometry, clarify Euclidean and non-Euclidean geometry, and whether or not other types of geometry should be included in a basic geometry course. Geometers, and teachers alike, should study different ways of analyzing geometry because in geometry, there can be many ways to solve a single problem. There also needs to be a change regarding the level at which geometry is taught, the quality of the education, and the amount of in-depth discussions within the class. In general, geometers believe that geometry success and overall understanding by students will increase if there is a set curriculum (Jones, Critical Issues in the Design of the School Geometry Curriculum, 2000).

One of the main goals of geometry is to learn problem solving skills, and the only way to do that is to simply practice problem solving; a student should do just that in a high school or a college geometry course. Ways to help instill problem solving skills in students are by the problems assigned. These problems are for students who are just starting out in a geometry sequence, as well as for students who are well versed in geometry. Recognition type problems can simply ask a student "How many triangles are in the following figure?"



Basic drill and algorithm practice problems call for students to simply apply given information into a formula that is already known from previous instruction. Common application problems can deal with Pythagorean Theorem applications, such as



where the student is given two lengths and would have to find the unknown length of the figure. Essentially, if the teacher finds a way to take the students' mind beyond mere textbook problems, then students will better understand the art of problem solving (Crowley, 1987).

An average geometry syllabus includes the essentials of the subject. First, students learn basic definitions, such as a line, line segment, point, and plane. After this is learned, students are introduced to basic logic skills including truth tables and conditional sentences. Following logic is a series of geometrical concepts including, but not limited to parallels, transversals, triangles, triangle inequalities, polygons, similarity, circles, area, surface area, and volume. However, this cannot be generalized to every introductory geometry class. There are some classes which focus solely on Euclidian geometry. Meanwhile, others spend time on Euclidian geometry while also familiarizing students with non Euclidian geometry as well.

Geometry at the University of Southern Mississippi

At the University of Southern Mississippi, all majors are required to take a minimum of three hours of mathematics. These classes include Quantitative Reasoning (MAT 100) (Quantitative Reasoning), College Algebra (MAT 101), or any other higher level mathematics class that is suitable for the student. Higher level mathematics classes include Plane Trigonometry (MAT 103), Brief Applied Calculus (MAT 102) and Calculus with Analytical Geometry (MAT 167). The classes' topics require the students to have an understanding of geometry. Plane Trigonometry incorporates many facets of geometry and Calculus involves analytical geometry. The only available geometry classes which focus solely on learning geometrical concepts are Introductory Geometry (MAT 370) and Modern Geometry (MAT 472) which require high level mathematics classes as prerequisites making it infeasible for students, other than mathematics majors, to partake in the class. Therefore, expanding a student's geometrical thinking at the University of Southern Mississippi is nearly impossible. Yes, geometry is an integral part of general Calculus and Algebra classes, but having a class solely devoted on geometry is important for a student's enhancement of spatial thinking.

As stated earlier, the University of Southern Mississippi offers two geometry courses, Introduction to Geometry and Modern Geometry. Introduction to Geometry requires a student to have taken Linear Algebra (MAT 326) and Discrete Mathematics (MAT 340), a class focused solely on proving mathematical theories. Not only is Euclidean geometry learned, but also students are exposed to non-Euclidean geometries. The exploration of these geometrical entities is in two and three dimensions while also focusing on proof writing skills. This class, however, is only available to those students who plan to teach geometry in a secondary education setting.

The Modern Geometry class offered is more in-depth. Just as the name suggests, modern geometry focuses on just that, modern geometry. The curriculum in place is based on projective or differential geometry, which is not covered in an introductory geometry course. Students in this class explore these topics of geometry heuristically, going a bit further than what is needed into the material, and analytically. Upon completing the requirements of this course, a student will have an extremely high level of geometrical thinking (Department, 2011).

Because geometry is required in secondary education, the majority of students enroll in this class in the tenth grade after taking an algebra class. After this introductory geometry class is completed, the student then completes his/her other required classes. If the student is not able to enroll in a general geometry class for college students, then he/she will not be able to reach the zenith of his/her mind's capability of logical/deductive thinking skills.

The Van Hiele Test

Dutch mathematicians Dina van Hiele-Geldof and Pierre van Hiele developed the idea of constructing a model of various levels of geometrical understanding. It is comprised of the following five levels.

Level 1: Basic Visualization

At this level, geometric concepts are seen as complete subjects within themselves, not a part of a greater body. Figures and polygons are recognized by their physical features rather than specific properties.

Level 2: Analysis

Students begin to learn the specific properties that were not known in level 1; therefore, they are able characterize shapes by their properties and parts.

Level 3: Abstractions

Students have learned basic definitions and can see relationships of properties of different figures. Most of all, the students are able to follow informal and formal proofs of geometrical concepts, but they do not know how to formulate these proofs individually.

Level 4: Deduction

Students can finally construct proofs by the information they have previously learned.

Level 5: Rigor

Students are capable of working in Euclidian and non-Euclidian geometry spaces. There is some controversy to the existence of a level 0. The van Hiele's do not speak on this existence; they merely start at the basic level of visualization. Level 0, however, can exist if a student does not understand the concept of the basic visualization stage (Crowley, 1987).

It has been studied that there are jumps in a learning curve because the process of learning is discontinuous. These "jumps" in learning are indicators of "levels." Hence, there are some properties that go along with these levels. First, each level must be conquered sequentially, and it is imperative for a student to actually advance to the next level. This is important because each level has its own language and symbols. Also, things learned in one level are the objects to be studied in the next; this is why passage of levels must be sequential. Something may be implicit at one level and explicit in the next. Consequently, a student may have to defend something that he/she was told to be true from one level to the next. Finally, if a student has a teacher who is not teaching on the same van Hiele level as him/her, advancement to the next level will not occur (Fuys, Geddes, & Tischler, 1988).

There are many activities that can be used in the classroom that are level-appropriate. For the visualization stage, students can simply be asked to create common shapes by folding and cutting construction paper. They can also be asked to create a copy of another shape, maybe a common object in the classroom. At the analysis stage, it would greatly benefit students to identify a shape for either visual clues or a list of given properties. Students who are able to informally deduce should practice providing more than one explanation to a problem, proof, or idea. On the other hand, students who are able to formally deduce should practice constructing proofs where the process, or answer, is not obvious. This is especially where critical thinking and problem solving come into play (Crowley, 1987).

First and foremost, the van Hiele's constructed this model to face the difficulties students encountered in secondary school geometry; they thought that if this model was followed through, students would perform better in school. They understood that secondary school geometry called for students to think on a level in which they had no previous practice or experience. The instruction of the teacher is what helps the student to enhance his/her level of thinking. Therefore, Dina van Hiele-Geldof created a didactic experiment to increase a student's level of thinking while Pierre created the structure of the levels, involving the thoughts and principles which will help students fully understand the concept of geometry (Fuys, Geddes, & Tischler, 1988).

Many educators use difficulty level, or discrimination index, to measure their tests. The formula for difficulty level is given as follows

$$D = (U_p - L_p)/U$$

where D is the difficulty level, U_p is the number of students who placed in the top half of test takers who answered a question correctly, L_p is the number of students who placed in the lower half of test takers who answered a question incorrectly, and U is the total number of test takers. When using this index, teachers analyze a single problem at a time to determine a problem's difficulty level. If more students in the upper half answer a problem correctly, the difficulty level increases. The difficulty level decreases if more students in the lower half answer a problem correctly. The higher the difficulty level, the harder the question. Up until Zalman Usiskin created a test to place students on these van Hiele levels, there was no set way to determine what level a student was on. Levels 1, 2, and 3 are easily tested, but levels 4 and 5 had questionable reliability. Van Hiele began to disregard levels 4 and 5 because of their unreliability. Additionally, the difficulty levels from 1 to 5 are not increasing. Levels 1 to 5 difficulty levels are -1.81, -0.32, 0.59, 1.07, and 0.46 respectively. Level five's difficulty level should be substantially higher given the difficulty of the previous levels (Wilson, 1990). Some geometers, however, disagree with van Hiele because that leaves the highest level of geometrical thinking at a level 3 which they believe to be entirely too simplistic. Usiskin has been the prominent force behind the rising popularity of the van Hiele test (Wilson, 1990).

Zalman Usiskin and the van Hiele Test

The van Hiele test, created by Usiskin from the University of Chicago which is based on the van Hiele model of geometric thinking, has been more widely used than expected. This test, however, was not written to determine levels. "A system that assigns levels to students solely on the basis of items with predetermined difficulty, is no theory at all" (Usiskin & Senk, Evaluating a Test of van Hiele Levels: A Response to Crowley and Wilson, 1990). The test items were written to correspond directly to the van Hiele characteristics for each level and what the student should be able to comprehend and perform. This is the sole purpose of the test. It is not the student's level that matters, but how he/she is able to think geometrically (Usiskin & Senk, Evaluating a Test of van Hiele Levels: A Response to Crowley and Wilson, 1990).

There are two ways to evaluate such a test. The test is composed of twenty five questions. Each level is divided into five equal sections. Level one spans questions one through five, level two spans questions 6-10, and so forth. A student is then scored on how well he/she performed in a certain bracket. Type I error allows a student to pass a level if he/she gets 4 out of 5 questions correct in that section. Type II error allows a student to pass if he/she gets 3 out of 5 questions correct. Once he/she does not pass a section, the other sections are not calculated. It must be recognized that if the criteria change, students' levels will also change as a result (Usiskin & Senk, Evaluating a Test of van Hiele Levels: A Response to Crowley and Wilson, 1990).

Zalman Usiskin, used the van Hiele model to develop a test for the placement of geometry knowledge. Usiskin, a world renowned geometer, knew of the dilemma with the curriculum, or lack thereof, for high school geometry. He attests that the problem starts when students are taught at a higher level than they understand. Therefore, he underwent a great task; Usiskin performed a research study to assess the progression of van Hiele levels in a one year geometry class.

Usiskin started by studying how long it takes to excel from one level to another via proper instruction. A study performed a couple of years after the van Hiele model was introduced said that it takes approximately half a year to advance from a level 1 to a level 3 (Fuys, Geddes, & Tischler, 1988).

The population for the study included all high school geometry students enrolled in a one year geometry class. The sample class included 13 schools consisting of 2699 students. Schools were not completely selected at random; they were selected based on certain socio-economic criteria and if the results obtained would be reliable based on certain aspects of the school and the students enrolled in the school. Care was taken to ensure diversity was present. Students came from different backgrounds, as well as different socioeconomic statuses. Though most students enrolled in high school geometry are in 10^{th} grade, students' grade levels ranged from 7^{th} to 12^{th}.

Usiskin used a pretest and posttest for his study. During the first week of school, not only did he administer a van Hiele test, but he also used an Entering Geometry Test. Towards the end of the school year, Usiskin administered a geometry proof test, the Comprehensive Assessment Program Geometry Test (CAP), and the van Hiele test again. This is the study where Usiskin first developed the van Hiele test from the van Hiele model of geometric thinking. The van Hiele test was administered in a 35 minute time frame, allowing for approximately 1.4 minutes per question. To accurately determine on what van Hiele level a student is performing, the student must get at least 3 out of 5 questions correct in that specific van Hiele level bracket.

Usiskin found that approximately 34% of the students entered performing on level 0 having little to no geometric thinking. He also found that if a student scored at a level k in the fall semester, by the spring semester, they will be on a level k+1. By the end of the spring semester, about 40% of student fell between levels 0 and 2. In this study, Usiskin neither made a correlation between the students' grade in the class, nor did he use non-identifying devices for the students (Usiskin, Van Hiele Levels and Achievement in Secondary School Geometry, 1982).

The van Hiele Test Used in College

A study was done at Kenyon College involving van Hiele levels and college students. The purpose of the study was to examine the effects of different levels of college mathematics courses on college students. The researcher wanted to observe if there were any differences in the van Hiele levels of students who had taken non-geometry logic based classes and those who had not. These groups were denoted as category I and II respectively. The study consisted of 149 students from varying areas of study. At the conclusion of the study, it was seen that students taking logic-based mathematics courses obtained higher van Hiele levels than those taking other mathematics courses, such as calculus and algebra. This study proved that a student's background in logic can be an indicator of how well they may perform on the van Hiele test (Aydin & Halat, 2009).

Methodology

Participants

In this study, the researcher used previously gathered information. The participants in the study were not a true random sample; these were the students available. Forty students were enrolled in Mathematics for Elementary Teachers III at the University of Southern Mississippi. These students have previously taken Mathematics for Elementary Teachers I and II where geometry is taught.

Data Sources

The teacher gave the students a geometry test called the Van Hiele Geometry Test. This test was administered in a single class period allowing the students 1 hour and 15 minutes to complete it. The Van Hiele Geometry Test consists of 25 multiple choice geometry questions. The test was given as a pretest at the beginning of the semester. Due to limitations, the researcher was not able to administer a post-test. This is a limiting factor on the conclusions of the study. At the end of the semester, each student's final exam grade is calculated. The final exam contains questions ranging from levels 1 to 3 of geometrical thinking. The identities of the students are unknown to the researcher. It is imperative not to generalize from this study. Possible ways to strengthen the study are discussed at the end.

Test Scoring Guide

In this study, the 1-5 level indicators were used. This allows the researcher to use a level 0 for students who do not master the basic level 1 of the van Hiele test. All of the participants' answer sheets were scored by the researcher and placed on an individual specialized level. Type II error will be used in calculating levels allowing students to answer 3/5 questions correct for master of that specific level.

The grading system used was on a ten point scale. An ‘A’ ranged from 90-100, ‘B’ ranged from 80-89, and so forth. Each student who completed the course has a grade of either A, B, C, D, or F.

Analysis of Data

The data consists of responses from the van Hiele tests at the beginning of the class and final exam grades in the course. The researcher calculated a Spearman correlation from the two sets of data to compare the two sets of data. A Spearman correlation was used because of its nonparametric properties. A perfect Spearman correlation is a result of two entities compared by any monotonic function, a function that is increasing or decreasing over its entire domain. It does not assume that the parameters used are of equal value, the values for the grades given. This correlation measures how closely related the two sets of data are; a linear relationship is not guaranteed. The Pearson correlation does guarantee a linear relationship.

Results

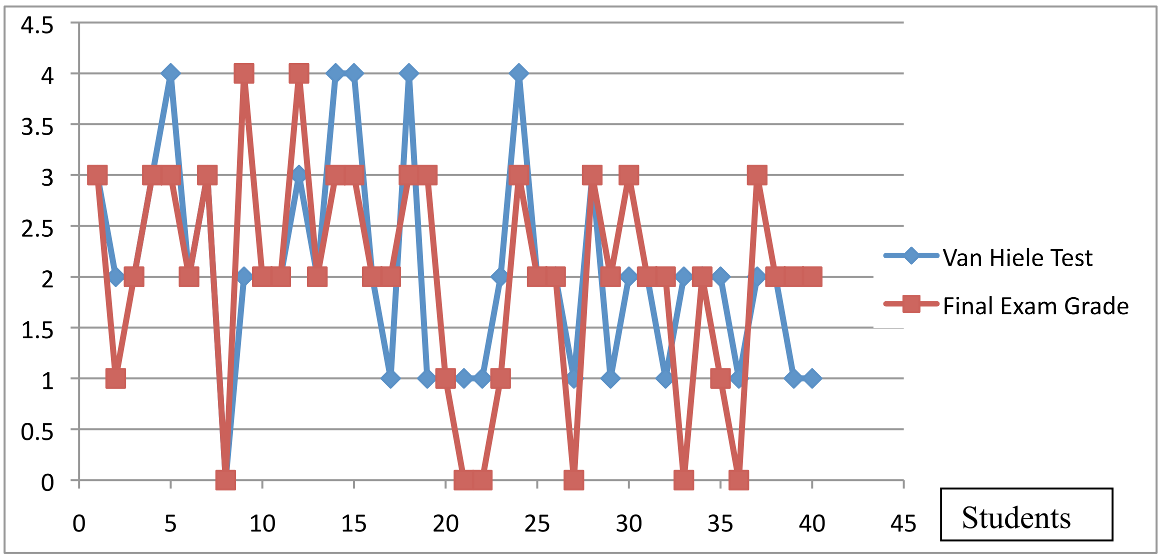
The researcher used a previously administered van Hiele test as a pretest for students enrolled in Mathematics for Elementary Teachers (MAT 310, Spring 2011) at the University of Southern Mississippi. The following table shows the student's van Hiele level and final exam grade in the class.

Table 1: Van Hiele Levels and Final Exam Grades

Van Hiele Level	Final Exam	Van Hiele Level	Final Exam
3	B	1	F
2	D	1	F
2	C	2	D
3	B	4	B
4	B	2	C
2	C	2	C
3	B	1	F
0	F	3	B
2	A	1	D
2	C	2	C
2	C	2	D
3	A	1	D
2	C	2	F
4	B	2	C
4	B	2	D
2	C	1	F
1	C	2	B
4	B	2	D
1	B	1	D
1	D	1	D

Table 3: Chart of Results

The following is a chart of the students' final exam grades and Van Hiele levels. Final exam grades of an A are noted as a "4", B as a "3", C as a "2", D as a "1", and F as a "0."



A Spearman coefficient was calculated to compare these two fields. The results are illustrated in Tables 4 and 5. This coefficient was calculated to be 0.742.

Table 4: Frequency Table

Level

	Frequency	Percent	Valid Percent	Cumulative Percent
Valid .00	1	2.5	2.5	2.5
1.00	11	27.5	27.5	30.0
2.00	18	45.0	45.0	75.0
3.00	5	12.5	12.5	87.5
4.00	5	12.5	12.5	100.00
Total	40	100.00	100.00	

Grade

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	.00 F	6	15.0	15.0	15.0
	1.00 D	10	25.0	25.0	40.0
	2.00 C	11	27.5	27.5	67.5
	3.00 B	11	37.5	27.5	95.0
	4.00 A	2	5.0	5.0	100.00
	Total	40	100.0	100.0	

Table 5: Nonparametric Correlations

Correlations

			Level	Grade
Spearman's rho	Level	Correlation Coefficient	1.000	.742**
		Sig. (2-tailed)		.000
		N	40	40
	Grade	Correlation Coefficient	.742**	1.00
		Sig. (2-tailed)	.000	
		N	40	40

**Correlation is significant at the 0.01 level (2-tailed).

Discussion

Distribution of Scores

The majority of students enrolled in this class scored on a van Hiele level of two or one, 45.0% and 27.5% respectively. In order to be accepted to the University of Southern Mississippi, each student must have taken a geometry course in high school. According to previous research (Crowley, 1987), the majority of high school geometry classes are taught on a level 3 of geometry thinking and upon exiting, students should master this stage. Only 12.5% of students performed on a level four. According to Usiskin's test corresponding to the van Hiele's model of geometrical thinking, these students were successful at constructing proofs individually. Students who performed on a level one are only capable of recognizing shapes as they appear; they are not known by their properties. Level two students begin to understand the different shapes have different properties. Because the class's levels of geometric thinking are spread out, the teacher was compelled to teach at the lowest level possible in order to academically reach all students.

The question is why these students are performing at such a low level of geometrical thinking. One student was not even able to think on the first level. The majority of students who enroll in geometry in high school take it in the tenth grade. The students in this specific mathematics class are, for the majority, sophomores or juniors. A plethora of students in college are not introduced to geometry again after high school. These students, however, have used geometrical skills in Mathematics for Elementary Teacher I and II. Therefore, it is nearly unexplainable as to why these students scored so low. There are, however, some suggestions. This van Hiele test was given at the beginning of the semester. At this time, the students' work may not be of the highest quality. Many students, at this time, are not yet focused on school. Therefore, the students' scores can depict this. Not only may students not be focused, but they may not have been worried about scoring high on the test because it was not a grade for the class.

Mathematics for Elementary Teachers III Curriculum

This class prepares future teachers to teach mathematics with an emphasis in geometry. Set up to provide an understanding of basic geometry, this course develops students' problem solving, critical, creative, quantitative, spatial and logical skills. Topics discussed are 2-dimensional and 3-dimensional geometry, motion geometry, and measurement. Logic is introduced in the class, but it is not required for students to be able to formally write proofs. Although, because of limitations, the researcher was not able to administer a posttest, it should be noted that if students were successful in the class, they should have at least performed on a level 3 of geometrical thinking.

Correlation

The Spearman correlation used for this research study is on a scale of 0.00 to 1.00. A perfect correlation coefficient is 1.00, and no correlation has a coefficient of 0.00. A low correlation coefficient ranges form 0.00-0.29, 0.30-0.59 is a moderate coefficient and a high coefficient ranges from 0.60-1.00. These numbers represent how closely related the two entities are that are being compared.

The correlation found in this study reported that there was a high correlation between the students' van Hiele level recorded at the beginning of the semester and their final exam grade; this correlation was 0.742. Because the students were matched to their respective grades, the results were matched as well.

Can the van Hiele Test Predict a Student's Success in a Class Based on Geometry Principles?

The researcher began to ask the question as to whether the van Hiele test, when taken as a pretest for a class heavily based on geometry, can be an effective predictor of a student's success in such class. From the results that the researcher found is this test can be such a predictor with 74.2% accuracy. This result is in favor of the researcher's hypothesis.

Limitations and Suggestions

For this study, only a pretest was given to the students. The researcher knew the van Hiele levels of the students at the start of the class. The only data located at the end of the class were the final exam grades. Because there was not a van Hiele test at the end of the class, there cannot be a correlation with pretest and posttest, or posttest and final exam grades. The researcher used previously administered data. The van Hiele test was initially used for the teacher's personal knowledge. Because of the already gathered data, this was most feasible for the researcher to use. If this study were to be retested, a suggestion would be to use a van Hiele test as a posttest as well to strengthen the results.

Conclusion

From this study, it can be concluded that a student's level of geometric thinking at the beginning of the semester is a major factor of a student's ability to perform well in that particular class. Because this study had limitations, it is suggested that if retested, a pretest, posttest, and final exam grade be recorded. This would provide more information to the researcher and greater validate these results.

Appendix:

Mathematics for Elementary Teachers III
MAT 310 SYLLABUS
Spring 2011

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Southern Hall 204

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Office Hours: MW 1:00-2:00
TTH 8:00-9:00
or by appointment

Prerequisites: C or better in both MAT 210 - (Mathematics for Elementary Teachers I) and MAT 101 (College Algebra)

Number of hours: 3

Nature of course: required for elementary licensure

Nature of students: undergraduate

Course generally offered: every semester

Format of course: lecture with small group activities and manipulative use

Required Textbook: Long, Calvin T. and DeTemple, Duane W. Mathematical Reasoning for Elementary Teachers, Custom ed. for USM from the 5th edition. Pearson Education, Inc. 2009.

Course description: This course is designed to turn students into practitioners of elementary mathematics with an emphasis in geometry. This course will develop an understanding of basic geometry. Problem solving will permeate the study of all concepts so that students develop the ability to think critically and creatively about quantitative, spatial and logical situations. Manipulatives and technology will be used to demonstrate and develop abstract concepts in order to meet the diverse learning styles and backgrounds of students. Students will build on skills learned in prerequisite courses, and to make connections between different topics. Students will also reflect on their learning and continually modify their learning processes to ensure that all students reach their maximum potential.

Catalog Description: The main topics discussed in MAT 310 are the basic concepts of 2-dimensional and 3-dimensional geometry, motion geometry, and measurement. (Open only to elementary and special education majors.)

Relationship of this course to the curriculum/program sequence: MAT 310 is the third in the Mathematics for Elementary teacher series. It is recommended that the courses are taken in order but students can take MAT 310 prior to taking MAT 309, Math for Elementary Teacher II.

Conceptual Framework: This course is one in a series of courses offered by the Mathematics Department that addresses the USM Professional Education Faculty Unit Vision of ***Freeing the Power of the Individual*** which includes the *Power of Knowledge to Inform, the Power to Inspire, the Power to Transform Lives, and to Empower a Community of Learners*.

Course Objectives: At the end of this course students will be able to:

1. Identify, describe, compare, and classify geometric figures.
2. Visualize and represent geometric figures with special attention to developing spatial sense.
3. Interpret and draw three-dimensional objects.
4. Represent and solve problems using geometric models.
5. Understand and apply geometric properties and relationships.
6. Extend their understanding of the process of measurement.
7. Understand the structure and use of systems of measurement.
8. Extend their understanding of the concepts of perimeter, area, volume, angle measure, capacity, and weight and mass.
9. Classify figures in terms of congruence and similarity and apply these relationships.

Teaching Strategies: This course will be taught using an inquiry-based approach in a laboratory setting with appropriate manipulatives and technology. Activities will be planned to provide the pre-service teachers an opportunity to explore and experiment. In-class teaching strategies used to accomplish the objectives of the course include guided teaching, whole group discussion, modeling of teaching methods, small group cooperative learning activities, student projects, presentations, individual reflections and individual writing assignments. Underlying the use of the varied teaching strategies is the need to address the diverse learning styles of all students.

Major Topics Covered:

1. Geometric figures and their properties to include figures in the plane, curves and polygons in the plane, and figures in space.

2. The measurement process, area, perimeter, the Pythagorean Theorem, surface area and volume.
3. Congruence, similarity and constructions.
4. Logic.

Course Policies:

1. Class attendance is very important. Many of the activities in class will be group activities involving manipulative materials that you will not have access to outside of class. Thus, there is an attendance/participation policy in this class. Each student is given thirty (30) points for attendance. This is 4.6% of your final grade and can easily make the difference in a letter grade. You will be allowed to miss one (1) day without penalty after which ten points will be deducted for each class period you miss for any reason, EXCUSED or UNEXCUSED. If you are tardy, it is **YOUR** responsibility to get the "absence" removed **THAT SAME DAY.** Otherwise, the absence remains.

2. Students should bring textbook, a straightedge, a 6-inch ruler marked in millimeters, a compass, a protractor, a calculator and various colored pens/pencils.

3. Students will not be allowed to drop this class after **Monday, February 28, 2011,** the last day to drop a class without academic penalty, unless the student is withdrawing from the university (all classes). The mathematics department strictly adheres to this policy.

4. It is expected that all assignments will be turned in on time without exception. LATE HOME WORK ASSIGNMENTS WILL NOT BE ACCEPTED. Any other assignment turned in late will be penalized by a reduction of 10% of the possible point value for each day it is late.

5. You are expected to take your tests on the day it is scheduled. NO-MAKE-UP TESTS will be given. If there is an emergency, you must contact me before the test is given to make arrangements for taking the test at an earlier time.

6. **MML homework.** You will have opportunities to use technology as a tool to complete assignments in this course. These technologies include use of the graphing calculator, interactive websites, and online homework in MyMathLab (MML). The MML course site will be used as a **course supplement** for this class. You should go to www.coursecompass.com, register, and add this course. Follow the steps outlined on the attached MML handout. In addition to providing online homework, important information will be conveyed to you through MML. You should communicate with your instructor and classmates via the MML mail tool.

Assignments:

1. **Homework**. Mathematics is NOT a spectator sport. You must do and understand your homework in order to do well in this class. I do include homework problems or problems

similar to homework problems on tests. Part of homework problems will be found online in MyMathLab (MML). You will need a **student access code** in order to enroll in MML. Directions for enrolling in MML will be given on a separate handout. Homework completed in MML will be included in the final grade and will be worth a possible 20 points. In addition to the MML homework, there will be homework problems assigned out of the textbook. These problems will be collected and graded for completion and correctness. The grade for the assignments out of the book will be averaged and will be worth a total of 50 points in the final grade. If your questions about homework are not answered in class, feel free to come by my office or the College of Science and Technology Learning Center (TEC 104) or Student Support Services (SSS) Program (601-266-6910) for additional help.

2. Dictionary. There will be a dictionary of terms that will need to be completed by Tuesday February 15th. The instructions are included on a separate handout. The dictionary will be worth 50 points.

3. There will be three tests during the semester that will count 100 points each.

4. The date for the final exam is Thursday, May 12th from 8:00-10:30. The final exam will be comprehensive and is worth 200 points.

Evaluation: The grades will be given according to a ten percentage points grading scale.

GRADE	TOTAL POINTS
A	585-650
B	520-584
C	455-519
F	0-454

The points will be earned in the following way:

Test 1	100
Test 2	100
Test 3	100
Final Exam	200
Homework	70
Dictionary	50
Attendance	30
Total possible	650

In order to receive a passing grade in this class, a student must score 70 or higher on at least one of the three in-class tests given during the semester. On the other hand, if a student scores 70 or higher on one of the in-class tests but fails to accumulate 70% of the total points, the student will not receive a passing grade.

<u>Academic honesty</u>: The following is from The University of Southern Mississippi Undergraduate Bulletin:

"When cheating is discovered, the faculty member may give the student an F on the work involved or in the course. If further disciplinary action is deemed appropriate, the student should be reported to the Dean of Students.

In addition to being a violation of academic honesty, cheating violates the Code of Student Conduct and may be grounds for probation, suspension, and/or expulsion. Students on disciplinary suspension may not enroll in any courses offered by The University of Southern Mississippi."

Students should understand that if they do not uphold the standards of academic honesty, the instructor will enforce all applicable punishment.

<u>Disabilities</u>: If a student has a disability that qualifies under the American with Disabilities Act (ADA) and requires accommodations, he/she should contact the Office for Disability Accommodations (ODA) for information on appropriate policies and procedures. Disabilities covered by ADA may include learning, psychiatric, physical disabilities, or chronic health disorders. Students can contact ODA if they are not certain whether a medical condition/disability qualifies. Address: The University of Southern Mississippi, Office for Disability Accommodations, 118 College Drive # 8586, Hattiesburg, MS 39406-0001. Voice Telephone: (601) 266-5024 or (228) 214-3232. Fax: (601) 266-6035. Individuals with hearing impairments can contact ODA using the Mississippi Relay Service at 1-800-582-2233 (TTY) or email Suzy Hebert at Suzanne.Hebert@usm.edu.

Van Hiele Test

Directions:

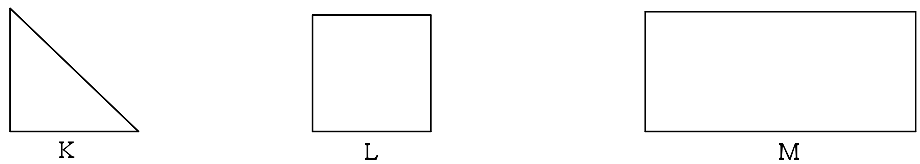
This test contains 25 questions. It is not expected that you know everything on this test. Please answer each question to the best of your ability.

When you are told to begin:

1. Read each question carefully.
2. Decide upon the answer you think is correct. There is only one correct answer to each question. Carefully circle in the correct answer on the answer sheet.
3. If you want to change an answer, please make sure you completely erase the first answer.
4. Use the scrap paper for finding the solution to a problem. DO NOT mark in the test booklet.
5. Use only a pencil to mark your answer on the answer sheet. If you need a new pencil, please raise your hand.

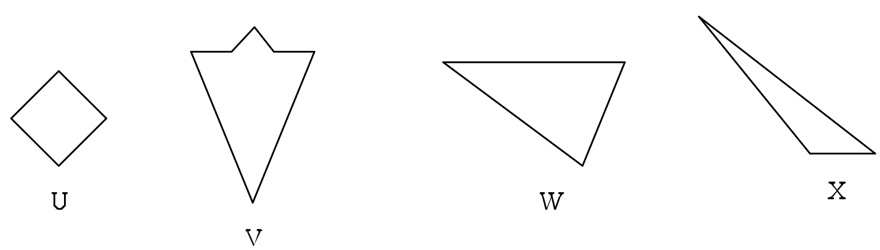
Van Hiele Geometry Test

1. Which of these are squares?



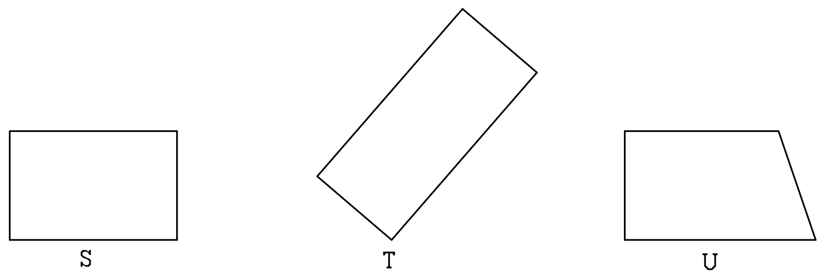
(A) K only
(B) L only
(C) M only
(D) L and M only
(E) All are squares

2. Which of these are triangles?



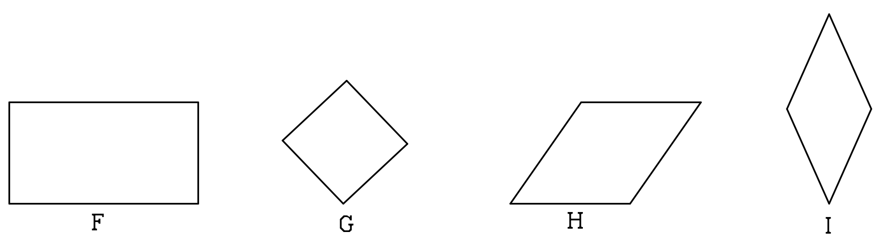
(A) None of these are triangles.
(B) V only
(C) W only
(D) W and X only
(E) V and W only

3. Which of these are rectangles?



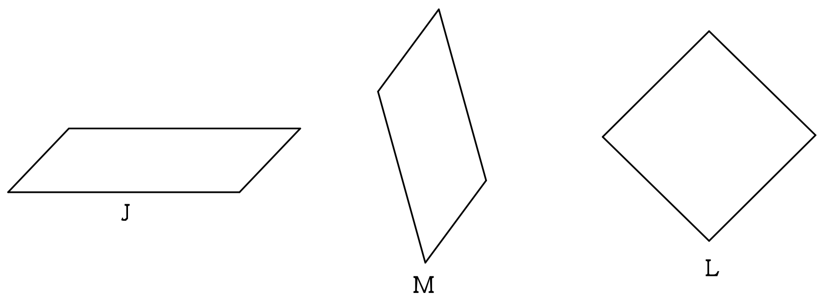
(A) S only
(B) T only
(C) S and T only
(D) S and U only
(E) All are rectangles

4. Which of these are squares?



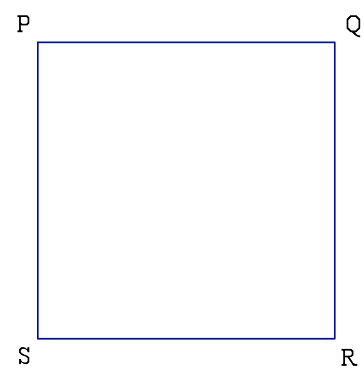
(A) None of these are squares.
(B) G only
(C) F and G only
(D) G and I only
(E) All are squares

5. Which of these are parallelograms?



(A) J only
(B) L only
(C) J and M only
(D) None of these are parallelograms
(E) All are parallelograms

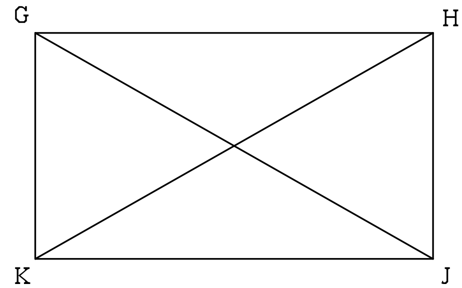
6. PQRS is a square



Which relationship is true in all squares?

(A) PR and RS have the same length.
(B) QS and PR are perpendicular.
(C) PS and QR are perpendicular.
(D) PS and QS have the same length.
(E) Angle Q is larger than angle R.

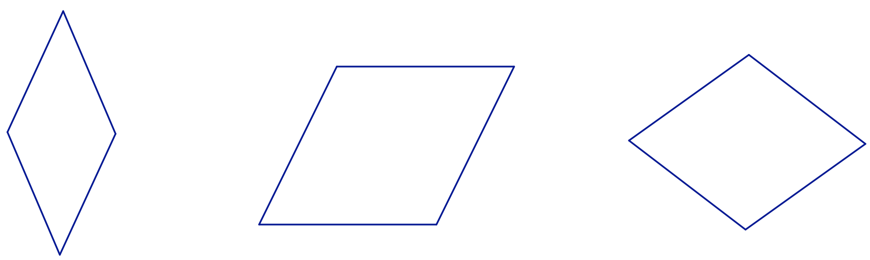
7. In the rectangle GHJK, GJ and HK are the diagonals.



Which of the (A) – (D) is not true in every rectangle?

(A) There are four right angles.
(B) There are four sides
(C) The diagonals have the same length.
(D) The opposite sides have the same length.
(E) All of (A) – (D) are true in every rectangle.

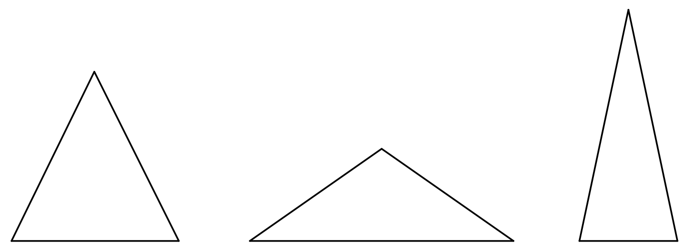
8. A rhombus is a four sided figure with all sides of the same length. Here are some examples.



Which of (A) – (D) is not true in every rhombus?

(A) The two diagonals have the same length.
(B) Each diagonal bisects two angles of the rhombus.
(C) The two diagonals are perpendicular.
(D) The opposite angles have the same measure.
(E) All of (A) – (D) are true in every rhombus.

9. An isosceles triangle is a triangle with two sides of equal length. Here are three examples.

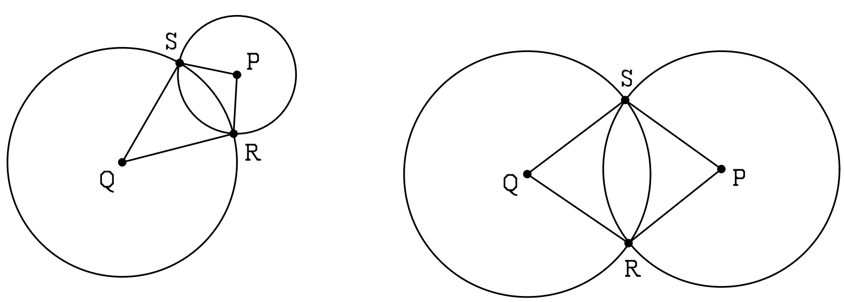


Which of (A) – (D) is true in every isosceles triangle?

(A) The three sides must have the same length.
(B) One side must have twice the length of another side.
(C) There must be at least two angles with the same measure.
(D) The three angles must have the same measure.
(E) None of (A) – (D) is true in every isosceles triangle.

10. Two circles with centers P and Q intersect at R and S to form a 4-sided figure PRQS.

Here are two examples.



Which of (A) – (D) is <u>not</u> always true?

(A) PRQS will always have two pair of sides of equal length.
(B) PRQS will have at least two angles of equal measure.
(C) PQ and RS will be perpendicular.
(D) Angles P and Q will have the same measure.

(E) All of (A) – (D) are true.

11. Here are two statements.

Statement 1: Figure F is a rectangle.

Statement 2: Figure F is a triangle.

Which is correct?

(A) If 1 is true, then 2 is true.
(B) If 1 is false, then 2 is true.
(C) 1 and 2 cannot both be true.
(D) 1 and 2 cannot both be false.
(E) None of (A) - (D) is correct.

12. Here are two statements.

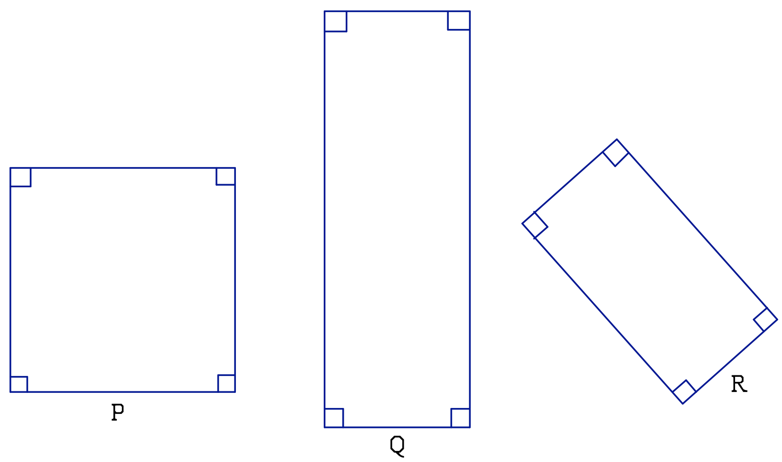
Statement S: ΔABC has three sides of the same length.

Statement T: In ΔABC, angle B and angle C have the same measure.

Which is correct?

(A) Statements S and T cannot both be true.
(B) If S is true, then T is true.
(C) If T is true, then S is true.
(D) If S is false, then T is false.
(E) None of (A) - (D) is correct.

13. Which of these can be called rectangles?



Which is correct?

(A) All can.
(B) Q only
(C) R only
(D) P and Q only
(E) Q and R only

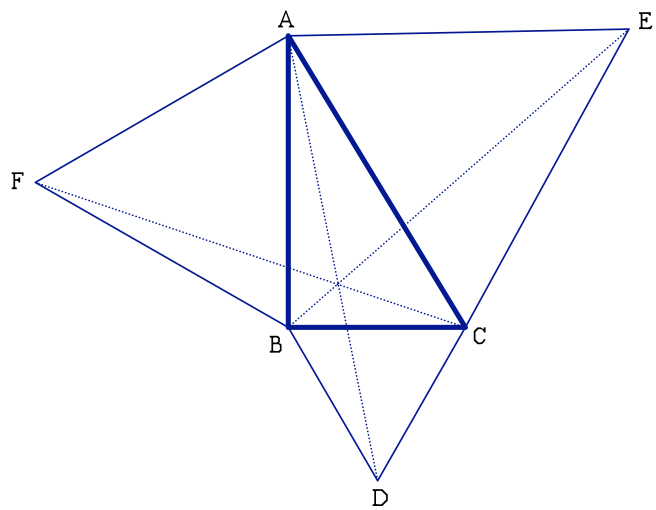
14. Which is true?

(A) All properties of rectangles are properties of all squares.
(B) All properties of squares are properties of all rectangles.
(C) All properties of rectangles are properties of all parallelograms.
(D) All properties of squares are properties of all parallelograms.
(E) None of (A) - (D) is true.

15. What do all rectangles have that some parallelograms do not have?

(A) Opposite sides equal.
(B) Diagonals equal.
(C) Opposite sides parallel.
(D) Opposite angles equal.
(E) None of (A) - (D).

16. Shown below is right triangle ABC. Equilateral triangles ACE, ABF, and BCD have been constructed on the sides of ABC.



From this information one can prove that □□, □□, and □□ have a point in common. What would this proof tell you?

(A) Only in this triangle drawn can we be sure that □□, □□, and □□ have a point in common.
(B) In some but not all right triangles, □□, □□, and □□ have a point in common.
(C) In any right triangle, □□, □□, and □□ have a point in common.
(D) In any triangle, □□, □□, and □□ have a point in common.
(E) In any equilateral triangle, □□, □□, and □□ have a point in common.

17. Here are three properties of a figure.

Property D: It has diagonals of equal length.

Property S: It is a square.

Property R: It is a rectangle.

Which is true?

(A) D implies S which implies R.
(B) D implies R which implies S.
(C) S implies R which implies D.
(D) R implies D which implies S.
(E) R implies S which implies D.

18. Here are two statements.

I: If a figure is a rectangle, then its diagonals bisect each other.

II: If the diagonals of a figure bisect each other, then the figure is a rectangle.

Which is correct?

(A) To prove I is true, it is enough to prove II is true.
(B) To prove II is true, it is enough to prove I is true.
(C) To prove II is true, it is enough to find one rectangle whose diagonals bisect each other.
(D) To prove II is false, it is enough to find one non-rectangle whose diagonals bisect each other.
(E) None of (A) - (D) is correct.

19. In geometry which of the following statements is correct.

(A) Every term can be defined and every true statement can be proved true.
(B) Every term can be defined but it is necessary to assume that certain statements are true.
(C) Some terms must be left undefined but every true statement can be proved true.
(D) Some terms must be left undefined and it is necessary to have some statements which are assumed true.

(E) None of (A) - (D) is correct.

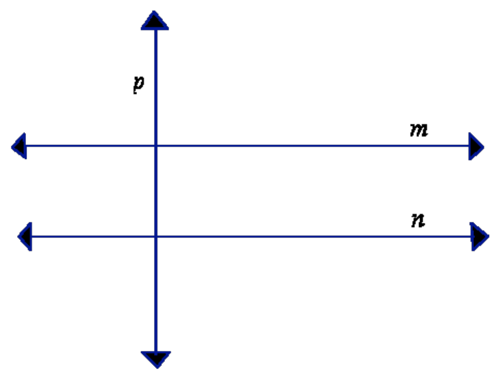
20. Examine these three sentences.

(1) Two lines perpendicular to the same line are parallel.

(2) A line that is perpendicular to one of two parallel lines is perpendicular to each other.

(3) If two lines are equidistant, then they are parallel.

In the figure below, it is given that lines *m* and *p* are perpendicular and lines *n* and *p* are perpendicular. Which of the above sentences could be the reason that line *m* is parallel to line *n*?



(A) (1) only
(B) (2) only
(C) (3) only
(D) Either (1) or (2)
(E) Either (2) or (3)

21. In F-geometry, one that is different from the one you are used to, there are exactly four points and six lines. Every line contains exactly two points. If the points are P, Q, R, and S, the lines are {P,Q}, {P,R}, {P,S}, {Q,R}, {Q,S}, and {R,S}.

Here are how the words "intersect" and "parallel" are used in F-geometry. The lines {P,Q} and {P,R} intersect at P because {P,Q} and {P,R} have point P in common. The lines {P,Q} and {R,S} are parallel because they have no points in common.

From this information, which is correct?

(A) {P,R} and {Q,S} intersect.
(B) {P,R} and {Q,S} are parallel.

(C) {Q,R} and {R,S} are parallel.
(D) {P,S} and {Q,R} intersect.
(E) None of (A) - (D) are correct.

22. To <u>trisect</u> an angle means to divide it into three parts of equal measure. In 1847, P.L. Wantzel proved that, in general, it is <u>impossible</u> to trisect angles using only a compass and an unmarked ruler. From this proof, what can you conclude?

(A) In general, it is impossible to <u>bisect</u> angles using only a compass and an unmarked ruler.
(B) In general, it is impossible to trisect angles using only a compass and a <u>marked</u> ruler.
(C) In general, it is impossible to trisect angles using any drawing instruments.
(D) It is still possible that in the future someone may find a general way to trisect angles using only a compass and an unmarked ruler.
(E) No one will ever be able to find a general method for trisecting angles using only a compass and an unmarked ruler.

23. There is a geometry invented by a mathematician J in which the following is true:

The sum of the measures of the angles of a triangle is less than 180°.

Which is correct?

(A) J made a mistake in measuring the angles of the triangle.
(B) J made a mistake in logical reasoning.
(C) J has a wrong idea of what is meant by “true”.
(D) J started with different assumptions than those in the usual geometry.
(E) None of (A) - (D) is correct.

24. Two geometry books define the word rectangle in different ways.

Which is true?

(A) One of the books has an error.
(B) One of the definitions is wrong. There cannot be two different definitions for a rectangle.
(C) The rectangles in one of the books must have different properties from those in the other book.
(D) The rectangles in one of the books must have the same properties as those in the other book.

(E) The properties of rectangles in the two books might be different.

25. Suppose you proved statements I and II.

I. If p, then q.

II. If s, then not q.

Which statement follows from statements I and II?

(A) If p, then s.
(B) If not p, then not q.
(C) If p or q, then s.
(D) If s, then not p.
(E) If not s, then p.

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