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Assessing Awareness, Interest, and Knowledge of Fractal Geometry among Secondary Mathematics Teachers in the United States and China

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ASSESSING AWARENESS, INTEREST, AND KNOWLEDGE OF FRACTAL GEOMETRY AMONG SECONDARY MATHEMATICS TEACHERS IN THE UNITED STATES AND CHINA

by

Suanrong Chen

Abstract of a Dissertation
Submitted to the Graduate School
of The University of Southern Mississippi
in Partial Fulfillment of the Requirements
for the Degree of Doctor of Philosophy

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ABSTRACT

ASSESSING AWARENESS, INTEREST, AND KNOWLEDGE OF FRACTAL GEOMETRY AMONG SECONDARY MATHEMATICS TEACHERS IN THE UNITED STATES AND CHINA

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Fractal geometry has gained great attention from mathematicians and scientists in the past three decades (Fraboni & Moller, 2008). As a new geometry language and subject, fractal geometry has significant value in teaching and learning secondary mathematics. The present study focused on investigating the current state of mathematics teachers' awareness, interest, and knowledge of fractal geometry in the United States (U.S.) and China, as well as the factors that influence them.

The instrument of the study included a survey and a test designed by the researcher and validated by five experts. The results of the study indicated that secondary math teachers in the U.S. and China had very low levels of awareness of fractals and lack the knowledge and skills of solving fractal problems, but they had a higher level of interest in fractals related to classroom teaching and professional development as compared with their levels of awareness. Furthermore, the results of this study indicated that the factor 'experience of learning fractals' had the most positive effect on the average score of awareness. The factor nationality (U.S.) had the most positive effect on the average score of interest. The factor nationality (U.S.) had the most negative effect on the average score of knowledge.

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DEDICATION

I dedicate this work to my parents and parents-in-law, father, Zhichu Chen; mother, Wanglian Sun; father-in-law, Kunyang Chen; and mother-in-law, Cuiying Sun.

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CHAPTER I

INTRODUCTION

Geometry has been developed and studied for more than two thousand years since Euclid, a Greek mathematician who lived in 300 BC, first developed [Euclidean] geometry. No doubt, the development of geometry has played a significant role in understanding, describing, and interacting with the space in which we live. In education, Euclidean geometry has been considered an important part of mathematics in K-12 and college in countries all over the world. Learning Euclidean geometry not only helps students to recognize the various man-made regular objects formed by line segments, angles, and planes in homes and offices, but also helps students to develop their logical thinking through learning its deductive and logical system.

However, Euclidean geometry does not explain the irregular shapes and objects that occur everywhere in the world. What are the structures of blood vessels, river networks, spirals, etc.? What is the length of a coast line? When math teachers are asked to answer these questions, Euclidean geometry does not help them give a clear explanation. These questions cannot be answered until mathematics teachers have a good grasp of a new geometry language and approach, which is named fractal geometry. Introduced by Benoit Mandelbrot (1977) about three decades ago (Vacc, 1999), it has become well established as a new subject in mathematics. Fractal geometry not only explains various natural phenomena that Euclidean geometry cannot deal with, but also provides a new way to think about geometry. The applications of fractal geometry can be found in many fields: art, astronomy, nature, computer science, fluid mechanics, telecommunications, surface physics, and medicine, among others.

Fractal geometry has attracted mathematicians and scientists because of its usefulness and wide ranging applications in many fields. In the education field, fractal geometry connects many mathematics concepts in the secondary math core curriculum (Common Core State Standards) and to other subject areas. Fractal geometry problems can be used as applications for many mathematics concepts in algebra and traditional geometry courses. Fractal problems can also be used for motivating students to conduct inquiry studies/active learning, inspiring students to discover novelties, and increasing interest in learning. In China, the topic "Coastal Lines and Fractal" has been listed as optional in China Mathematics Curriculum Standards (10-12) (2003). Optional topics are meant to broaden students' horizons, increase students' historical knowledge, and enlighten students' innovative consciousness. In cross-curriculum, fractals have been connected to the courses of music, paint, physics, chemistry, and other disciplines. For example, Padula (2005, 2009) illustrated how music teachers teach fractal geometry with music. Moreover, the development of technology tools provides more possibilities for exploring the dynamics of geometry and displaying the beauty of fractal art. Most importantly, fractal geometry changes the view and the way that students think about geometry and explains many natural phenomena that Euclidean geometry cannot explain.

Research has shown that some mathematics educators and teachers have explored how fractal geometry can be integrated into students' math learning even at the elementary level (Adams & Russ, 1992; Fraboni & Moller, 2008; Siegrist, Dover, & Piccolino, 2009; Vacc, 1999). However, the treasure of fractal geometry still has not been widely recognized and discovered in secondary mathematics education. This can be demonstrated by at least the following two facts. First, fractal geometry is not specifically incorporated into the Common Core State Standards for Mathematics (CCSSM) (2015) in the United States nor in many other countries. For example, Singapore Mathematics Curriculum Standards (2006), Korea Mathematics Curriculum Standards (n.d.), and England mathematics Curriculum Standards (2014). Second, since most math teachers have not learned fractal geometry, they may not know its values and may lack the ability to integrate fractal geometry into the math curriculum. Thus, most secondary mathematics teachers may not have the experiences of: 1) exploring how the math core curriculum supports teaching fractal geometry at secondary school level; 2) exploring how fractal geometry as a supplementary material influences the way students learn and think about geometry; 3) exploring how fractal geometry supports teaching mathematics from very basic concepts to the most advanced concepts; 4) exploring how fractal geometry supports learning across disciplines; and 5) exploring how fractal geometry inspires students' motivation, interest, and curiosity in learning mathematics.

The International Commission on Mathematical Instruction (ICMI) (1995) argued that the teaching of geometry must reflect the actual and potential needs of society and proposed the question "would it be possible and advisable also to include some elements of non-Euclidean geometries into curricula?" (p. 96). This indicates that incorporating non-Euclidean geometries will become a new direction, and then mathematics teachers will be required to equip themselves with knowledge of non-Euclidean geometry. The challenges of the teaching of geometry for the future would be rising. Do mathematics teachers' preparations prepare them for the challenges? Are mathematics teachers willing to be challenged and to learn new things in teaching geometry? No studies have answered these questions.

Since fractal geometry is not a required element in U.S. math core curriculum, there are no references about the state of awareness, interest, and knowledge of fractal geometry in secondary mathematics teachers at the present time. The National Curriculum Standards in China has incorporated fractal geometry as optional studies. Would the incorporation of these studies influence secondary mathematics teachers' awareness, interest, and knowledge of fractal geometry? Moreover, are there any differences between those who have educational experience of fractal geometry and those who have not? Educational experience may influence teachers' preparation in teaching fractal geometry. The answers to those questions are very crucial in deciding whether fractal geometry needs to be incorporated into pre- and in- service teachers' educational experience, and whether fractal geometry needs to be incorporated into the K-12 math curriculum.

Objectives of the Study

The purpose of this study is to investigate awareness, interest, and knowledge of fractal geometry in U.S. and Chinese secondary mathematics teachers, as well as any factors that affect teachers' awareness, interest, and knowledge of fractal geometry.

The specific objectives:

- To assess the awareness level of secondary mathematics teachers about fractal geometry.
- To assess the interest level of secondary mathematics teachers in fractal geometry and its integration in their classrooms.
- To assess the knowledge of secondary mathematics teachers in fractal geometry.

- To investigate whether there are differences between Chinese and American secondary mathematics teachers' awareness, interest, and knowledge of fractal geometry.
- To investigate the factors that may contribute to teachers' awareness, interest levels, and knowledge of fractal geometry.

Statement of the Problem

The development of a global economy highly connects the development and updating of knowledge, which challenges school education to conduct innovation in both curriculum development and instructional methodology. School geometry education should also keep a watchful eye on the future development of modern geometry when emphasizing the missions of traditional geometry. Fractal geometry, as a subject rising in prominence within mathematics proper and across scholarly and popular domains, has gained great attention from mathematicians and scientists (Davis & Sumara, 2000). Although fractals started to be incorporated in mathematics and science courses, mostly at the college level, and usually in courses on topics in geometry, physics, or computer science, fractal geometry has resonated with a wider audience, including secondary students and even elementary students (Mandelbrot & Michael, 2002). Research has shown the value of integrating fractal geometry into secondary math curricula (Adams & Russ, 1992; Fraboni & Moller, 2008; Siegrist et al., 2009; Vacc, 1999).

But fractal geometry is not widely acknowledged, appreciated, or even understood in school mathematics education because it has not been incorporated in the math core curriculum. Further, no investigation has been done on teachers' preparedness to integrate fractal geometry into the secondary core mathematics curriculum. This study is designed to build a foundation on assessing the secondary mathematics teachers' preparation in teaching fractal geometry by examining their awareness of the basic concepts of fractal geometry, interest in integrating fractal geometry into the core curriculum, and basic knowledge of fractal geometry through their performance on the test.

The survey will be used to gauge teachers' levels of awareness and interest. The test will be used to examine the knowledge of fractal geometry that secondary teachers currently have. This study will further examine the influence of various demographics on the level of awareness, interest and knowledge.

Research Questions and Hypotheses

Overarching Research Questions: What is the current state of the awareness of, interest in, and knowledge of fractal geometry in U.S. and Chinese secondary mathematics teachers and what factors affect the levels of their awareness, interest, and knowledge?

Specific Research Question One: What factors (nationality, gender, age, degree, the years of teaching math, grade levels, experience of learning fractals, and experience of integrating fractals) contribute the most and to what extent to teachers' awareness levels in fractal geometry?

- H_{a1}: The factors degree, grade levels, experience of learning fractals, and experience of integrating fractals will account for more variance than gender, age, the years of teaching math, and nationality in the overall mean awareness survey scores.

Specific Research Question Two: What factors (nationality, gender, age, degree, the years of teaching math, grade levels, experience of learning fractals, and experience of integrating fractals) contribute the most and to what extent to teachers' interest levels in integrating fractal geometry in the math core curriculum?

- H_{a2}: The factors degree, grade levels, experience of learning fractals, and experience of integrating fractals will account for more variance than gender, age, the years of teaching math, and nationality in the overall mean interest in integrating fractal geometry in the math core curriculum.

Specific Research Question Three: What factors (nationality, gender, age, degree, the years of teaching math, grade levels, experience of learning fractals, and experience of integrating fractals) contribute the most and to what extent to teachers' knowledge of test scores in performing fractal geometry problems?

- H_{a3}: The factors nationality, degree, experience of learning fractals, and experience of integrating fractals will account for more variance than gender, age, grade levels, and the years of teaching math in the overall mean knowledge test scores in performing fractal geometry problems.

Significance of the Study

The results of this study can be an important reference for curriculum designers: K-12 mathematics education designers, the geometry course designers for pre-service teachers, and the professional development designers for in-service teachers. It is crucial to know the current state of the awareness, interest, and knowledge of fractal geometry that secondary mathematics teachers have so that curriculum designers and teacher educators can rethink whether it is necessary to incorporate fractal geometry into the core math curriculum and math teachers' educational experience. No doubt, what should be taught is a daily question that mathematics teachers must ask. But what should be taught is first determined by what teachers have been taught or learned. This study will open a door for all mathematics educators, teachers, and other curriculum designers to consider redesigning math education with fractal geometry in schools.

Definition of Terms

The following terms are defined as they are used in this study.

- Core Curriculum - In this study, the Core Curriculum refers to the Common Core State Standard in the United States and the National Curriculum Standards in China.
- Awareness - In the context of this study, awareness means the state of being aware of the basic concepts of fractal geometry.
- Interest - In the context of this study, interest is specified as how much interest secondary mathematics teachers have in integrating fractal geometry into the core curriculum.
- Basic Concepts of Fractal Geometry – In this study, there are basic fundamental concepts that form the basis for the survey instrument (Appendix A) developed by the researcher and validated by three mathematics professors. The essential concepts in fractal geometry include geometric transformation, geometric sequences, similar figures, geometric iteration, self-similarity, fractal dimension, and mathematical fractal.
- Fundamental Knowledge of Fractal Geometry - In this study, there is fundamental knowledge forming the basis for the test instrument (Appendix B) developed by

the researcher to assess teachers' mathematical skills in performing problems in fractal geometry.

Delimitations

The results of this study were limited to the particular secondary mathematics teachers who were in-service teachers teaching 6th grade through 12th grade in the 2014-2015 academic year in the greater metropolitan area of Hattiesburg, MS, and the greater metropolitan area of Shanghai, China. The participants were asked to complete the awareness and interest survey and the basic knowledge test of fractal geometry within a given time.

Limitations and Discussion

This study was limited by the number of teachers and geographic regions of the countries where the teachers live, who could be contacted for inclusion in the study, and who responded. A potential limitation of this study included attitudes of teachers during completion of the survey and tests. Teachers had the potential of responding without thoroughly reading and thinking through each question.

Assumptions

The study assumed that the participants were representative of the populations in the U.S. and China. Another assumption was that participants would thoughtfully and accurately respond to the survey and seriously take the test.

Justification

The literature on fractal geometry has demonstrated its value in teaching and learning mathematics at the secondary level (for example, Jarry-Shore, 2013). Chinese National Curriculum Standards have incorporated fractal topics in optional studies (http://hrd.apec.org/images/2/29/54.3.pdf). Some U.S. mathematics college educators and secondary teachers have begun to integrate fractal geometry into the math core curriculum (Adams & Russ, 1992; Fraboni & Moller, 2008; Siegrist et al., 2009; Vacc, 1999). Incorporating fractal geometry is gaining attention from both countries. Further, the researcher has been living and studying in the U.S for six years. Prior to 2009, the researcher taught in Shanghai, China, for 13 years and worked as a teacher trainer for 7 of those years. Thus, the researcher has the potential resources to conduct this research.

The purpose of this study is to gain knowledge and information about the level of secondary mathematics teachers' awareness of the basic concepts of fractal geometry, their level of interest in integrating fractals into the math core curriculum, and their level of knowledge of fractal geometry. The study will also reveal information on factors that impact mathematics teachers' awareness, interest, and knowledge of fractal geometry.

The results of the study will be reported to the mathematics education communities through publications. It will provide important references for math educators to think about integrating fractals into pre-service teachers' math curriculum or in-service teachers' professional development. It will also indirectly benefit our students if math teachers gain the knowledge and abilities to integrate fractal geometry into the classrooms.

CHAPTER II

REVIEW OF RELATED LITERATURE

Introduction

What should be taught is an ongoing topic due to developments in technology and emerging knowledge in the world. Mathematics college educators and secondary teachers must also think about what they should bring to students so that our students can face future challenges in dealing with nature, science, and humanities when they enter higher education or society. Fractal geometry, as a very new subject, has shown its prominent applications in wide ranges of science, engineering, and many other fields, which, in turn, has attracted many mathematicians and scientists to work in the field (Fraboni, & Moller, 2008). The power, beauty, and complexity of fractal knowledge have also attracted some mathematics educators and teachers to study its various educational values in teaching and learning mathematics (Peitgen, Jϋgens, & Saupe, 1992). Does fractal geometry need to be incorporated in K-12 mathematics education? There is no agreement on it (Davis & Sumara, 2000). In China, some topics of fractal geometry have been incorporated into the optional curriculum of the National Curriculum Standards. Although the topics of fractal geometry have not been incorporated in the Common Core State Standards (2015) in the U.S., they have been studied as the applications of other mathematics subjects and cross subjects in the research community.

For fractal geometry to be included as a topic in secondary mathematics courses, mathematics teachers must have the knowledge and ability to integrate fractal geometry into the math core curriculum. Thus, it is very important to know the current level of mathematics teachers' awareness of the concepts of fractal geometry, the level of interest in integrating fractal geometry into the math core curriculum, and the knowledge that mathematics teachers have in performing fractal problems. As no other studies have done this, the present study conducted this task and built the corresponding references for the research community.

In light of the research objectives and the instruments being used in the present study, the literature review focused on the fundamental knowledge of fractal geometry, support from the math core curriculum, values of teaching and learning fractal geometry, necessity of integrating fractal geometry, and research on fractal geometry in K-12 mathematics education.

Fundamental Knowledge of Fractal Geometry

What is a fractal? According to Debnath (2006), a fractal is defined as “a geometrical curve that consists of an identical shape repeating on an ever decreasing scale” (p. 30). To put it simply, a fractal is formed by small copies of itself. The important characteristic of fractals is self-similarity. This characteristic distinguishes fractals from most conventional Euclidean figures and makes them attractive (Fraboni & Moller, 2008). Euclidean geometry, studied for more than two thousand years, cannot describe many phenomena of nature which are so irregular and complex; Euclidean geometry seems “cold” and “dry,” and needs to be refreshed. Benoit Mandelbrot was the first to recognize this point and first introduced the concept of fractals (Debnath, 2006). Fractal geometry is the geometry whose structures are the connection of order to chaos (Peitgen et al., 1992). Fractal geometry plays a significant role in bridging order and chaos and reveals a new, fantastic area of geometry. Peitgen and colleagues (1992) described that “Fractal geometry is first and foremost a new ‘language’ used to describe the complex forms found in nature" (p. viii). Fractal geometry contains several essential elements: fractals, chaos, bifurcations, and Hausdroff dimension (Debnath, 2006). Many classical examples, such as the Cantor set, the Sierpinsiki Gasket, the Koch Curve, etc., can well demonstrate the essential elements of fractal geometry (see Debnath 2006; Peitgen et al., 1992; Peitgen, Jügens, & Saupe, 2004). Next, this section will introduce some basic knowledge of fractals: the principle of fractals, self-similarity of fractals, and dimensions of fractals.

The Principle of Fractals

When one sees a beautiful and complex picture of fractals, one must be curious about the process that is used to form this incredible image. One should also think that the process must be complex due to the complexity of the picture. The surprise is that a very simple process is responsible for the complex pattern in the picture, which impacts fractal geometry and chaos theory. Three terms--integrator, feedback, and dynamic law--can be used to describe this process. Peitgen et al. (2004) made an effort to introduce the process. The basic and important idea was described by the Feedback Machine, with IU = input unit, CU = control unit, PU = processing unit, and OU = output unit (see Figure 1). The Feedback Machine contains four storage units (IU, CU, PU and OU), and one processor, which are collected by transmission lines. How does this feedback machine work? Peitgen and colleagues (2004) described the following process:

> The whole unit is run by a clock, which monitors the action in each component and counts cycles. The control unit acts like a gear shift in an engine. That is, we can shift the iterator into a particular state and then run the unit. There are

preparatory cycles and running cycles, each of which can be broken down into elementary steps:

Preparatory cycle:

Step 1: load information into IU

Step 2: load information into CU

Step 3: transmit the content of CU into PU

Running cycle:

Step 1: transmit content of IU and load into PU

Step 2: process the input from IU

Step 3: transmit the result and load into OU

Step 4: transmit the content from OU and load into IU

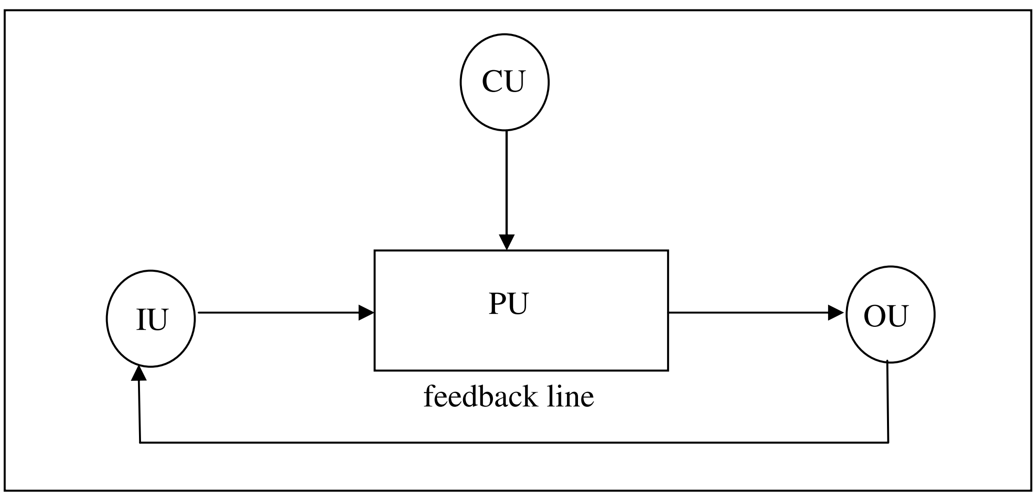


Figure 1. The Feedback Machine. Reprinted from "Chaos and Fractals." by Peitgen et al., 2004, p. 17.

To initiate the operation of the machine, we run one preparatory cycle. Then we start the running cycles and execute a certain number of them, the count of which may depend on observations which we make by monitoring the actual output. Execution of one running cycle is sometimes called iteration (Peitgen et al., 2004, pp. 17-18).

How does this Feedback Machine produce a beautiful picture of a fractal? Providing an example may help explain the process. An equilateral triangle (called the seed) is in the plane (IU). Pick the midpoints of its three sides, connect the midpoints, and drop the center triangle, which is one of the four congruent triangles defined by the old vertices of the original triangle and the three midpoints (CU). After loading information to IU and CU, transmit the content of CU to PU. We complete the preparatory cycle. Then, follow with the running cycle: transmit content of IU and load into PU, process the input from IU, transmit the result and load into OU (three congruent triangles, having half of the sides of the original triangle sides, are formed), and transmit the content from OU and load into IU. After the first execution of the running cycle, follow the same procedure with the three remaining triangles and repeatedly execute the running cycle, and a beautiful fractal picture will be formed. This process will produce the classical fractal, the Sierpinski Gasket, first introduced by the great Polish mathematician Waclaw Sierpinski (Peitgen et al., 2004). In explanation, each new triangle is scaled down by half, based on the former triangle. One triangle is transformed into three, three triangles are transformed into nine, nine triangles are transformed into twenty-seven, etc. The resulting triangles are similar and the numbers of them are increased by geometric growth (see Figure 2).

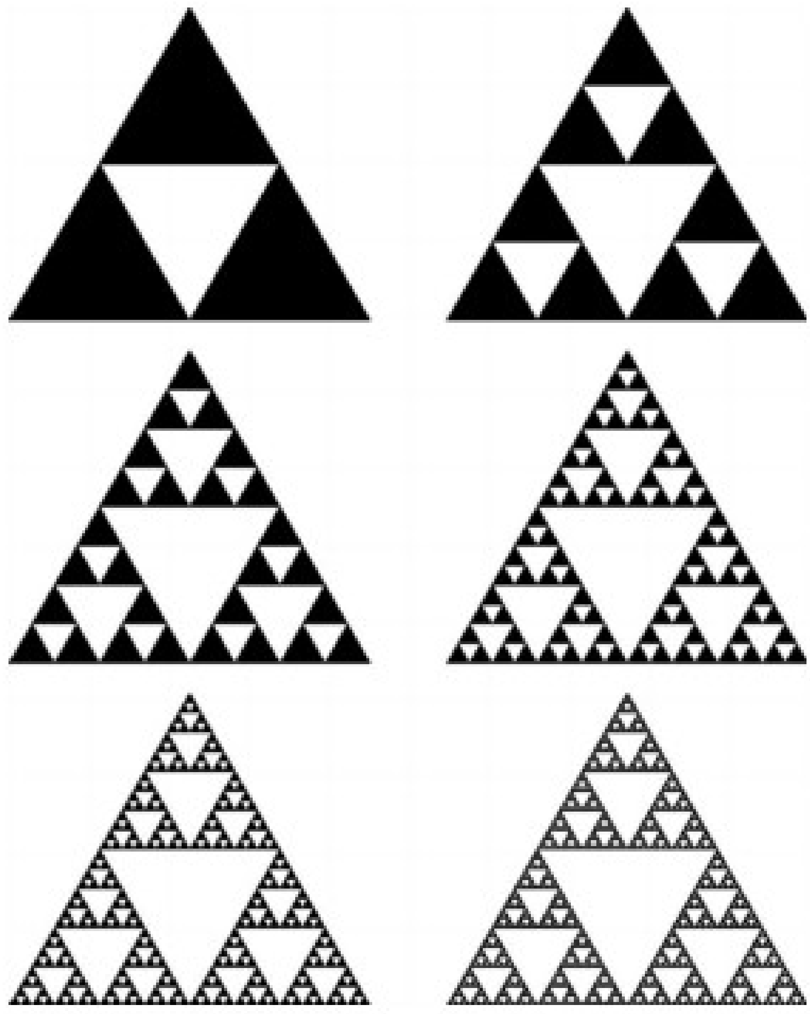


Figure 2. The Sierpinski Gasket.

Self-Similarity of Fractals

The example of the Sierpinski Gasket shows the important feature of fractals, self-similarity, because all produced triangles are similar. Similarity is the most fruitful notion of elementary geometry, and self-similarity is an extension of the notion (Peitgen et al., 2004). For any two objects, if they have the same shape, they are similar regardless of size. If they have angles and line segments, the corresponding angles must be the same; the ratio of the corresponding sides must also be the same. In the example of the Sierpinski Gasket, the ratio of the newly produced sides of the triangles and the former triangle is always 1/2, which is also called the scaling factor. Each two similar objects have a scaling factor that means that one object can be transformed from another by the same scale horizontally and vertically, and the transformation between these two objects is said to be a similarity transformation. In geometry, a transformation often includes three basic motions: a translation, a rotation, and a reflection. Similarity transformation is a transformation involved in scaling. The following is a specific similarity transformation in which a scaling, a rotation, and a translation are applied.

Pick any point $P(x, y)$ as a vertex of a triangle in the x-y plane. When this triangle is conducted by a transformation with a scaling, a rotation, and a translation, any point on this triangle must be given the same action. Thus, we only need to see what is done for the point $P(x, y)$ when conducting the transformation. First, a scaling operation, denoted as S ($S > 0$, and $S \neq 0$), is applied to the point P , and then a new point $P_1(x_1, y_1)$ is produced such that $x_1 = Sx$, and $y_1 = Sy$. Next, a counterclockwise rotation by an angle θ acts on to the point $P_1(x_1, y_1)$, which yields a new point $P_2(x_2, y_2)$ such that:

$$x_2 = \cos\theta \cdot x_1 - \sin\theta \cdot y_1,$$

$$y_2 = \sin\theta \cdot x_1 + \cos\theta \cdot y_1.$$

Now, apply a translation $T(T_x, T_y)$ to the point $P_2(x_2, y_2)$, yielding a new point $P_3(x_3, y_3)$ such that:

$$x_3 = x_2 + T_x,$$

$$y_3 = y_2 + T_y.$$

Thus, after the transformation operations of scaling, rotation, and translation, the original point $P(x, y)$ is transformed into the point $P_3(x_3, y_3)$. The relation between these two points can be described as the formula:

$$x_3 = S\cos\theta \cdot x - S\sin\theta \cdot y + T_x,$$

$$y_3 = S\sin\theta \cdot x - S\cos\theta \cdot y + T_y.$$

This formula can be applied to all points of the triangle, since $P(x, y)$ is the arbitrary point of the triangle. A new triangle is produced in the plane, which is similar to the original triangle. This is a two-dimension case, and it can be extended to the one-dimension and three-dimension cases. It is not difficult to find that if an object with area A is transformed by a scaling factor S , the area of the resulting object should be $S^2 \cdot A$; if an object with volume V is transformed by a scaling factor S , the volume of the resulting object should be $S^3 \cdot V$.

Self-similarity is self-explanatory in a certain sense, but it is hard to define in words. According to Peitgen and colleagues (2004), there are different degrees of self-similarity: self-similarity at a point (self-similarity property can only be found at a particular point), self-affinity (the property of self-similarity only held by part of the object), and strict self-similarity. The previous example, the Sierpinski Gasket, is a strict self-similarity fractal, in which one can find copies of the whole in nearly every point of it. The two-branched tree (see Figure 3) is self-affinity in which the set of leaves are self-similar, and the stem is not similar to the whole tree, but it can be interpreted as affine copy which is compressed to a line.

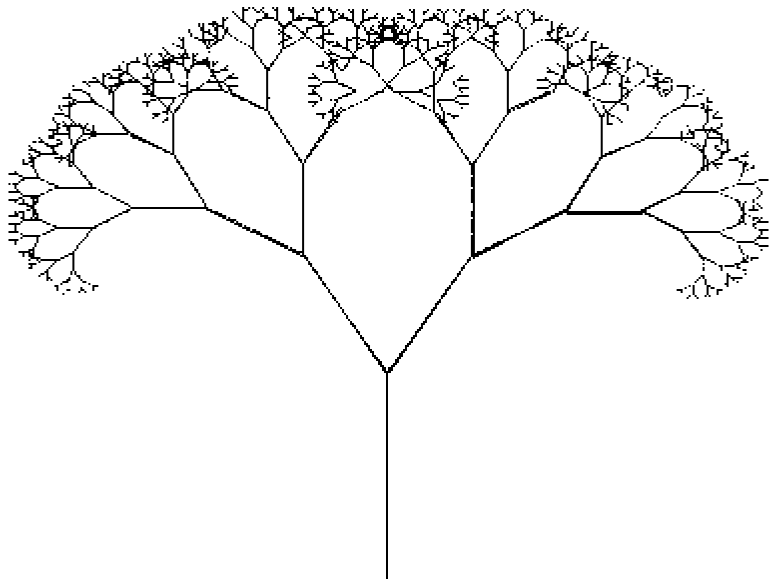


Figure 3. The Two Branches Tree.

The book design (see Figure 4) shows the self-similarity at a point that is the limit point at which the size of the copies tends to zero.

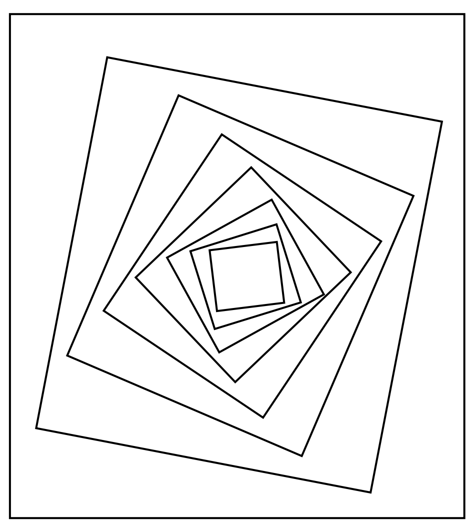


Figure 4. The Book Design.

It is worth noting that a fractal has the property of self-similarity, but an object with a self-similar structure may not be a fractal. For example, a square can be broken into small copies, and these copies can be obtained by similarity transformations; however, this structure is not a fractal (see Figure 5).

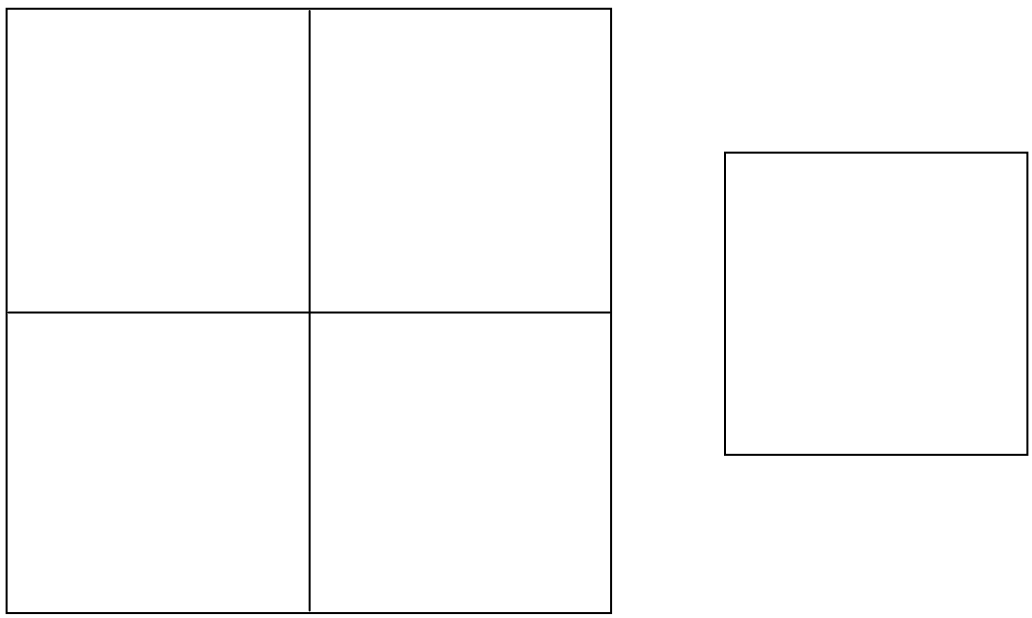


Figure 5. The Self-Similarity Square.

Dimensions of Fractals

In Euclidean space, the topological dimensions are defined as natural numbers. A point has zero dimensions, a straight line has one dimension, a plane has two dimensions, and a volume has three dimensions. We have understood and accepted the notion of dimension. However, what dimensions does a fractal have? Would it be an integer? Peitgen et al. (2004) stated that there are many different notions of dimension: topological dimension, Hausdorff dimension, fractal dimension, self-similarity dimension, box-counting dimension, capacity dimension, information dimension, Euclidean dimension, etc. They are all related, and sometimes they are the same. The concepts of fractals and the fractal dimension were first introduced by Benoit Mandelbrot, and the fractal dimension was based on a definition of Hausdorff in 1919 (Debnath, 2006). According to Debnath (2006), Benoit Mandelbrot was the first to recognize that the topological dimension cannot be applied to some sets, including fractals, and he defined fractals as a set with non-integer Hausdorff dimension.

A simplification of the Hausdorff dimension is the capacity dimension. Debnath (2006) introduced how the fractal dimension is defined from the capacity dimension. For a bounded set S in R^n, if the minimum number of balls of radius r required to cover the set S is $N(r)$, which is plotted against their unitary length r in a bi-logarithmic diagram so that it gives a straight line whose slope is $-D$, then there is a fundamental relation:

$$N(r)r^D = 1.$$

This relation results in:

$$D = \lim_{r \to 0} \frac{\log N(r)}{\log(1/r)}.$$

This result is generalized by the capacity dimension D of S, defined as:

$$D = \lim_{r \to 0} \inf \frac{\log N(r)}{\log(1/r)}.$$

The Hausdorff dimension is a fractal measure including all covers of S with balls of radius less than 1, which is often equal to the capacity dimension, called the fractal dimension. The Hausdorff dimension for self-similar sets can be well applied to the fractal dimension. In the condition that the length of l initiator is not equal to one, the above fundamental relation becomes:

$$N(r)r^D = l^D.$$

Then, the dimension of self-similar fractals results in:

$$D = \lim_{r \to 0} \frac{\log N(r)}{\log(l/r)}.$$

In general, give a self-similar structure with the reduction factor s and the number of pieces p into which the structure can be divided. Then, in the fundamental relation, $N(r)$ can be replaced as p, and r can be replaced as s. The self-similarity dimension can be simplified as:

$$D = \frac{\log p}{\log(1/s)}.$$

In the fractal of the Sierpinski Gasket, the reduction factor $s$is 1/2, and the structure can be divided into 3 pieces (p) each time. Applying this formula to the Sierpinski Gasket, the dimension of this fractal would be:

$$D = \frac{\log 3}{\log 2} \approx 1.58496$$

Due to the purpose of this study, more information about fractal dimension will not be introduced here.

Support from the Math Core Curriculum

Fractal geometry is new and different from Euclidean geometry because of the property and structure of fractals, which are different from Euclidean geometric shapes. But the knowledge of fractal geometry is grounded on the knowledge of Euclidean geometry. First, the creation of every geometric fractal is based on a basic geometric shape such as a line segment, triangle, quadrilateral, etc., which is called the seed. The properties of those basic geometric shapes and the theorems related to those geometric shapes are also related to the created corresponding fractal. Euclidean geometry is a requirement for study in the math core curriculum, which in turn is the fundamental basis of supporting fractal learning. The following examples are used to demonstrate this point.

Fraboni and Moller (2008) introduced the example of the Sierpiński triangle and its utilization in the K-12 classroom. To create a Sierpiński triangle, an equilateral triangle and its interior are used as the seed. The geometric iteration rule is: Remove the triangle formed from the middle points of each side of the original triangle so that each side is one-half of the original one and three other congruent triangles remain. During this repeated process of creating the Sierpiński triangle, students must connect to and understand the theorem, "the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length," which is in Geometry 10 of Common Core State Standard (G.CO.10, http://www.corestandards.org/) in the U.S. and in Geometry 7-9 of National Curriculum Standards (G.NC.7-9, http://hrd.apec.org/images/e/e4/54.2.pdf) in China. Then students must also understand why the remaining three triangles are congruent to one another and they are still equilateral triangles and similar to the original triangles. Again the concept of congruence and similarity are required in both U.S. and Chinese math core curricula. Further, several questions can be developed by teachers, as Fraboni and Moller (2008) stated. One question is "what is the triangle's area?" (Fraboni & Moller, 2008, p. 198). If the side of the original triangle is 1, the students must apply the properties of equilateral triangles to find the height of this triangle and use the area formula of a triangle to find the area of the original triangles, and then they can find the area of each stage of the Sierpiński triangle. More questions can be developed, such as what geometric transformations can be conducted from one piece of the Sierpiński triangle to another piece. Again all the above knowledge is required by the math core curricula both in the U.S. and China.

Ding and Li (2009) studied the dimension of Sierpiński pedal triangle. Although secondary general students may not be able to reach the knowledge of fractal dimension, the structure of Sierpiński pedal triangle contains many Euclidean geometry concepts and theorems that are required by the math core curriculum. Given the original triangle ABC, draw three altitudes: AA_1, $BB_1$and CC_1. The pedal triangle $A_1B_1C_1$ is constructed by joining the three feet A_1, B_1, and C_1. Repeating the same process, the pedal triangle $A_nB_nC_n$ can be constructed from the previous pedal triangle $A_{n-1}B_{n-1}C_{n-1}$. If the original triangle is an acute triangle, the pedal triangle $A_nB_nC_n$ in each stage should be inside of the previous pedal triangle $A_{n-1}B_{n-1}C_{n-1}$. What is the relationship between the previous pedal triangle $A_{n-1}B_{n-1}C_{n-1}$ and the remaining three triangles after removing the middle of pedal triangle $A_nB_nC_n$ inside of triangle $A_{n-1}B_{n-1}C_{n-1}$? And what is the relationship between the area of pedal triangle $A_nB_nC_n$ and the pedal triangle $A_{n-1}B_{n-1}C_{n-1}$? Actually, many questions which are supported by knowledge of Euclidean geometry can be created.

Secondly, fractal geometry knowledge can connect the study of numbers, ratios (proportions), algebraic expressions, patterns, functions, exponents, logarithms, geometric sequences, etc., which are the requirements of the math core curricula both in the U.S. and China. The example of the Sierpiński triangle (Fraboni & Moller, 2008) can be used to create the following questions: What is the proportion of the first removed middle triangle from the original triangle? What is the total number of the removed middle triangles after the 3rd iteration, after the 20th iteration, and after the nth iteration? What is the total area of the removed middle triangles after the nth iteration? What is the total area of the removed middle triangles after the infinite iteration? All of these questions connect the knowledge proportion, numbers (count), exponent, algebraic expression (exponent function/pattern), sum of geometric sequence, and limit, respectively. If we ask what fractal dimension that the Sierpiński triangle is, we then make a connection to logarithmic functions. Further, the example of the Sierpiński triangle (Fraboni & Moller, 2008) is a special case of the example of the Sierpiński pedal triangle (Ding & Li, 2009) when the original triangle is an equilateral triangle. In this case, the foot of the altitude of each side is the midpoint of each side according to this theorem of isosceles triangle. Ding and Li (2009) demonstrated that "the fractal dimension of the Sierpiński triangle be strictly less than that of any other Sierpiński pedal triangle associated with an acute triangle" (p. 43). This demonstration requires knowledge of trigonometry and other college-level mathematics knowledge.

Overall, fractal geometry connects to almost all required knowledge in the math core curriculum in both the U.S. and China. Since the principle of fractal is easily explained by teachers and understood by students in a short time, fractal geometry can be integrated into the math core curriculum as an application of mathematical concepts and theorems (Fraboni & Moller, 2008). The next section gives an in-depth literature review of the values of teaching and learning fractal geometry.

Value of Teaching and Learning Fractal Geometry

Euclidean geometry has played a significant role in mathematics education for developing students' logical thinking and problem-solving in a life full of lines, triangles, and circles. It is still dominant in geometry courses and continues to structure contemporary thinking (Davis & Sumara, 2000). However, what is the shape of a cloud, a mountain, a coastline, or a tree? Classical geometry does not have answers to these questions because the ideal basis for understanding geography is formed from the axiomatic study of lines, triangles, and circles. Clouds, mountains, coastlines, and trees cannot be applied to explain any shapes in classical geometry. The nature of the world is often irregular and complex. Fractals, a new field, arose in recent years and came to explain that kind of natural phenomena. The elements of fractals are not lines and circles but iterations and self-similarity; the surfaces of fractals are not smooth but jagged; the features of fractals are not perfect but broken (Frame & Mandelbrot, 2002). Fractal geometry creates an alternative to Euclidean geometry, rising to prominence with mathematics and across scholarly and popular domains (Davis & Sumara, 2000). The extraordinary utility for explaining the nature of the world phenomena in many fields would be a sufficient warrant for fractals to be awarded a prominent role in mathematics education (Frame & Mandelbrot, 2002). The value of teaching and learning fractal geometry would be more than the mere function of explaining the world.

Inspiring Students' Learning Interest and Curiosity

Natural fractals can be found everywhere in the world: in the shape of a mountain range and in the windings of a coastline. Mathematically structured fractals can also be found in the arts, engineering, physics, and chemistry fields. The images of fractals show incredible beauty and complexity, while the principle of fractals is very simple by doing iteration and self-similarity. When one asks students, "How do you measure the length of the British coastline?" students should not quickly answer this question because classical geometry knowledge cannot help them to find solutions. They should, however, have great curiosity about this problem. The inability to explain and solve this kind of natural world problem makes students feel that classical geometry is "dry" and "cold" (Peitgen & Richter, 1986). The field of fractals not only inspires mathematicians and scientists to explore the unknown in natural world phenomena but also inspires students' interest in and curiosity for exploring the secrets of nature.

At the early stage, the beauty of fractals was the favorite reason mathematicians and math educators studied them (Peitgen & Richter, 1986). According to Mandelbrot and Frame (2002), "the popularity of elementary courses using fractals was largely credited to the surprising beauty of fractal pictures and the centrality of the computer to instruction in what lies behind those pictures" (p. 3). Both were convinced that the beauty of fractals attracts mathematicians and math educators to study and teach fractals. For students, the attraction of learning fractals is far more than appreciating their beauty. The voices of teachers about teaching and learning fractals fully illustrated the above point:

- Simple ideas lead to unexpected complexity. Fractals are more life-like than objects studied in other parts of mathematics; thus, they appeal to many students who find traditional mathematics cold and austere.
- Many easy problems remain unsolved. Fractals are rich in open conjectures that lead to deep mathematics. Moreover, the distance from elementary steps to unsolved problems is very short.
- Careful inspection yields immediate rewards. Insights and conjectures arise readily when our well-developed visual intuition is applied to fractal images. In studying fractals, children can see and conjecture as well as adults.

- Computers enhance learning. The visual impact of computer graphics makes fractal images unforgettable, while the unforgiving logical demands of computer programs yield important lessons in the value of rigorous thinking. (p. x).

Needless to say, during teaching and learning fractals, the treasures, unforgettable fractal pictures, unexpected complexity of fractal structure, and unsolvable possibilities and implied conjectures when exploring fractals become students' infinite motivation. Their interest and curiosity would be largely expanded.

Creating an Exploratory Learning Environment

Inquiry-based learning or active learning has become popular in recent years. Fractal geometry offers a great opportunity for K-12 and college students to explore knowledge and develop their ability to discover new ideas. Researchers are convinced that exploratory activities can be created through the learning of basic concepts of fractal geometry to discover the deep application of those concepts. Students gain some basic knowledge of fractals by observing the objects, collecting and analyzing data, looking for patterns, making conjectures, and thinking about proofs.

In Vacc (1999), elementary students gained the basic concepts of fractal geometry by their exploration of the created appropriate activities. Students were asked to describe, identify, and measure fractal patterns and explore the attribute of self-similarity in some fractal sets without being told what the knowledge is. For example, to gain the knowledge of the shape of the fractal, a sample of a natural fractal was given to small groups of children or individuals. All students were required to observe the natural fractal sets and describe the shape and characteristics of the objects by using appropriate descriptors to develop the terms of fractals such as irregular and fragmented spatial patterns. To identify fractal sets, students were asked to generate lists of fractals and explain to others why the objects were identified as fractal patterns. To know the ideas of measuring fractals, students were asked to measure the line segments and the path of a coastline using three different units of measurement within small groups. In each activity, a set of open-ended questions was designed to assist students' mathematical thinking. Through the series of exploratory activities, elementary students can build the basic concepts of fractals.

In Jarry-Shore (2013), eighth grade students gained the methods of creating fractals by their practice exploration in the designed activities. Five selected fractal patterns, X-out, Lonpland, the Sierpinski triangle, the Von Koch curve, and Squareflake were provided for students to analyze, investigate and ultimately re-create. Students were grouped by two or three in each class and were required to work collaboratively in an effort to re-create a rough design of their assigned fractal. A set of tools was needed for this activity: rulers, right-angle tools, protractors, pencils, construction paper of varying colors, scissors, and glue. The re-creation activity provided an opportunity for enhancing measurement skills, as well as developing "their appreciation for the exponential growth in the number of shapes at each successive stage of their fractal" (p. 36). After completing the re-creation of the assigned fractals, students were asked:

- to show how they re-created their fractal,
- to discuss the reason behind their choice of dimensions,
- to explain what made their pattern a fractal, and
- to identify which fraction/percentage was applied in the re-creation of their pattern and how. (p.37).

This activity can be extended to 9^{th} -12^{th} grade students by examining fractal patterns in the growth of numbers, length, area, and volume in each stage of the fractal using different fractal objects. Fraboni and Moller (2008) introduced the example of designing exploratory activities to investigate the exponential growth of the number of triangles and the area of the removed triangles for the Sierpinski triangle. Students must be surprised that the Sierpinski triangle has zero area demonstrated by the geometric series that they have explored. This activity would lead students to discover more about the Sierpinski triangle, like its perimeter. Gluchoff (2006) introduced how the hands-on activity for constructing fractal images created students' engagement in discussing not only the fractals themselves but also the processes that give rise to them.

In Mandelbrot and Frame (2002), college students discovered the connectivity of gasket relatives by being assigned an open-ended project. Both in-class and out-of-class exploratory activities were designed for students applying the transformations of the Sierpinski gasket to other figures. The transformations of the gasket are:

$$T_1(x, y) = (\frac{x}{2}, \frac{y}{2}),$$

$$T_2(x, y) = (\frac{x}{2}, \frac{y}{2}) + (\frac{1}{2}, 0),$$

$$T_3(x, y) = (\frac{x}{2}, \frac{y}{2}) + (0, \frac{1}{2}).$$

If the transformations are applied the unit square $S = \{(x, y): 0 \le x \le 1; 0 \le y \le 1\}$, different fractals will be produced. In a self-directed investigation, students speculated a universal shape existed among fractals: Cantor dust, dendrite, multiply connected, and hybrid.

The above examples illustrate that fractal geometry can provide rich materials to develop discovery learning. Math teachers can design the discovery activities based on the grade level of students and the content level of fractals. This section is concluded with words from math teachers about the exploration of fractals:

- The first steps are so much fun. Exploring fractals creates unprecedented enthusiasm for discovery learning among teachers and students.
- Fractals are beautiful. Stunning visuals appeal to the mind's eye and create contagious demand for continued exploration.
- Anyone can play. Exploration of fractal geometry appeals to students of every age, from primary school through college and beyond. (Frame & Mandelbrot, 2002, p. x).

Necessity of Integrating Fractal Geometry

Fractals as a learning subject began appearing in mathematics and science education mostly at the college level. Mandelbrot and Frame (2002) claimed that college students' reactions to fractal learning often were extremely positive, and because of this, entire courses on fractal geometry have been developed for college students. Mandelbrot and Frame (2002) also pointed out that fractal geometry has worked its way into general mathematics education and science curricula, and into parts of the high school curriculum. Actually, several studies (Fraboni & Moller, 2008; Jarry-Shore, 2013; Vacc, 1999) have shown that teaching and learning fractals can be conducted from K-12. Fraboni and Moller (2008) argued that "fractals offer much to explore for even very young students" (p. 197) and "fractal activities can be found that address most NCTM Standards" (p. 199). Vacc (1999) made an argument that both fractal patterns and classical geometric shapes exist in the daily environment of children; our educators cannot overlook fractal patterns because a new mathematical language can be applied to highly irregular and fragmented special patterns; this new mathematics can be addressed in the K-12 curriculum. The discussions of the necessity of incorporating fractal geometry into the math curriculum will be concentrated on two aspects: first, fractal geometry offers an extension and an alternative to classical geometry; second, fractal geometry builds a connection with many other subject fields.

Offering an Extension and Alternative to Classical Geometry

When talking about geometry, the images of line segments, triangles, and circles would be evoked. These images are some important basic forms of classical geometries, Euclidean geometries that influence human thinking and viewing of life. The language of classical geometry has been used to communicate the designs of technological products and to approximate some forms of natural creations. However, classical geometry has constraints because it cannot study the roughness of the world that can be seen everywhere. In other words, the roughness of the world cannot be precisely explained by classical geometry. The emergence of fractal geometry provides a new scientific way to think about the nature of the world. Hence, fractal geometry is an extension of classical geometry, which cannot be ignored by educators (McHugh, 2006).

Fractal geometry is an extension of classical geometry, which also can be illustrated through the connection between fractal geometry and classical geometry. In the early section, basic knowledge of fractals and fractal geometry, we have discussed that self-similarity is one important property of fractals. In classical geometry, students have learned about the familiar forms of symmetry: translation, rotation, and reflection that are the elements of similarity. Self-similarity is a kind of similarity. The foundation knowledge of translation, rotation, and reflection is also important to fractal geometry. Thus, fractal geometry is not an interrupter of classical geometry, but an extension. Because of this, fractal geometry is easy to start to learn even for elementary students.

Fractal geometry is an alternative of classical geometry. According to Davis and Sumara (2000), there is no doubt that classical geometry has played a significant role in shaping the sensibilities, practices, and structures of much of curriculum discourse and developing human logical thinking, and continues to play this role. However, to illustrate the pervasiveness and the constraining tendencies of classical geometry, a new mathematical language is needed. Fractal geometry offers an alternative way of thinking about and describing the nature of the world by using its new mathematical language. The two significant properties of fractal figures scale independence and self-similarity, display striking levels of complexity at all levels of magnification and reduction that classical geometry cannot explain. In Davis and Sumara (2000), a brief examination is made to show how fractal geometry can complement and inform other emergent sensibilities in the curriculum, especially to discuss criticism of the linear structures associated with classical logic. Peitgen et al. (1992) claimed that to students, mathematics is brought out of the realm of ancient history and into the twenty-first century by fractals; to teachers, a unique, innovative opportunity to illustrate both the dynamics of mathematics and its many interconnecting links is provided.

Building a Wide Connection with Many Other Subject Fields

Fractals connect many different aspects of mathematics such as computing, algebra, calculus and more advanced mathematics. For example, Habecker and Crannell

(2004) introduced how fractals and the iterated function system were used to motivate some of the foundational concepts of linear algebra. Fraboni and Moller (2008) reached a conclusion that “through fractal geometry, students will investigate a range of topics, including sequences, symmetry, ratio and proportions, measurement, and fractions” and “at a higher level, tools such as logarithm, the composition of functions, Pascal’s triangle, arithmetic in different bases, and the complex numbers can be applied” (p. 199). Thus, fractal geometry can build a network among different math learning topics. Fractals not only can be taught as a separate topic but also can be incorporated as examples into traditional lessons (Fraboni & Moller, 2008).

Fractals also widely connect to many other subject fields such science, music, art, etc. For example, in engineering, copying natural fractals for inspiration has been used by human engineers to build successful devices; fractal art has been used by artists to create arts and crafts. Taylor (1985) argued that fractals as a mathematical model can be appropriately used to describe many of the phenomena in the scientific disciplines, from astronomy to fluid mechanics to biology to economics. Mathematics as an important tool has been applied to all different fields and has become a significant foundation of many other subjects. Fractals as a vital part of mathematics knowledge and its various usages must be incorporated into math curricula to meet the needs of the development of society.

Further, fractal geometry also can build a good connection with many other education fields. For example, Padua (2005) described how mathematics teachers use their knowledge of fractals to create electronic music that most students are interested in learning. Padua (2009) introduced how mathematics teachers teach fractal geometry by combining computer-based music and how fractal patterns connect to music patterns.

Gluchoff (2006) described how the processes of creating fractals can be used to build cartoons and art through hands-on activities. Eglash, Krishnamoorthy, and Sanchez (2011) introduced how the fractal art, African design, can be used in teaching computing. The connections between fractals and other subjects indicate the necessity of incorporating fractal geometry into math curricula; it is important for developing students' mathematical thinking and understanding and also for developing their interest and motivation in learning mathematics and applying mathematics to their lives. Habecker and Crannell (2004) illustrated that by combining fractals into linear algebra learning, students can gain a powerful understanding of both concepts of fractals and linear algebra. In the study of Eglash et al. (2011), the use of fractal simulation of African design in a high school computing class provided evidence that African students' achievements and attitudes gained significant improvement in the experimental class. The simulation of fractal structures used in this experiment both enhanced the engagement and performance of under-represented students in computing.

Research on Fractal Geometry in K-12

Since Benoit Mandelbrot first coined the word "fractal" to describe shapes which are detailed at all scales in 1975 (McHugh, 2006), mathematics educators became interested in this field. Although fractal geometry is a relatively new teaching and learning field and was first explored as a learning course started at the college level, fractal geometry has gained great attention in K-12 math education in recent years. The exploration of fractal geometry in the classroom covers kindergarten through graduate school now (Gluchoff, 2006). Research on fractal geometry in K-12 mostly focuses on how some concrete fractal examples are modified to develop teaching and learning activities based on the grade level of the students, and it is associated with some discussions about the appropriateness and usefulness of creating these activities (for example, Barton 2003; Fraboni & Moller, 2008; Jarry-Shore, 2013; McHugh, 2006; Vacc, 1999). There are very few studies which focus on the theoretical discussions about incorporating fractal geometry in math curricula (for example, Davis & Sumara, 2000). Because fractal geometry is a new mathematical teaching and learning area, research topics on this area still need to be developed both in practical and theoretical levels.

On the practical level, the existing studies have made contributions to the incorporation of some certain fractal content knowledge in K-12 math curricula and the creation of the corresponding exploratory learning activities for developing students' mathematical thinking and conceptual understanding. Many examples can support this point. Vacc (1999) demonstrated the awareness of possible applications of fractal geometry with children and the appropriateness of including this new mathematics in elementary curricula by creating activities for children to explore basic concepts of fractal geometry. In the study of McHugh (2006), a unit of teaching fractals was incorporated into a geometry course, in which the Sierpinski Triangle and the Cantor Set were chosen as learning materials for third and fourth grade students. McHugh (2006) illustrated how elementary students were exposed to the vocabulary of working with fractals and how elementary students learned to create fractals through hands-on activities and were pushed to think new thoughts about the order and chaos in nature. Fraboni and Moller (2008) presented an example of how the iteration, the main tool for creating a fractal, is used to construct fractals and look for patterns associated with the numbers and the areas at each stage of the generated fractals. Jarry-Shore (2013) discussed how an integrated project involving fractals helps students understand several mathematics concepts and create fractal art. Barton (2003) studied how Pascal's Triangle can provide another opportunity to connect young children to deep mathematical truth; the numeric form of Pascal's Triangle is filled with hidden relationships and connections to deeper mathematics concepts.

These studies opened a door of how we can integrate fractal geometry in the K-12 math curriculum. But, it is the tip of the iceberg for fractal geometry studies. The topics that have been studied only touched a small part of materials and resources of fractal geometry, some basic concepts and terminology and some classical fractal structures; the topics that have been studied only connected to some points of the other K-12 math content. Thus, the connections between fractal geometry and other K-12 math content need more empirical development; the appropriateness of fractal geometry at different levels of K-12 needs more empirical examination; the assessment of teaching and learning fractal geometry needs more empirical guidance.

On the theoretical level, the existing studies have made contributions to the illustration of why fractal geometry needs to be incorporated in math curricula, and how fractal geometry can complement and inform other emergent sensibilities in the curricula. For example, Davis and Sumara (2000) gave a deeper discussion and analysis about the limitations of classical geometry for describing the shape and character of natural objects and the emergent changes of fractal geometry for solving the problems that classical geometry is unable to do. Davis and Sumara (2000) also gave an examination of the associated notions of fractal geometry for rethinking curriculum and schooling. However, along with the needs for practice on fractal geometry, the theory of teaching and learning fractal geometry needs to be developed in many aspects. Many questions need to be answered. What is the domain of teaching and learning fractal geometry in K-12 math curricula? How does this domain consistently incorporate into each math grade level? What pedagogical knowledge should teachers have in order to integrate fractal geometry into class teaching? What should be changed in current curricula and textbooks for geometry courses? How do we assess the curriculum, textbook, workbook, and the effectiveness of teaching and learning within the scope of fractal geometry? What is the role of technology in teaching and learning fractal geometry?

Although fractal geometry has received widespread attention in recent years, teaching and learning fractal geometry in the classroom is not universal in the U.S. and China. Only those teachers who are interested in and willing to make changes in geometry teaching put their efforts into integrating fractal geometry into math curricula. If the theory of teaching and learning fractal geometry can be more available to guide math teachers and educators in practice, the time of universally integrating fractal geometry into the classroom would be soon. But, the primary problem needed to be solved currently is to provide an important reference to curriculum designers or mathematics educators about the current state of secondary mathematics teachers' awareness, interest, and knowledge of fractal geometry.

Summary

The purpose of the literature review is to give an understanding of fractal geometry and its educational values in K-12. This is a new teaching and learning area, which is highly connected to the math core curriculum. To demonstrate fractal geometry being a valuable teaching and learning area, three dimensions have been discussed: the value of teaching and learning fractal geometry, the necessity of incorporating fractal geometry into math curricula, and the discussions about research on fractal geometry. Teaching and learning fractal geometry is valuable because it can inspire students' learning interests and curiosity and create an exploratory environment for all students' learning; teaching and learning fractal geometry is necessary because it can offer an extension of and an alternative to classical geometry, which is unable to explain the roughness of the world and build a great connection within mathematical knowledge and many other subjects. To illustrate each point, this chapter presented a series of empirical research evidence. The discussions about research on fractal geometry focused on both practical and theoretical levels to address the possibilities of topics needed to be studied in the future. This literature review also opened a dialogue of the value and necessity of teaching and learning fractal geometry in the K-12 grades, but it is still worthy of continuation because of its new and immeasurable value.

Rationale for the Study

Any secondary mathematics teachers should have been educated at least at the college level in mathematics. They should have the knowledge that is required to learn by the math core curriculum. As the above literature demonstrated, fractal geometry is heavily supported by the math core curriculum, and the whole of Euclidean geometry is the fundamental basis of fractal knowledge. Thus, all secondary mathematics teachers should have the fundamental basis and other related knowledge of fractals, at least at the K-12 level. Although not all secondary mathematics teachers have had educational experience with fractal geometry, they might study fractals by themselves because of self-interest or teaching experience. Their awareness, interest, and knowledge of fractal geometry may relate to their demographic background and working experience. More specifically, mathematics teachers who have higher degrees like master, specialist, and doctor, may have higher levels of awareness, interest, and knowledge about fractal geometry than those who only have bachelor's degrees. Mathematics teachers who have experience in teaching fractals may have higher levels of awareness of, interest in, and knowledge about fractal geometry than those who do not. There are many possible factors that contribute to the state of mathematics teachers' awareness of, interest in, and knowledge about fractal geometry.

The present study is to examine the current state of secondary mathematics teachers' awareness of, interest in, and knowledge about fractal geometry, as well as the factors that influence the level of awareness, interest, and knowledge. This will provide a first-hand reference for curriculum designers and mathematics educators to think about fractal geometry in K-12 education, and even at the college level, as well as in service teachers' professional development.

CHAPTER III

METHODOLOGY

Introduction

The purpose of this study was to investigate the levels of Chinese and U.S. secondary mathematics teachers' awareness of, interest in, and knowledge of fractal geometry, as well as any factors that influence that level of awareness, interest, and knowledge. This chapter describes the research design, participants, instrumentation, procedure, and analysis of data.

Research Questions and Hypothesis

The data was analyzed to address the following research questions and hypotheses:

Overarching Research Questions: What is the current state of the awareness of, interest in, and knowledge of fractal geometry in U.S. and Chinese secondary mathematics teachers and what factors affect the levels of their awareness, interest, and knowledge?

Specific Research Question One: What factors (nationality, gender, age, degree, the years of teaching math, grade levels, experience of learning fractals, and experience of integrating fractals) contribute the most and to what extent to teachers' awareness levels in fractal geometry?

- H_{a1}: The factors degree, grade levels, experience of learning fractals, and experience of integrating fractals will account for more variance than gender, age, the years of teaching math, and nationality in the overall mean awareness survey scores.

Specific Research Question Two: What factors (nationality, gender, age, degree, the years of teaching math, grade levels, experience of learning fractals, and experience of integrating fractals) contribute the most and to what extent to teachers' interest levels in integrating fractal geometry in the math core curriculum?

- H_{a2}: The factors degree, grade levels, experience of learning fractals, and experience of integrating fractals will account for more variance than gender, age, the years of teaching math, and nationality in the overall mean interest in integrating fractal geometry in the math core curriculum.

Specific Research Question Three: What factors (nationality, gender, age, degree, the years of teaching math, grade levels, experience of learning fractals, and experience of integrating fractals) contribute the most and to what extent to teachers' knowledge of test scores in performing fractal geometry problems?

- H_{a3}: The factors nationality, degree, experience of learning fractals, and experience of integrating fractals will account for more variance than gender, age, grade levels, and the years of teaching math in the overall mean knowledge test scores in performing fractal geometry problems.

Research Design

A quantitative research design was used to address the research questions. The research questions were investigated using a survey followed by a test of knowledge specifically designed by the researcher and based on the review of literature (Appendix A and Appendix B). There are two versions of the survey and test of knowledge: one is the English version and the other is the Chinese version, which was used for U.S. and Chinese secondary mathematics teachers separately. The survey and test of knowledge were examined by a panel of experts and revised according to suggested changes. The panel of experts included two science and mathematics education professors, one professor of educational research, and three mathematics professors at the University of Southern Mississippi. The Chinese version was examined by three educated mathematics teachers to ensure the accuracy of translation from English to Chinese.

The survey and test was taken by math teachers only once. A number of statistical techniques were used to answer the research questions. The independent variables that came from the survey instrument were nationality, gender, age, degree, the years of teaching math, grade levels, experience of learning fractals, and experience of integrating fractals. The survey instrument in this research design included the dependent variables of awareness, interest, and knowledge of fractal geometry and the related mathematical skills.

Descriptive analysis of the data was conducted to determine mean and standard deviation, which then was used in the statistical analysis. Three multiple linear regressions were used to determine if demographic data significantly impacts secondary mathematics teachers' awareness, interest, and knowledge of fractal geometry. Both the survey and test were conducted after receiving permission from the University of Southern Mississippi's Institutional Review Board and in accordance with IRB standards (Appendix C).

Participants

Participants included male and female secondary mathematics teachers who were teaching math courses from grades 6 to 12 in the academic year 2014-2015 in the United States and China. All the participants were over 18 years of age, and therefore, parent/guardian permission was not necessary. Participants were not excluded due to gender, race, or ethnicity. The participants represented a wide range of ethnicities, socioeconomic groups, and ages. The researcher anticipated a sample size of approximately 160 respondents according to the result of g power analysis with 8 predictors and an effect size f^2 0.15, but received 167 responses from the U.S. and China. Specifically, 82 responses were from the U.S. and 85 responses were from China. There were about 20 participants who dropped out from participating in the program at the very beginning of their participations. The ultimate sample size is 166 because one participant from the U.S. did not respond to questions 18, 19, and 20. After data screening, it was deleted from the sample since the power of analysis would not be reduced too much due to one deletion from one hundred sixty seven participants.

Instrumentation

The survey instrument (Appendix A) was designed by the researcher. The survey instrument included three parts. The first part was the participants' common demographic data such as gender (0 = "male," 1 = "female"), age (0 = "20-25," 1 = "26-30," 2 = "31-35," 3 = "36-40," 4 = "41-45," 5 = "46 or above"), degree (0 = "bachelor degree," 1 = "M.S. degree," 2 = "specialist degree," 3 = "doctoral degree," 4 = "other type"), the years of teaching math (0 = "0-5," 1 = "6-10," 2 = "11-15," 3 = "16-20," 4 = "21-25," 5 = "26 or above"), grade levels (0 = "6-9," 1 = "10-12"), experience of teaching geometry (0 = "yes," 1 = "no"), experience of learning fractals (0 = "yes," 1 = "no"), experience of integrating fractals (0 = "yes, 1 = "no"), the amount of knowing examples of fractals (0 = "none," 1 = "only one," 2 = "more than one"), active learning (0 = "yes," 1 = "no"), location (0 = "rural," 1 = "suburban," 2 = "city") , and nationality (0 = "China", 1 = "the

U.S.”). The items “experience of teaching geometry,” “active learning,” “the amount of knowing examples,” and “the location” were not chosen as the predictors, but they were included in the survey. The item “experience of teaching geometry” was planned as a predictor, but in fact, geometry content is presented in any secondary math course. Therefore it was not meaningful to use it as a predictor. Thus, the independent variables were nationality, gender, age, degree, the years of teaching math, grade levels (grade levels that the participant taught), experience of learning fractals, and experience of integrating fractals.

The second part of the survey instrument (Appendix A) was used to examine the level of secondary mathematics teachers’ awareness of the basic concepts of fractal geometry. It included eight items. Aware13 = “I know the concept of geometric transformation.” Aware14 = “I know the concept of geometric sequences.” Aware15 = “I know the concept of similar figures.” Aware16 = “I know the concept of geometric iteration.” Aware17 = “I know the concept of self-similarity.” Aware18 = “I know the concept of magnification factor of fractals.” Aware19 = “I know the concept of fractal dimension.” Aware20 = “I know how to create a geometric fractal.” Participants were asked to rate their level of awareness of fractal geometry on a scale of 1-4: 1 = completely unknowing, 2 = somewhat unknowing, 3 = somewhat knowing, and 4 = completely knowing. The reliability test was conducted for examining the level of internal consistency for the four levels scale with the sample of 166. The Cronbach’s alpha was 0.80, which indicated a high level of internal consistency for the scale that was used in this instrument. The test result also revealed that the removal of any items from Aware13 to Aware20 would result in a lower Cronbach’s alpha. See Table 1. Therefore we would not remove any items of awareness in this instrument when conducting data analysis. The average of a total of eight item scores was used as the score of dependent "awareness" for each participant when running multiple linear regressions in SPSS software.

Table 1

Item-Total Statistics of Awareness

Items	*Cronbach's Alpha if Item Deleted*
Aware13	0.79
Aware14	0.79
Aware15	0.79
Aware16	0.77
Aware17	0.76
Aware18	0.77
Aware19	0.77
Aware20	0.77

The third part of the survey instrument included eight items to investigate participants' interest in integrating fractal geometry into the math core curriculum. Interest21 = "I would like to know how Common Core (National Standard was used to replace Common Core in China) supports teaching fractal geometry at the secondary school level." Interest22 = "I would like to know how fractal geometry as a supplementary material influences the way that students learn and think about geometry." Interest23 = "I would like to know how fractal geometry supports teaching mathematics from very basic concepts to the most advanced concepts." Interest24 = "I would like to know how fractal geometry supports learning across disciplines." Interest25 = "I would like to know how fractal geometry inspires students' motivation, interest, and curiosity in teaching and learning mathematics." Interest26 = "I would like to know how inquiry study/active learning can be used when integrating fractal geometry for students' learning in my class." Interest27 = "If I had the opportunity to learn fractal geometry in a professional development program, I would like to participate in the program." Interest28 = "If I had the knowledge and ability to integrate fractal geometry into the core curriculum, I would like to integrate fractals in my curriculum." The scale of measuring level of interest was: 1 = completely disagree, 2 = somewhat disagree, 3 = somewhat agree, and 4 = completely agree.

Table 2

Item-Total Statistics of Interest

Items	*Cronbach's Alpha if Item Deleted*
Interest21	0.93
Interest22	0.93
Interest23	0.93
Interest24	0.93
Interest25	0.93
Interest26	0.93
Interest27	0.93
Interest28	0.94

The reliability analysis was also conducted for examining the level of internal consistency for the four levels of the scale with the sample of 166. The Cronbach's alpha was 0.94, which also indicated a high level of internal consistency for the scale that was used in this instrument. The test result also revealed that the removal of any items from Interest21 to Interest28 would result in a lower or equal Cronbach's alpha. See Table 2 (above). Therefore we would not remove any items of interest in this instrument when conducting data analysis. The average of a total of eight item scores was used as the score of the dependent "interest" when running multiple linear regressions.

The 16-problem (from item 29 to item 44) test of knowledge (Appendix B) was used to determine the level of the fundamental knowledge of fractal geometry and the related mathematical skills. Each problem was valued as 1 if it was correct and as zero if it was incorrect. Problem43 and Problem44 were not used in the data analysis because the knowledge being used to answer these questions do not directly reflect the knowledge of fractals, which reflects the knowledge and skills of plane geometry proofs. Plane geometry proofs are heavily required in the Chinese secondary mathematics curriculum, while plane geometry proofs are not very required in the U.S. mathematics secondary curriculum. Reliability of the test of knowledge was accomplished using the test-retest method with 10 secondary mathematics teachers (5 middle school teachers and 5 high school teachers). The test-retest study results revealed an overall Cronbach's alpha score of 0.90 within subjects and an overall Cronbach's alpha score of 0.94 within items. Both scores were greater than 0.7, which indicated the consistency of the test instrument was very good. The total score valued from Problem29 through Problem42 for each participant was converted to a score with a one hundred scale when conducting multiple linear regression analysis.

Procedure

The researcher created the survey and test using both the English and Chinese languages separately. The survey was reviewed by a professor of science education; the test was reviewed for face validity by three math professors. The Chinese translation was approved by a math professor from China. The test were piloted with 5 middle school and 5 high school math teachers using the test-retest method.

Upon approval from the University of Southern Mississippi's IRB (Appendix C), the English version was used for U.S. secondary mathematics teachers; the Chinese version was used for Chinese secondary mathematics teachers. Three recruitment ways were used to distribute the surveys and tests: electronic copies, online versions (on Quatrics.com), and hard copies. A recruitment letter was sent to the participants with the electronic copies, the link of online versions, and the hard copies for participating in the survey and test. After the participants completed the survey and test, it meant that the participants had given their consent to participate in the study. Participants were allowed to withdraw from this study at any time and for any reason without repercussions. Confidentiality was strictly maintained throughout the study; all information remained anonymous. All hard copies of the responded surveys and tests were securely locked in a file cabinet in the researcher's university office. Electronic data was kept in a password protected file. Data was reported anonymously so that readers cannot identify any particular teacher or associate that teacher with any specific response. The teachers were given approximately thirty to fifty minutes to complete the survey and test. The survey and tests were turned in to the specified data collector if hard copies were used.

Data Analysis

The participants' responses from the survey instrument were entered into SPSS version 22 by the researcher. The researcher conducted data screening and analyzed the three hypotheses by conducting three multiple linear regression tests. In this analysis, the independent variables were nationality, gender, age, degree, the years of teaching math, grade levels, experience of learning fractals, and experience of integrating fractals. The dependent variables were awareness, interest, and knowledge.

CHAPTER IV

ANALYSIS OF DATA

Introduction

The main purpose of this research was to investigate the levels of U.S. and Chinese secondary mathematics teachers' awareness of, interest in, and knowledge of fractal geometry, as well as any factors that influence that level of awareness, interest, and knowledge. The researcher developed a survey instrument (Appendix A) and a test instrument (Appendix B). The survey instrument included three parts: teachers' demographic information, awareness of the basic concepts of fractals, and interest in learning and teaching fractals. The test instrument was designed to examine math teachers' knowledge of fractal geometry and the related mathematical skills. The survey and test instrument were distributed to teachers who were teaching math from grades 6 to 12 in the greater metropolitan area of Hattiesburg, MS, and the greater metropolitan area of Shanghai, China by three different ways: electronic copies, hard copies, and the online link.

There were a total of 167 responses, but one respondent had missing values on 3 awareness items (18, 19, and 20) which resulted in invalid data for the awareness variable. Therefore, there were a total of 166 valid responses with 88 of electronic copies, 62 hard copies, and 16 copies from the online link.

In order to achieve the goal of this study, descriptive data was needed to describe the current levels of secondary math teachers' awareness of, interest in, and knowledge of fractal geometry. Three multiple regression analyses with SPSS were used to test of three hypotheses.

Data Screening

Data screening was conducted to check missing values, outliers (univariate and multivariate outliers), normality, linearity, homoscedasticity, and Multicollinearity. Therefore, “Frequency,” “Explore,” “Regression,” and “Graphs” with SPSS were used in this study.

Missing Values

Data was collected from middle and high schools in the U.S. (48.8%) and China (51.2%). The percentage of males was 43.4%; the percentage of females was 56.6%; no missing values were found for this item. The percentages of age intervals: 20-25 (10.8%), 26-30 (13.3%), 31-35 (22.3%), 36-40 (17.5%), 41-45 (15.1%), and 46 or above (21.1%); no missing values were found for this item. The percentages of mathematics teachers’ degree earned: bachelor (56.0%), master (38.0%), specialist (1.2%), and other types (4.8%); no missing values were found for this item. The percentages of the year interval that teachers have taught math: 0-5 (24.7%), 6-10 (18.7%), 11-15 (22.3%), 16-20 (15.7%), 21-25 (6.0%), and 26 or above (12.7%); no missing values were found for this item. The percentages of the grade level interval that teachers taught in: grade 6-9 (51.2%) and grade 10-12 (48.8%); no missing values were found for this item. The percentages of the item “have you ever taught geometry in any of your math courses”: yes (84.3%) and no (15.7%); no missing values were found for this item. The percentages of the item “have you ever learned fractals in your educational experience”: yes (33.7%) and no (66.3%); no missing values were found for this item. The percentages of the item “have you ever integrated fractal geometry in your math curriculum”: yes (22.9%) and no (77.1%); no missing values were found for this item. The percentages of the amount fractal examples that teacher knew: none (47.0%), only one (10.8%), more than one (41.6%). There was one missing value (0.6%) on this item. The percentages of conducting active learning: yes (77.1%) and no (21.7%). There were two missing values (1.2%) on this item. The percentages of the locations that teachers' schools were: rural (56.6%), suburban (21.1%), and city (22.3%). One missing value on the item "the amount of example" and two missing values on the item "active learning" would not influence the analysis of this study because these two items were not chosen as predictors.

After deleting the responses with invalid data for awareness, no missing values were shown on three continuous dependent variables: awareness, interest, and knowledge among 166 valid responses.

Normality of the Three Continuous Variables

"Explore" with SPSS was conducted to check normality and univariate outliers for the selected continuous variables. Table 3 provided the statistics of testing normality for the three continuous variables. Both skewness and kurtosis of these three variables were within the range of -1 to 1. Thus, the normality of awareness, interest, and knowledge was acceptable for the purpose of the study.

Table 3

Normality of the Three Continuous Variables

	Skewness	*Kurtosis*
Awareness	0.21	-0.22
Interest	-0.36	-0.43
Knowledge	0.17	-0.35

Outliers of the Three Continuous Variables

Univariate outliers were checked by using “Explore” with SPSS. The two boxplots of Figure 6 and Figure 7 described that no univariate outliers were shown on the variables awareness and interest. The boxplot of Figure 8 described that case 22, 23, 24, and 127 were univariate outliers of the variable knowledge. Theoretically, they should be deleted from the sample, but they were kept for this study because the sample size was not very big. If these four cases were deleted from the sample, the power of analysis would be reduced by deletion.

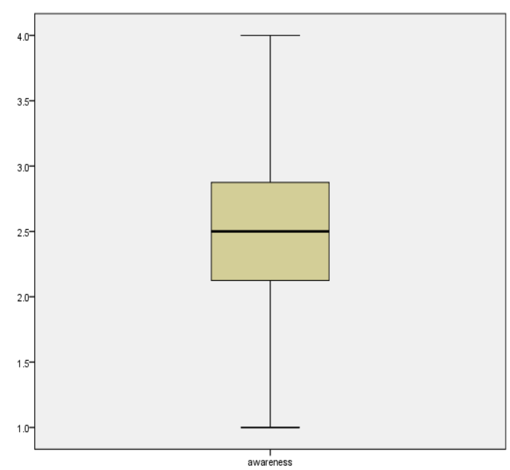


Figure 6. The Boxplot of Awareness

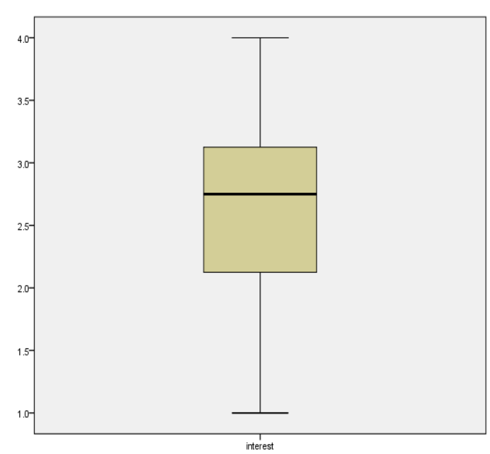


Figure 7. The Boxplot of Interest

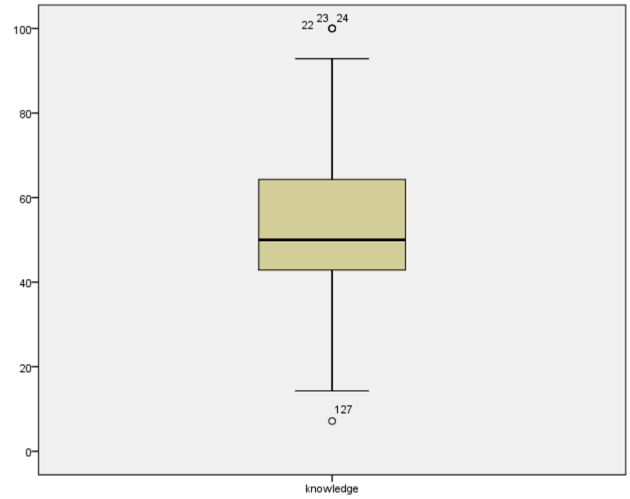


Figure 8. The Boxplot of Knowledge

Multivariate outliers were checked by using Mahalanobis distance through "Regression" and "Explore" with SPSS. Figure 7 showed the statistics for extreme values. The boxplot (see Figure 9) marked the multivariate outliers. These outliers numbered cases 24, 51, 123, and 126 were above the upper inner fence. Theoretically, all multivariate outliers were possible candidates for deletion. For this study, these cases would not be deleted from the sample considering the reduction of the power of analysis.

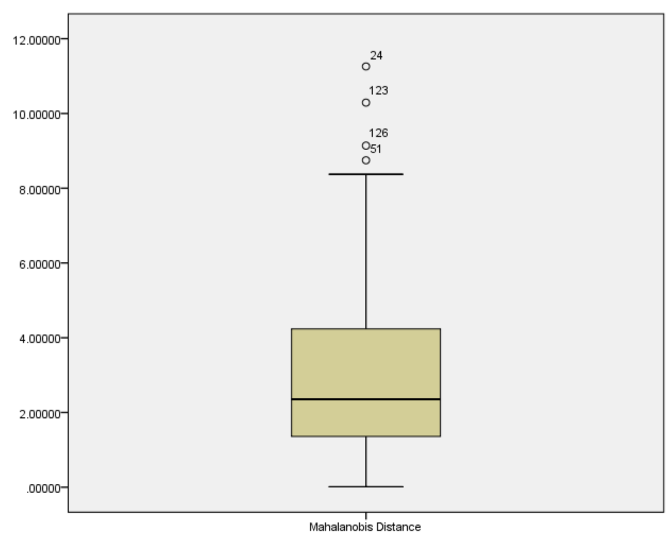


Figure 9. The Boxplot of Mahalanobis Distance

Linearity

"Graphs" with SPSS was conducted to determine if the selected continuous variables are linearly related to each other. Figure 10 was the scatterplots matrix which was used to examine the shape of the bivariate scatterplots for each combination of the selected continuous variables. The shapes of these scatterplots were certainly good. The normality of each variable had been checked and no violations were found. Therefore the linearity describing the scatter plot would be suitable to proceed with the analysis.

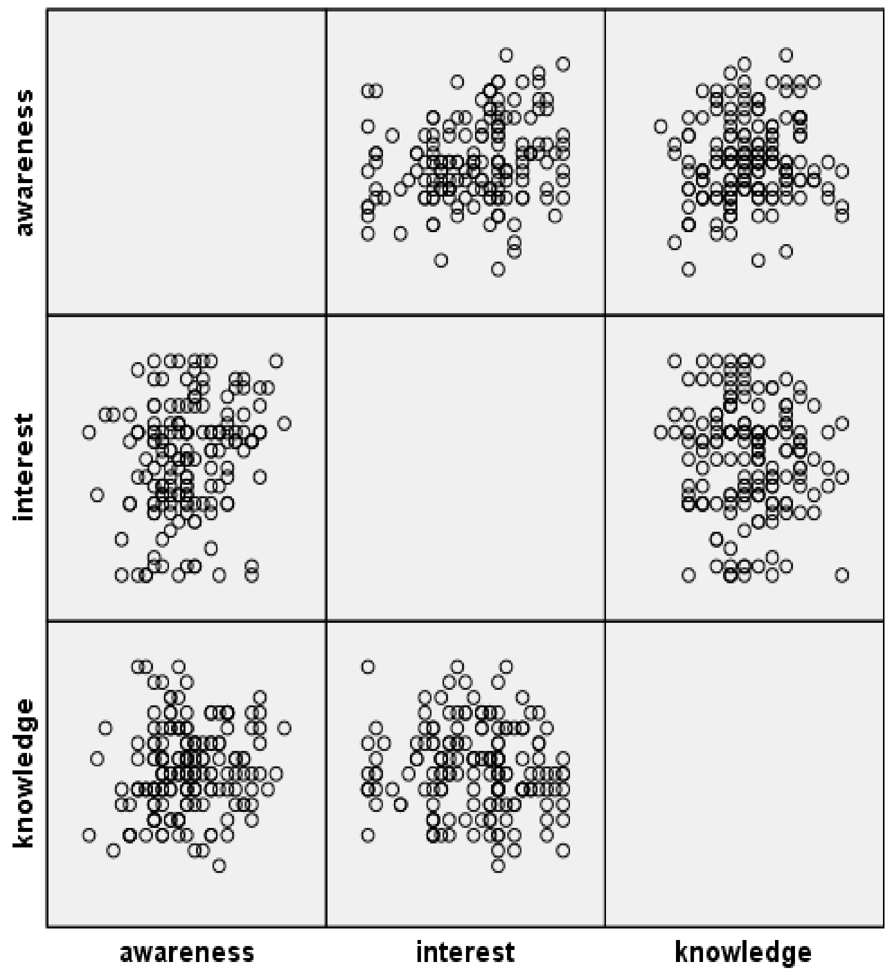


Figure 10. Scatterplots Matrix for the Selected Continuous Variables with Each Other.

Homoscedasticity

“Regression” with SPSS was conducted to test the homoscedasticity. The residuals scatterplot was used to detect the homoscedasticity multiple regression violation. Figure 11, 12, and 13 were the scatterplots of standardized residuals for awareness, interest, and knowledge respectively. One could see that the regression standardized residuals of all cases in these three figures were within the range of -3 to 3. No residual outliers existed in the data set. Further the three scatterplots looked very good because the plots came up with consistent patterns and they were flat. Therefore the homoscedasticity of the dependent variables awareness, interest, and knowledge was not violated in this study.

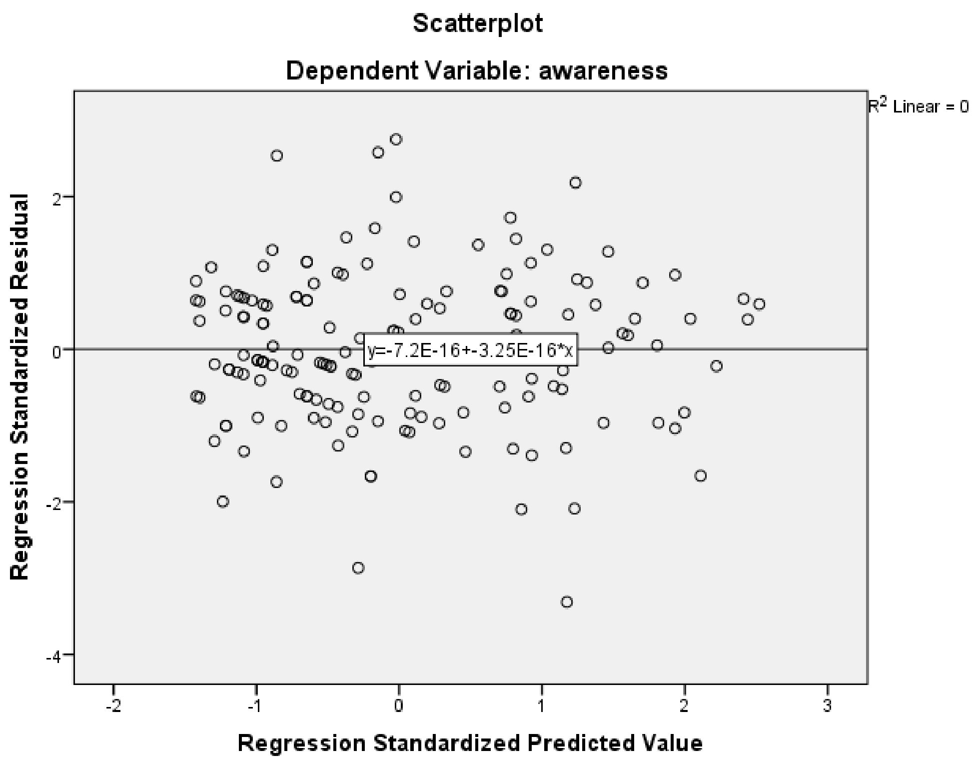


Figure 11. Residual Scatterplot of Awareness

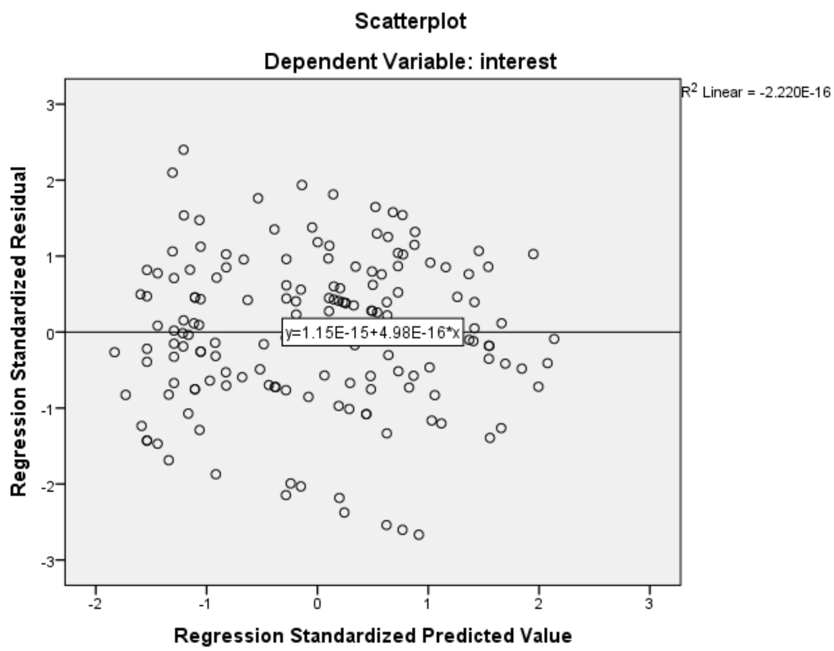


Figure 12. Residual Scatterplot of Interest

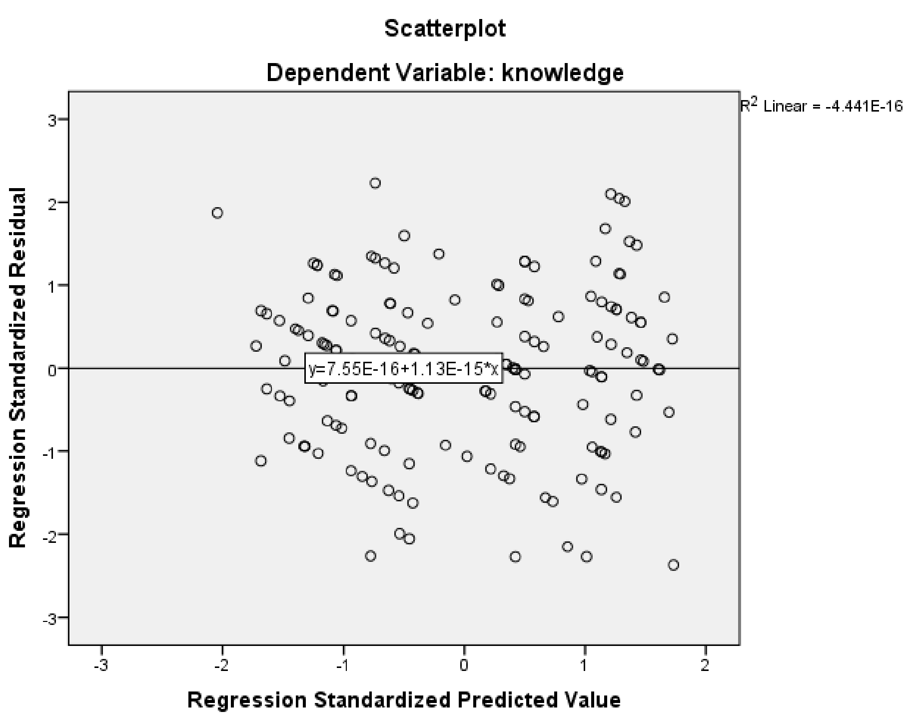


Figure 13. Residual Scatterplot of Knowledge

Multicollinearity

The multicollinearity of the predictors was tested by running "Linear Regression" with SPSS and using the statistics of Variable Inflation Factors (VIF). Table 4 gave the VIF values when using one predictor as a dependent variable (the first row) and the other predictors as independent variables (the first column). All the VIF values were less than 3 or just a little above 3 which indicated that the multicollinearity of the predictors was not a problem.

Table 4

VIF Statistics of the Predictors

	Nation	*Gender*	*Age*	*Degree*	*Years*[a]	*Level*[b]	*LeFra*[c]	*InFra*[d]
Nation		1.12	1.16	1.22	1.14	1.22	1.21	1.20
Gender	1.01		1.10	1.11	1.10	1.10	1.11	1.10
Age	3.19	3.35		3.33	1.08	3.35	3.37	3.22
Degree	1.07	1.07	1.06		1.07	1.05	1.05	1.06
Years[a]	3.24	3.46	1.11	3.47		3.45	3.46	3.30
Level[b]	1.13	1.13	1.13	1.11	1.12		1.09	1.10
LeFra[c]	1.33	1.34	1.34	1.32	1.34	1.30		1.09
InFra[d]	1.39	1.41	1.35	1.40	1.34	1.41	1.14	

Note: [a] refers the years of teaching math; [b] refers to grade levels; [c] refers to experience of learning fractals; [d] refers to experience of integrating fractals

Being comprehensively considered the analyses of univariate outliers, multivariate outliers, and residual outliers, a total of 166 cases were good to proceed with the analysis and no multiple linear regression violations were found in this data set. Thus, the final resulted sample was 166 and the number of the predictors was 8.

Descriptive Statistics

In order to acquire the detailed information of data, descriptive data analyses were separately conducted for categorical variables, ordinal variables, and continuous variables. Table 5 gave the frequency statistics of gender which indicated that more male math teachers (57.6%) participated in the study in China and less male math teachers participated in the study in the U.S. (28.4%) compared with females.

Table 5

Frequency Table of Gender

		Frequency	*Percent*
China (N=85)	male	49	57.6%
	female	36	42.4%
U.S. (N=81)	male	23	28.4%
	female	58	71.6%

Table 6 gave the frequency statistics of age range in both the U.S. and China. The largest percentage of participants in the U.S. group was the interval 46 or above, and the smallest percentage was the interval 41-45. The largest percentage of participants in the Chinese group was the interval 31-35, and the smallest percentage was the interval 20-25.

Table 6

Frequency Table of Age

	China (N=85)		*U.S. (N=81)*	
	Frequency	*Percent*	*Frequency*	*Percent*
20-25	6	7.1%	12	14.8%
26-30	9	10.6%	13	16.0%
31-35	25	29.4%	12	14.8%
36-40	17	20.0%	12	14.8%
41-45	17	20.0%	8	9.9%
46 or above	11	12.9%	24	29.6%

From Table 7, the majority of participants from the U.S. earned a master degree (51.9%) and the majority of participants from China earned a bachelor degree (65.9%). No participants earned doctorate degrees in either country. No participants earned the specialist degree in China, and no participants answered other types in the U.S. Later, in the section of data analysis, the degree variable was considered as two cases: bachelor's and master's since the percentage of other types were below 10%.

Table 7

Frequency Table of Degree

	China (N=85)		*U.S. (N=81)*	
	Frequency	*Percent*	*Frequency*	*Percent*
Bachelor	56	65.9%	37	45.7%
Master	21	24.7%	42	51.9%
Specialist	0	0.0%	2	2.5%
Doctorate	0	0.0%	0	0.0%
Other types	8	9.4%	0	0.0%

Table 8 showed that the highest percentage of the years of teaching math was a range of 11-15 (32.9%) from China and a range of 0-5 (37.0%) from the U.S. The lowest percentage of the years of teaching math was a range of 21-25 (32.9%) from both China (4.7%) and the U.S. (7.4%).

Table 8

Frequency Table of the Years of Teaching Math (Years)

	China (N=85)		*U.S. (N=81)*	
	Frequency	*Percent*	*Frequency*	*Percent*
0-5	11	12.9%	30	37.0%
6-10	15	17.6%	16	19.8%
11-15	28	32.9%	9	11.1%
16-20	16	18.8%	10	12.3%
21-25	4	4.7%	6	7.4%
26 or above	11	12.9%	10	12.3%

Table 9 showed that the percentages of grade 6-9 and grade 10-12 participants were very close from both China and the U.S. But the ninth grade in China belongs to middle school level, while it belongs to high school level in the U.S. The participants who taught in middle schools and high schools were very similar percentages from both China and the U.S.

Table 9

Frequency Table of Grade Levels

	China (N=85)		*U.S. (N=81)*	
	Frequency	*Percent*	*Frequency*	*Percent*
6-9	42	49.4%	43	53.1%
10-12	43	50.6%	38	46.9%

Table 10 demonstrated that the participants who had experience of learning fractals had very similar percentages between China (34.1%) and the U.S. (33.3%).

Table 10

Frequency Table of Experience of Learning Fractals

	China (N=85)		*U.S. (N=81)*	
	Frequency	*Percent*	*Frequency*	*Percent*
yes	29	34.1%	27	33.3%
no	56	65.9%	54	66.7%

From Table 11, the percentage of the participants who had the experience of integrating fractals in China (30.6%) was higher than the percentage of the participants who had the experience of integrating fractals in the U.S. (14.8%).

Table 11

Frequency Table of Experience of Integrating Fractals

	China (N=85)		*U.S. (N=81)*	
	Frequency	*Percent*	*Frequency*	*Percent*
yes	26	30.6%	12	14.8%
no	59	69.4%	69	85.2%

Table 12 showed that most of math teachers were aware of the concept of geometric transformation (item 13), geometric sequences (item 14), and similar figures (item 15), but were barely aware of the concept of geometric iteration (item 16), self-similarity (item 17), magnification factor of a fractal (item 18), fractal dimension (item 19), and how to create a fractal (item20).

Table 12

Descriptive Statistics of Awareness of Fractal Geometry

Items	*China (N=85)*			*U.S (N=81)*		
	Mode[a]	*Mean*	*SD*	*Mode*[a]	*Mean*	*SD*
13	4	3.51	0.73	4	3.25	1.03
14	4	3.85	0.48	4	3.23	0.88
15	4	3.82	0.47	4	3.62	0.70
16	2	2.32	1.07	1	2.22	1.08
17	1	2.09	1.08	2	2.19	1.01
18	1	1.58	0.89	1	1.69	0.96
19	1	1.51	0.85	1	1.74	0.89
20	1	1.86	0.95	1	1.75	0.92

Note: The awareness scores were based on a 4 point scale, where 1= “completely unknowing,” 2 = “somewhat unknowing,” 3 = “somewhat knowing,” 4 = “completely knowing.”

The results from this table revealed that the participants in China were a little more aware of geometric transformation, geometric sequences, similar figures, geometric iteration, and how to create a fractal than the participants in the U.S., but were a little less aware of self-similarity, magnification factor of fractals, and fractal dimension than the participants in the U.S.

Table 13 described the current level of interest on a scale of 4 that the participants would like to do the things demonstrated in the items from 21 to 28. The statistics of the mode and mean in Table 13 indicated that the participants from China intended to be some-what not interested in the things described in the items from 21 to 24 (mode = 2, means were close to 2), while the participants from the U.S. intended to be somewhat interested in the things described in the items from 21 to 24 (mode = 3, means were close to 3). The participants from both China and the U.S. showed a very similar level on the items from 25 to 28, but each mean of these items showed that the participants from the U.S. were a little more interested in the things described in items 25 to 28 than the participants from China.

Table 13

Descriptive Statistics of Interest in Fractal Geometry

Items	*China (N=85)*			*U.S (N=81)*		
	Mode^b	*Mean*	*SD*	*Mode*^{b}	*Mean*	*SD*
21	2	2.26	0.92	3	2.82	0.91
22	2	2.32	0.80	3	2.91	0.84
23	2	2.14	0.97	3	2.96	0.90
24	2	2.25	0.82	3	3.02	0.87
25	3	2.52	0.96	3	2.86	0.89
26	3	2.54	0.92	3	2.81	0.82
27	3	2.75	0.87	3	2.81	0.96
28	3	2.60	0.90	3	2.78	0.88

Note: The interest scores were based on a 4 point scale, where 1 = “completely disagree,” 2 = “somewhat disagree,” 3 = “somewhat agree,” 4 = “completely agree.”

Table 14 described the average score and standard deviation of each problem from both Chinese and U.S. participants. The participants from the U.S. got little higher average scores on problems 29 (0.93), 30 (0.89), and 33 (0.77) than the participants from China (0.87, 0.82, and 0.75 respectively). Problems 29 and 30 were used to test the knowledge of geometric translation and reflection. Problem 33 was used to test the concept of self-similarity. The participants from the U.S. got lower average scores on all other problems than the participants from China.

Table 14

Descriptive Statistics of Knowledge of Fractal Geometry

Problems	*Mean*		*SD*	
	China (N=85)	*U.S. (N=81)*	*China(N=85)*	*U.S.(N=81)*
29	0.87	0.93	0.34	0.26
30	0.82	0.89	0.38	0.31
31	0.79	0.43	0.41	0.50
32	0.52	0.33	0.50	0.47
33	0.75	0.77	0.43	0.43
34	0.80	0.46	0.40	0.50
35	0.53	0.22	0.50	0.42
36	0.69	0.42	0.46	0.50
37	0.62	0.25	0.49	0.43
38	0.68	0.17	0.47	0.38
39	0.32	0.16	0.47	0.37
40	0.49	0.26	0.50	0.44
41	0.52	0.32	0.50	0.47
42	0.35	0.19	0.48	0.39

Table 15 gave the statistics mean and standard deviation of the three dependent variables awareness, interest, and knowledge. The participants from China appeared to be more aware of the concepts than the participants from the U.S., but less interested in learning and integrating fractals than the participants from the U.S. The participants from China appeared to have more knowledge of fractals than the participants from the U.S. from Table 15, which matched the information of awareness from Table 14. Additionally, the standard deviations of knowledge were very large: China (SD = 17.69) and the U.S. (SD = 15.06) compared to awareness' and interests' standard deviations.

Table 15

Descriptive Statistics of the Three Dependent Variables

	Mean		*SD*	
	China (N=85)	*U.S. (N=81)*	*China(N=85)*	*U.S.(N=81)*
Awareness	2.57	2.46	0.49	0.66
Interest	2.42	2.88	0.72	0.76
Knowledge	62.60	41.36	17.69	15.06

Test of Hypothesis

After the data screening, the final data set resulted in a sample of N=166. A standard multiple regression model was selected for this study in order to know the effect of each predictor in the criterion. Linear Regression with SPSS was conducted to run three standard regression analyses. In order to run the multiple regressions, "Recode" with SPSS was used to transfer categorical variables into dichotomies. According to Meyers, Gamst, and Guarino (2006), the reference group should have a relatively large sample size. According to frequencies of analyses, the group of China should be the reference group of nationality. The female group should be the reference group of gender. The group of ages 31-35 should be the reference group of age. The bachelor degree groups should be the reference group of degree. The years 0-5 group should be the reference group of the years of teaching math. The grades 6-9 should be the reference group of grade levels. The answer "no" group should be the reference group of experience of learning fractals and experience of integrating fractals. After recoding into different names, the new independent variables were: U.S. (renamed from nation), male (renamed from gender), age 20-25, age 26-30, age 36-40, age 41-45, age above 45, M.S. degree (renamed from degree), years 6-10, years 11-15, years 16-20, years 21-25, years above 25, high levels (renamed from grade levels), LearnedFra (renamed from experience of learning fractals), and IntegratedFra (renamed from experience of integrating fractals).

Test of Ha1

To test the first hypothesis Ha1, awareness was used as a dependent variable. Table 16 provided the statistics of collinearity for each independent variable. The tolerance within the range 0.2 or less, or the VIF greater than 5 indicates that multicollinearity is present. From Table 16, all the values of tolerance were greater than 0.2 and all the values of the VIF were less than 5. Hence, both indexes of each variable were within the normal bounds. This indicated that no multicollinearity was present in this study. Additionally, from the output table of Collinearity Diagnostics, the greatest value of the condition index was 12.57 and the smallest value of the condition index was 1.00. According to Meyers, Gamst, and Guarino (2006), if all the values of the condition index were within a range less than 30, it indicates that multicollinearity is not present. Again, this index also verified that there was no multicollinearity shown in this study.

Table 17 provided the statistics from the output tables of Model Summary and ANOVA. The significant F value, $F (16, 149) = 4.90$, $p < 0.01$ (critical alpha value was set 0.05 in this study), showed that there was a significant relationship between the weighted linear composite of the predictors as specified by the model and the criterion. The R^2 coefficient of 0.35 indicated that 35% of the variance of the criterion variable was explained by the combination of the selected independent variables in this model.

Table 16

Collinearity Statistics of the Standard Multiple Regression Analysis

	Collinearity Statistics	
	Tolerance	*VIF*
U.S.	0.65	1.55
Male	0.83	1.21
Age 20-25	0.46	2.20
Age 26-30	0.57	1.77
Age 36-40	0.54	1.85
Age 41-45	0.42	2.38
Age above 45	0.26	3.90
M.S. degree	0.85	1.17
Years 6-10	0.44	2.26
Years 11-15	0.30	3.35
Years 16-20	0.27	3.72
Years 21-25	0.43	2.32
Years above25	0.22	4.49
High levels	0.79	1.27
LearnedFra	0.73	1.37
IntegratedFra	0.69	1.46

Table 17

ANOVA Summary Table of the Standard Multiple Regression (DV = Awareness)

Source	*SS*	*df*	*MS*	*F*	*p*	R^2
Regression	19.30	16	1.21	4.90	0.01*	0.35
Residual	36.68	149	0.25			
Total	55.97	165				

*Refers to less than 0.01

Table 18 provided some of the information of the SPSS Coefficients table and Correlations table, which gave us the Pearson correlation r, the unstandardized b coefficients, the standardized beta coefficients, the squared semipartial correlations, the structure coefficients, and the t values and their corresponding p values. The t and p values showed that the variable U.S. (t = -0.05, p = 0.96), male (t = 1.32, p = 0.19), age

20-25 (t = 0.61, p = 0.55), age 26-30 (t = 1.36, p = 0.18), age 36-40 (t = -0.74, p = 0.46), age 41-45 (t = -1.30, p = 0.20), age above 45 (t = 0.36, p = 0.72), M.S. degree (t = 1.33, p = 0.19), years 6-10 (t = 1.00, p = 0.32), years above 25 (t = 1.36, p = 0.18), and high levels (t = 0.76, p = 0.45) statistically had no significant relationship with the criterion. The other t and p values, years 11-15 (t = 2.23, p = 0.03), years 16-20 (t = 2.62, p = 0.01), years 21-25 (t = 2.66, p = 0.01), LearnedFra (t = 3.72, p < 0.01), and IntegratedFra (t = 2.92, p < 0.01) showed that all these independent variables had significant relationships with the criterion.

Further, the values of the Pearson correlations showed that the variable “age 26-30” (r = -0.01) had the least correlation with the criterion, and then the next was “years 11-15” (r = 0.04). The variable “LearnedFra” (r = 0.43) had the most correlation with the criterion, and then the variable “IntegratedFra” (r = 0.40) had the second place of correlations with the criterion. The structure coefficients were calculated by hand using the formula:

$$\text{Structure Coefficients} = \frac{r_{IV \times DV}}{R}.$$

Meyers, Gamst, and Guarino (2006) argued that, ‘‘the structure coefficient indexes the correlation between the predictor and the variate; stronger correlations indicates that the predictor is a stronger reflection of the construct underlying the variate” (p. 163). The order of the structure coefficients in Table 18 showed that “LearnedFra” (structure coefficient = 0.74) was the strongest reflection of the construct underlying the variate; the variable “IntegratedFra” (structure coefficient = 0.69) had the second place; the variable “high levels” (structure coefficient = 0.39) had the third place.

The squared semipartial correlation indexes (see Table 18, they were calculated by squaring the index of part correlation in the output of Coefficients table) described the variance accounted for uniquely by each predictor in the full model. The top five of the predictors' unique contributions to the prediction model from the largest to smallest was "LearnedFra" ($Sr^2 = 0.06$), "IntegratedFra" ($Sr^2 = 0.04$), "Years 21-25" ($Sr^2 = 0.03$), "Years 16-20" ($Sr^2 = 0.03$) and "Years 11-15" ($Sr^2 = 0.02$).

Table 18

Summary Table of the Standard Multiple Regression (DV=Awareness)

Variable	*b*	*beta*	*r*	Sr^2	*Structure coefficient*	*t*	*p*
U.S.	-.01	-.01*	-.09	.01*	-.15	-.05	.96
Male	.11	.10	.09	.01	.16	1.32	.19
Age 20-25	.11	.06	-.09	.01*	-.16	.61	.55
Age 26-30	.21	.12	-.01	.01	-.01	1.36	.18
Age 36-40	-.10	-.07	-.06	.01*	-.10	- .74	.46
Age 41-45	-.22	-.13	.07	.01	.12	-1.30	.20
Age above 45	.07	.05	.14	.01*	.24	.36	.72
M.S. degree	.12	.10	.12	.01	.20	1.33	.19
Years 6-10	.15	.10	-.14	.01*	-.24	1.00	.32
Years 11-15	.38	.27	.04	.02	.07	2.23	.03
Years 16-20	.53	.33	.12	.03	.20	2.62	.01
Years 21-25	.66	.27	.14	.03	.24	2.66	.01
Years above[b]	.33	.19	.13	.01	.22	1.36	.18
High levels	.07	.06	.23	.01*	.39	.76	.45
LearnedFra	.36	.29	.43	.06	.74	3.72	.01*
IntegratedFra	.32	.23	.40	.04	.69	2.92	.01*
Constant	1.92						

Note: Sr^2 refers to squared semi-partial correlation; [b] refers to the years of teaching above 25; * refers to less than 0.01.

My hypothesis was: the factors degree, grade levels, experience of learning fractals, and experience of integrating fractals will account for more variance than gender, age, the years of teaching math, and nationality in the overall mean awareness survey scores. The actual statistics results indicated that the factors experience of learning fractals, experience of integrating fractals, and the years of teaching math accounted for more variance than degree, grade levels, gender, age, and nationality in the overall mean awareness survey scores.

The regression equation was produced below by using b weights from Table 18.

The raw score equation is as follows:

Awareness (predicted) = 1.92 – (0.01) (U.S.) + (0.11) (Male) + (0.11) (age 20-25) +(0.21) (age 26-30) – (0.10) (age 36-40) – (0.22)(age 41-45) + (0.07) (age above 45) + (0.12) (M.S. degree) + (0.15) (years 6-10) + (0.38) (years 11-15) + (0.53) (years 16-20) + (0.66) (years 21-25) + (0.33) (years above 25) + (0.07) (high levels) + (0.36) (LearnedFra) + (0.32) (IntegratedFra).

The explanation of b weights of the predictors was made as follows:

1. When controlling for the other predictors, the group of U.S. mathematics teachers scored awareness on average 0.01 times less than the group of Chinese mathematics teachers.
2. When controlling for the other predictors, the male group scored awareness on average 0.11 times greater than the female group.
3. When controlling for the other predictors, the group of age 20-25 scored awareness on average 0.11 times greater than the group of age30-35.
4. When controlling for the other predictors, the group of age 26-30 scored awareness on average 0.21 times greater than the group of age 30-35.
5. When controlling for the other predictors, the group of age 36-40 scored awareness on average 0.10 times less than the group of age30-35.

6. When controlling for the other predictors, the group of age 41-45 scored awareness on average 0.22 times less than the group of age30-35.
7. When controlling for the other predictors, the group of age above 45 scored awareness on average 0.07 times greater than the group of age30-35.
8. When controlling for the other predictors, the group of mathematics teachers who earned a master’s degree scored awareness on average 0.12 times greater than the group of mathematics teachers who earned a bachelor’s degree.
9. When controlling for the other predictors, the group of mathematics teachers who have taught math from 6-10 years scored awareness on average 0.15 times greater than the group of mathematics teachers who have taught math from 0-5 years.
10. When controlling for the other predictors, the group of mathematics teachers who have taught math from 11-15 years scored awareness on average 0.38 times greater than the group of mathematics teachers who have taught math from 0-5 years.
11. When controlling for the other predictors, the group of mathematics teachers who have taught math from 16-20 years scored awareness on average 0.53 times greater than the group of mathematics teachers who have taught math from 0-5 years.
12. When controlling for the other predictors, the group of mathematics teachers who have taught math from 21-25 years scored awareness on average 0.66 times greater than the group of mathematics teachers who have taught math from 0-5 years.

13. When controlling for the other predictors, the group of mathematics teachers who have taught math 25 years or more scored awareness on average 0.33 times greater than the group of mathematics teachers who have taught math from 0-5 years.
14. When controlling for the other predictors, the group of mathematics teachers who taught math in grade levels 10-12 scored awareness on average 0.07 times greater than the group of mathematics teachers who have taught math in grade levels 6-9.
15. When controlling for the other predictors, the group of mathematics teachers who had experience of learning fractals scored awareness on average 0.36 times greater than the group of mathematics teachers who had no experience of learning fractals.
16. When controlling for the other predictors, the group of mathematics teachers who had experience of integrating fractals in class scored awareness on average 0.32 times greater than the group of mathematics teachers who had no experience of integrating fractals in class.

Test of Ha2

To test the second hypothesis Ha2, interest was used as a dependent variable. The independent variables were now U.S., male, age 20-25, age 26-30, age 36-40, age 41-45, age above 45, M.S. degree, years 6-10, years 11-15, years 16-20, years 21-25, years above 25, high levels, LearnedFra, and IntegratedFra.

The statistics of collinearity for each independent variable was provided in Table 16 and analyzed before. Table 19 provided the statistics from the output tables of Model Summary and ANOVA. The significant F value, $F(16, 149) = 3.00$, $p<0.01$, showed that there was a significant relationship between the weighted linear composite of the predictors as specified by the model and the criterion. The R^2 coefficient of 0.24 indicated that 24% of the variance of the criterion variable was explained by the combination of the selected independent variables in this model.

Table 19

ANOVA Summary Table of the Standard Multiple Regression (DV = Interest)

Source	*SS*	*df*	*MS*	*F*	*p*	R^2
Regression	24.01	16	1.50	3.00	0.01*	0.24
Residual	74.60	149	0.50			
Total	98.62	165				

*Refers to less than 0.001

Table 20 provided some of the information of the SPSS Coefficients table and Correlations table, which gave us the Pearson correlation r, the unstandardized b coefficients, the standardized beta coefficients, the squared semipartial correlations, the structure coefficients, and the t values and their corresponding p values. The t and p values showed that the variables, male (t = 0.19, p = 0.85), age 20-25 (t = -0.37, p = 0.72), age 26-30 (t = -0.57, p = 0.57), age 41-45 (t = 0.80, p = 0.42), age above 45 (t = 1.44, p = 0.15), M.S. degree (t = 1.62, p = 0.11), years 6-10 (t = 0.42, p = 0.68), years 11-15 (t = -1.96, p = 0.05), years 16-20 (t = -0.23, p = 0.82), years 21-25 (t = -0.45, p = 0.66), years above 25 (t = -1.04, p = 0.30), high levels (t = -0.61, p = 0.54), and LearnedFra (t = 1.50, p = 0.14), statistically had no significant relationship with the criterion. The other t and p values, U.S. (t = 2.54, p = 0.01), age 36-40 (t = 2.22, p = 0.03) and IntegratedFra (t = 2.31, p = 0.02) showed that these three independent variables had significant relationships with the criterion.

Further, the values of the Pearson correlations showed that the variables age 41-45" and age 20-25 ($r = 0.01$) had the least correlation with the criterion. The independent variable "U.S." ($r = 0.29$) had the most correlation with the criterion, and then the independent variable "LearnedFra" ($r = 0.18$) had the second place of correlations with the criterion. The order of the structure coefficients in Table 20 showed that "U.S." (structure coefficient = 0.61) was the strongest reflection of the construct underlying the variate; the variable "master degree" (structure coefficient = 0.42) had the second place; the variable "LearnedFra" (structure coefficient = 0.37) had the third place.

The squared semipartial correlation indexes (see Table 20) were calculated by squaring the index of part correlation in the output of Coefficients table described the variance accounted for uniquely by each predictor in the full model. The top four of the predictors' unique contributions to the prediction model from the largest to smallest was "U.S." ($Sr^2 = 0.03$), "IntegratedFra" ($Sr^2 = 0.03$), "age 26-30" ($Sr^2 = 0.03$), and "years 11-15" ($Sr^2 = 0.02$). My hypothesis was: the factors degree, grade levels, experience of learning fractals, and experience of integrating fractals will account for more variance than gender, age, the years of teaching math, and nationality in the overall mean interest in integrating fractal geometry in the math core curriculum. The actual statistics results indicated that the factors nationality, experience of integrating fractals, age, and the years of teaching math accounted for more variance than degree, experience of learning fractals, grade levels, and gender in the overall mean interest survey scores.

Table 20

Summary Table of the Standard Multiple Regression (DV= Interest)

Variable	*b*	*beta*	*r*	*Sr^2*	*Structure*	*t*	*p*

					coefficient		
U.S.	.35	.23	.29	.03	.61	2.54	.01
Male	.02	.02	-.09	.01*	-.19	.19	.85
Age 20-25	-.10	-.04	.01	.01*	.02	-.37	.72
Age 26-30	-.12	-.05	-.02	.01*	-.05	-.57	.57
Age 36-40	.44	.22	.07	.03	.15	2.22	.03
Age 41-45	.19	.09	.01	.01*	.02	.80	.42
Age above 45	.38	.20	.14	.01	.28	1.44	.15
M.S. degree	.20	.13	.21	.01	.42	1.62	.11
Years 6-10	.09	.05	.07	.01*	.15	.42	.68
Years 11-15	-.47	-.26	-.26	.02	-.53	-1.96	.05
Years 16-20	-.07	-.03	.12	.01*	.13	-.23	.82
Years 21-25	-.16	-.05	.05	.01*	.11	-.45	.66
Years above[b]	-.36	-.16	.04	.01	.08	-1.04	.30
High levels	-.08	-.05	.07	.01*	.14	-.61	.54
LearnedFra	.20	.13	.18	.01	.37	1.50	.14
IntegratedFra	.37	.20	.16	.03	.31	2.31	.02
Constant	2.27						

Note: Sr^2 refers to squared semi-partial correlation; [b] refers to the years of teaching above 25.

The regression equation was produced below by using b weights from Table 20.

The raw score equation is as follows:

Interested (predicted) = 2.27 + (0.35) (U.S.) + (0.02) (Male) – (0.10) (age 20-25) – (0.12) (age 26-30) + (0.44) (age 36-40) + (0.19)(age 41-45) + (0.38) (age above 45) + (0.20) (M.S. degree) + (0.09) (years 6-10) – (0.47) (years 11-15) - (0.07) (years 16-20) – (0.16) (years 21-25) – (0.36) (years above 25) – (0.08) (high levels) + (0.20) (LearnedFra) + (0.37) (IntegratedFra).

The explanation of b weights of the predictors was made as follows:

1. When controlling for the other predictors, the group of U.S. mathematics teachers scored interest on average 0.35 times greater than the group of Chinese mathematics teachers.
2. When controlling for the other predictors, the male group scored interest on average 0.02 times greater than the female group.
3. When controlling for the other predictors, the group of age 20-25 scored interest on average 0.10 times less than the group of age30-35.
4. When controlling for the other predictors, the group of age 26-30 scored interest on average 0.12 times less than the group of age30-35.
5. When controlling for the other predictors, the group of age 36-40 scored interest on average 0.44 times greater than the group of age30-35.
6. When controlling for the other predictors, the group of age 41-45 scored interest on average 0.19 times greater than the group of age30-35.
7. When controlling for the other predictors, the group of age above 45 scored interest on average 0.38 times greater than the group of age30-35.
8. When controlling for the other predictors, the group of mathematics teachers who earned a master's degree scored interest on average 0.20 times greater than the group of mathematics teachers who earned a bachelor's degree.
9. When controlling for the other predictors, the group of mathematics teachers who have taught math from 6-10 years scored interest on average 0.09 times greater than the group of mathematics teachers who have taught math from 0-5 years.

10. When controlling for the other predictors, the group of mathematics teachers who have taught math from 11-15 years scored interest on average 0.47 times less than the group of mathematics teachers who have taught math from 0-5 years.
11. When controlling for the other predictors, the group of mathematics teachers who have taught math from 16-20 years scored interest on average 0.07 times less than the group of mathematics teachers who have taught math from 0-5 years.
12. When controlling for the other predictors, the group of mathematics teachers who have taught math from 21-25 years scored interest on average 0.16 times greater than the group of mathematics teachers who have taught math from 0-5 years.
13. When controlling for the other predictors, the group of mathematics teachers who have taught math 25 years or more scored interest on average 0.36 times less than the group of mathematics teachers who have taught math from 0-5 years.
14. When controlling for the other predictors, the group of mathematics teachers who taught math in grade levels 10-12 scored interest on average 0.08 times less than the group of mathematics teachers who have taught math in grade levels 6-9.
15. When controlling for the other predictors, the group of mathematics teachers who had experience of learning fractals scored interest on average 0.20 times greater than the group of mathematics teachers who had no experience of learning fractals.
16. When controlling for the other predictors, the group of mathematics teachers who had experience of integrating fractals in class scored interest on average 0.37 times greater than the group of mathematics teachers who had no experience of integrating fractals in class.

Test of Ha3

To test the third hypothesis Ha3, knowledge was used as a dependent variable. The independent variables were U.S., male, age 20-25, age 26-30, age 36-40, age 41-45, age above 45, master, years 6-10, years 11-15, years 16-20, years 21-25, years above 25, high levels, LearnedFra, and IntegratedFra.

Table 21 provided the statistics from the output tables of Model Summary and ANOVA. The significant F value, $F(16, 149) = 6.32$, $p < 0.01$, showed that there was a significant relationship between the weighted linear composite of the predictors as specified by the model and the criterion. The R^2 coefficient of 0.40 indicated that 40% of the variance of the criterion variable was explained by the combination of the selected independent variables in this model.

Table 21

ANOVA Summary Table of the Standard Multiple Regression (DV = Knowledge)

Source	*SS*	*df*	*MS*	*F*	*p*	R^2
Regression	25522.05	16	1595.13	6.32	0.01*	0.40
Residual	37626.04	149	252.52			
Total	63148.09	165				

* refers to less than 0.01

Table 22 provided some of the information of the SPSS Coefficients table and Correlations table, which gave us the Pearson correlation r, the unstandardized b coefficients, the standardized beta coefficients, the squared semipartial correlations, the structure coefficients, and the t values and their corresponding p values. The t and p values showed that the variable male ($t = -0.74$, $p = 0.46$), age 20-25 ($t = -0.03$, $p = 0.98$), age 26-30 ($t = -0.39$, $p = 0.70$), age 36-40 ($t = -1.63$, $p = 0.11$), age 41-45 ($t = -1,28$, $p = 0.20$), age above 45 ($t = -1.22$, $p = 0.23$), M.S. degree ($t = -0.01$, $p = 0.99$), years 6-10 ($t = -0.56$, $p = 0.58$), years 11-15 ($t = 0.61$, $p = 0.55$), years 16-20 ($t = -0.41$, $p = 0.69$), years

21-25 ($t = -0.34$, $p = 0.74$), years above 25 ($t = 0.64$, $p = 0.52$), LearnedFra ($t = -0.09$, $p = 0.93$), IntegratedFra ($t = 0.70$, $p = 0.49$) statistically had no significant relationship with the criterion. The other t and p values, U.S. ($t = -6.52$, $p < 0.01$) and high levels ($t = 4.36$, $p < 0.01$) showed that these two independent variables statistically had significant relationships with the criterion.

Further, the values of the Pearson correlations showed that the variable "years 16-20" ($r = 0.01$) had the least correlation with the criterion, and then the next was "age 36-40" ($r = - 0.02$). The independent variable "U.S." ($r = -0.55$) had the most correlation with the criterion, and then the independent variable "high levels" ($r = 0.26$) had the second place of correlations with the criterion. The order of the structure coefficients in Table 20 showed that "U.S." (structure coefficient = -0.86) was the negative strongest reflection of the construct underlying the variate; the variable "high levels" (structure coefficient = 0.41) had the positive strongest reflection of the construct underlying the variate; the variable "IntegratedFra" (structure coefficient = 0.33) had the positive second place.

The squared semipartial correlation indexes (see Table 20) described the variance accounted for uniquely by each predictor in the full model. The top five of the predictors' unique contributions to the prediction model from the largest to smallest was "U.S." ($Sr^2 = 0.17$), "high levels" ($Sr^2 = 0.08$), "age 36-40" ($Sr^2 = 0.01$), "age 41-45" ($Sr^2 = 0.01$), and "age above 45" ($Sr^2 = 0.01$). My hypothesis was: the factors nationality, degree, experience of learning fractals, and experience of integrating fractals, will account for more variance than gender, age, grade levels, and the years of teaching math in the overall mean knowledge test scores in performing fractal geometry problems. The actual statistics results indicated that the factors nationality, grade levels, and age accounted for more variance than gender, the years of teaching math, degree, experience of learning fractals, and experience of integrating fractals in the overall mean interest survey scores.

Table 22

Summary Table of the Standard Multiple Regression (DV= Knowledge)

Variable	*b*	*beta*	*r*	*Sr*^{2}	*Structure coefficient*	*t*	*p*
U.S.	-20.02	-.51	-.55	.17	-.86	-6.52	.01*
Male	-2.02	-.05	.15	.01*	.23	-.74	.46
Age 20-25	-0.17	-.01*	-.03	.01*	-.05	-.03	.98
Age 26-30	-1.87	-.03	-.05	.01*	-.07	-.39	.70
Age 36-40	-7.19	-.14	-.02	.01	-.04	-1.63	.11
Age 41-45	-6.78	-.12	.05	.01	.08	-1.28	.20
Age above 45	-7.29	-.15	-.12	.01	-.20	-1.22	.23
M.S. degree	-.03	-.01*	-.13	.01*	-.20	-.01	.99
Years 6-10	-2.65	-.05	-.08	.01*	-.13	-.56	.58
Years 11-15	3.29	.07	.20	.01*	.31	.61	.55
Years 16-20	-2.66	-.05	.01	.01*	.01	-.41	.69
Years 21-25	-2.68	-.03	-.04	.01*	-.06	-.34	.74
Years above[b]	5.04	.09	.04	.01*	.06	.64	.52
High levels	12.13	.31	.26	.08	.41	4.36	.01*
Learned Fra	-.27	-.01	.09	.01*	.14	-.09	.93
Integrated Fra	2.47	.05	.21	.01*	.33	.70	.49
Constant	60.28						

Note: Sr^2 refers to squared semi-partial correlation; [b] refers to the years of teaching above 25; * refers to less than 0.01.

The regression equation was produced below by using b weights from Table 22.

The raw score equation is as follows:

Knowledge (predicted) = 60.28 – (20.02) (U.S.) – (2.02) (Male) – (0.17) (age 20-25) – (1.87) (age 26-30) – (7.19) (age 36-40) – (6.78)(age 41-45) – (7.29) (age above 45) – (0.03) (M.S. degree) – (2.65) (years 6-10) + (3.29) (years 11-15) – (2.66) (years 16-20) – (2.68) (years 21-25) + (5.04) (years above 26) + (12.13) (high levels) – (0.27) (LearnedFra) + (2.47) (IntegratedFra).

The explanation of b weights of the predictors was made as follows:

1. When controlling for the other predictors, the group of U.S. mathematics teachers scored knowledge on average 20.02 times less than the group of Chinese mathematics teachers.
2. When controlling for the other predictors, the male group scored knowledge on average 2.02 times less than the female group.
3. When controlling for the other predictors, the group of age 20-25 scored knowledge on average 0.17 times less than the group of age30-35.
4. When controlling for the other predictors, the group of age 26-30 scored knowledge on average 1.87 times less than the group of age30-35.
5. When controlling for the other predictors, the group of age 36-40 scored knowledge on average 7.19 times less than the group of age30-35.
6. When controlling for the other predictors, the group of age 41-45 scored knowledge on average 6.78 times less than the group of age30-35.
7. When controlling for the other predictors, the group of age above 45 scored knowledge on average 7.29 times less than the group of age 30-35.
8. When controlling for the other predictors, the group of mathematics teachers who earned a master's degree scored knowledge on average 0.03 times less than the group of mathematics teachers who earned a bachelor's degree.
9. When controlling for the other predictors, the group of mathematics teachers who have taught math from 6-10 years scored knowledge on average 2.65 times less than the group of mathematics teachers who have taught math from 0-5 years.

10. When controlling for the other predictors, the group of mathematics teachers who have taught math from 11-15 years scored knowledge on average 3.29 times greater than the group of mathematics teachers who have taught math from 0-5 years.
11. When controlling for the other predictors, the group of mathematics teachers who have taught math from 16-20 years scored knowledge on average 2.66 times less than the group of mathematics teachers who have taught math from 0-5 years.
12. When controlling for the other predictors, the group of mathematics teachers who have taught math from 21-25 years scored knowledge on average 2.68 times less than the group of mathematics teachers who have taught math from 0-5 years.
13. When controlling for the other predictors, the group of mathematics teachers who have taught math 25 years or more scored knowledge on average 5.04 times greater than the group of mathematics teachers who have taught math from 0-5 years.
14. When controlling for the other predictors, the group of mathematics teachers who taught math in grade levels 10-12 scored knowledge on average 12.13 times greater than the group of mathematics teachers who have taught math in grade levels 6-9.
15. When controlling for the other predictors, the group of mathematics teachers who had experience of learning fractals scored knowledge on average 0.27

times less than the group of mathematics teachers who had no experience of learning fractals.

16. When controlling for the other predictors, the group of mathematics teachers who had experience of integrating fractals in class scored knowledge on average 2.47 times greater than the group of mathematics teachers who had no experience of integrating fractals in class.

Summary

In summary, the statistical analysis from this study as shown in Table 17 revealed that 35% of the variance in awareness scores could be accounted for by the respondent's nationality, gender, age, degree, the years of teaching math, grade levels, experience of learning fractals, and experience of integrating fractals. It was also found that there was a statistically significant relationship between the weighted linear composite of the predictors as specified by the model and the criterion. The first hypothesis Ha1 was not fully supported by the results shown in Table 18. The factors experience of learning math, experience of integrating math, and the years of teaching math accounted for more variance on awareness scores than the other factors.

The statistical analysis from this study, as shown in Table 19, revealed that 24% of the variance in interest scores could be accounted for by the respondent's nationality, gender, age, degree, the years of teaching math, grade levels, experience of learning fractals, and experience of integrating fractals. It was also found that there was a statistically significant relationship between the weighted linear composite of the predictors as specified by the model and the criterion. The second hypothesis Ha2 was not fully supported by the results from Table 20. The factors nationality, experience of integrating math, age, the years of teaching math, and degree accounted for more variance on interest scores than other factors.

The statistical analysis from this study, as shown in Table 21, revealed that 40% of the variance in knowledge scores could be accounted for by the respondent's nationality, gender, age, degree, the years of teaching math, grade levels, experience of learning fractals, and experience of integrating fractals. It was also found that there was a statistically significant relationship between the weighted linear composite of the predictors as specified by the model and the criterion. The third hypothesis Ha3 was not fully supported by the results from Table 22. The factors nationality, grade levels, and age accounted for more variance on knowledge scores than other factors.

Finally, three linear models described how the b weight of each predictor revealed the relationship between the reference group and the targeted group when controlling for the other predictors.

CHAPTER V

CONCLUSION AND DISCUSSION

Introduction

The study was set out to investigate the levels of Chinese and U.S. secondary math teachers' awareness of, interest in, and knowledge of fractal geometry, as well as any factors that influence that level of awareness, interest, and knowledge. To achieve this goal, the researcher developed the survey and test instruments to gather the information of secondary mathematics teachers' awareness of, interest in, and knowledge of fractal geometry, as well as their demographic information. Then the researcher conducted three multiple regression analyses with SPSS to get the statistical results and findings. This chapter mainly focused on drawing conclusions and making discussions from the statistical results and findings. Additionally, some limitations and suggestions were made for the references of future studies.

Summary of Procedures

This study was conducted with mathematics teachers at secondary schools in the greater metropolitan area of Hattiesburg, MS, and the greater metropolitan area of Shanghai, China. The data was collected from January to March when secondary mathematics teachers were in sessions in both the U.S. and China in 2015. The instruments were developed by the researcher based on the literature review. The survey instrument was examined by two science education professors and one educational statistics professor. The test instrument was examined by three mathematics professors. The reliability tests of the survey of awareness and interest were conducted with a sample of 166. The overall Cronbach's alphas of both tests were above 0.7 (0.80 and 0.94

respectively). The test-retest method with 10 secondary mathematics teachers was used to test the reliability of the test instrument. The overall Cronbrach's alpha was 0.90.

Data was collected in three different ways: electronic copies, hard copies, and online. The researcher developed two language versions, English and Chinese, respectively used for U.S. and Chinese secondary teachers. For the electronic and hard copies, the specified data collectors in different schools were responsible for distributing the recruitment letter and gathering the responded electronic or hard copies, and then returning them to the researcher. Data was reported anonymously and secured in a locked file cabinet and a laptop with password protection. The researcher input the data into IBM SPSS statistics version 22. After the data screening and test of assumptions, the researcher performed three multiple linear regression analyses. The significance was defined at the 0.05 level in this study. The results and findings were reported in Chapter IV. Conclusions and discussions were made in the next two sections, based on the statistical results and findings.

Conclusions

Conclusion 1

Most of the participating secondary math teachers lacked the awareness of the concepts directly relating to the concepts of fractals. However, they were well aware of the concepts indirectly related to the concepts of fractals which were required to be known under the secondary math curriculum. This conclusion was demonstrated by the descriptive data analyses of awareness.

U.S. and Chinese secondary mathematics teachers were not very aware of the concepts of fractals based on their average score of awareness ($M_{Chinese} = 2.57$, $M_{u.s.} =$

2.46) because 2 was ranked as somewhat unknowing and 3 was ranked somewhat knowing, especially for the items 16 “I know the concept of geometric iteration,” 17 “I know the concept of self-similarity,” 18 “I know the concept of magnification factor of fractals,” 19 “I know the concept of fractal dimension,” and 20 “I know how to create a geometric fractal.” The mode of the item 16 was 1 in the U.S. group and 2 in the Chinese group. The mode of the item 17 was 2 in the U.S. group and 1 in the Chinese group. The mode of the item 18 was 1 in U.S. and Chinese groups, as well as the items 19 and 20. Their average levels of awareness on these items were mostly below 2. Most of teachers were barely aware of these concepts. The mode of the items 13 was 4 in U.S. and Chinese groups, as well as 14, and 15. Both groups got the high rank level 4 as the mode and high average on the concepts geometric transformation, geometric sequences, and similar figures because they were required concepts to be understood by students in the secondary math curriculum, as well as teachers. The standard deviations of all items were below or a little above 1 which indicated that they were not having various opinions on each item.

Conclusion 2

Compared to the levels of the awareness, mathematics teachers showed a higher level of interest in learning and integrating fractals in their classroom because the average level of interest for each item was above 2 in both U.S. and Chinese groups. The U.S. group (M = 2.88) showed a little more interest in fractals than the Chinese group (M = 2.42) regarding the level of the mode and the average scores on each item (See Table 13). All standard deviations of all items from 21 to 28 were below 1 in both the U.S. and

Chinese groups which indicated that the participating secondary math teachers did not have various opinions about their interests on each item.

The U.S. group ranked the highest average level on the item 24 “I would like to know how fractal geometry supports learning across disciplines” (M = 3.02, mode = 3). The second place was the item 23 “I would like to know how fractal geometry supports teaching mathematics from very basic concepts to the most advanced concepts” (M= 2.96, mode = 3), while the items 23 and 24 were ranked as the last two places in the Chinese group. The Chinese group ranked the highest average level on the item 27 “If I had the knowledge and ability to integrate fractal geometry in a professional program, I would like to participate in the program” (M = 2.75, mode = 3). The second place was the item 28 “If I had the knowledge and ability to integrate fractal geometry into the core curriculum, I would like to integrate fractals in my curriculum” (M =2.60, mode = 3), while the items 27 and 28 were ranked as the last two places in the U.S. group. But the average level of scores on the items 27 and 28 in the Chinese group were still lower than in the U.S. group. From this point, we could draw a conclusion that the participants showed that they were willing to learn fractals and integrate fractals into their curriculum.

Conclusion 3

The U.S. and Chinese groups both lacked the knowledge and skills to solve fractal problems. They had relatively low average scores on knowledge of fractal geometry ($M_{U.S.}$ = 41.36, $M_{Chinese}$ = 62.60) based on a 100 scale. The participants showed various abilities and skills in solving the problems related to fractals because both standard deviations were very high.

Additionally, the statistics of means also demonstrated that the Chinese group showed higher abilities and skills in solving fractal problems than the U.S. group. But the U.S. group had a little higher average score on problem 29 than the Chinese group, as well as on problems 30 and 33. These three problems were about translation, reflection, and self-similarity. Observation was the major skill necessary to solve them based on the understanding of the fractal concepts. Many of the other problems involved more complicated analytical and computational skills with the fractal concepts relating to traditional geometry and algebra. Therefore, this may indicate that Chinese secondary math teachers have more background knowledge and problem-solving skills in traditional geometry and algebra than U.S. secondary math teachers, but this does not mean Chinese teachers know more about fractals than U.S. teachers.

Conclusion 4

Each of the three multiple linear regression models as a whole had statistically significant predictive capability in this study. This conclusion came from the statistical F values with its p values from the summary of model tables. $F_{awareness}$ (16, 149) = 4.90, p < 0.01; $F_{interest}$ (16, 149) = 3.00, p < 0.01; $F_{knowledge}$ (16, 149) = 6.32, p < 0.01. Therefore, statistically there was a significant effect of nationality, gender, age, degree, the years of teaching math, grade levels, experience of learning fractals, and experience of integrating fractal on awareness, as well as interest and knowledge. The combination of the selected independent variables explained 35% of the variance of awareness, 24% of the variance of interest, and 40% of the variance of knowledge.

However, not all predictors had statistically unique significant relationships with the dependent variables. From the t and p values of Table 18, only the variables years 11-

15 ($t = 2.23$, $p = 0.03$), years 16-20 ($t = 2.62$, $p = 0.01$), years 21-25 ($t = 2.66$, $p = 0.01$), LearnedFra ($t = 3.72$, $p < 0.01$), and IntegratedFra ($t = 2.92$, $p < 0.01$) statistically showed a significant relationship to awareness. From the t and p values of Table 20, only the variables U.S. ($t = 2.54$, $p = 0.01$), age 36-40 ($t = 2.22$, $p = 0.03$) and IntegratedFra ($t = 2.31$, $p = 0.02$) statistically showed a significant relationship to interest. From the t and p values of Table 22, only the variables U.S. ($t = -6.52$, $p < 0.01$) and high levels ($t = 4.36$, $p < 0.01$) statistically showed a significant relationship to knowledge.

Conclusion 5

The factor LearnedFra (experience of learning fractals answered "yes") had the most positive effect on the dependent variable awareness according to the structure coefficients (structure coefficient = 0.74) shown in Table 18, and then the factor IntegratedFra (experience of integrating factors answered "yes") (structure coefficient = 0.69) and high levels (grade levels from 10 to 12) (structure coefficient = 0.39) followed. The structure coefficients explained the bivariate correlation between an independent variable and dependent variable, which shows the independent variable's direct effect on the dependent variable. There were a total of five factors having a negative effect on the dependent variable awareness. The factors years 6-10 had the greatest negative effect on the dependent variable awareness (structure coefficient = -0.24), and then the factor age 20-25 (structure coefficient = -0.16) and U.S (structure coefficient = -0.15) followed. The variables years 6-10 ($r = -0.14$), age 20-25 ($r = -0.09$), U.S. ($r = -0.09$), age 36-40 ($r = -0.06$), and age 26-30 ($r = -0.01$) had negative correlations with the dependent variable awareness according to the statistics Pearson r indexes. All of the other variables LearnedFra ($r = 0.43$), IntegratedFra ($r = 0.40$), high levels ($r = 0.23$), years 21-25 ($r =$

0.14), age above 45 (r = 0.14), years above 25 (r = 0.13), M.S. degree (r = 0.12), years 16-20 (r = 0.12), male (r = 0.09), age 41-45 (r = 0.07), and years 11-15 (r = 0.04) had positive correlations with the dependent variable awareness. According to the squared semipartial correlation indexes, my hypothesis of Ha1 was not fully supported. The factors LearnedFra (Sr^2 = 0.06), IntegratedFra (Sr^2 = 0.04), years 21-25 (Sr^2 = 0.03), and years 16-20 (Sr^2 = 0.03) accounted for more variance to the model of awareness than the other variables. The factor, the years of teaching math, was not in the proposed list of accounting more variance in Ha1, while the independent variables, high levels and M.S. degree, were in the proposed list.

The factor U.S. (nationality) had the most positive effect on the dependent variable interest according to the structure coefficients (structure coefficient = 0.61) shown in Table 20, then the factors M.S. degree (structure coefficient = 0.42) and LearnedFra (structure coefficient = 0.37) followed. Only three factors had negative effects on the dependent variable interest. The factor years 11-15 (structure coefficient = -0.53) had the most negative effect on the dependent variable interest, and then the factors male (structure coefficient = -0.19) and age 26-30 (structure coefficient = -0.05) followed. The variables years 11-15 (r = -0.26), male (r = -0.09), and age 26-30 (r = -0.02) had negative correlations with the dependent variable interest according to the statistics Pearson r indexes, and all of the other variables U.S. (r = 0.29), M.S. degree (r = 0.21), LearnedFra (r = 0.18), IntegratedFra (r = 0.16), age above 45 (r = 0.14), years 16-20 (r = 0.12), years 6-10 (r = 0.07), age 36-40 (r = 0.07), high levels (r = 0.07), years 21-25 (r = 0.05), years above 45 (r = 0.04), age 20-25 (r = 0.01), and age 41-45 (r = 0.01) had positive correlations with the dependent variable interest. According to the squared semipartial correlation indexes, my hypothesis of Ha2 was not fully supported. The factors U.S. (Sr^2 = 0.03), IntegratedFra (Sr^2 = 0.03), age 36-40 (Sr^2 = 0.03), and years 11-15 (Sr^2 = 0.02) accounted for more variance to the model of interest than the other variables. The factors U.S. (nationality), age 36-40 (age), years 11-15 (the years of teaching math) were not in the proposed list of accounting more variance in Ha2, while LearnedFra (experience of learning fractals), degree, and grade levels were in the proposed list.

The factor high levels had the most positive effect on the dependent variable knowledge according to the structure coefficients (structure coefficient = 0.41) shown in Table 22, then the factors IntegratedFra (structure coefficient = 0.33) and years 11-15 (structure coefficient = 0.31) followed. Eight factors had negative effects on the dependent variable interest. The factor U.S. (structure coefficient = -0.86) had the most negative effect on the dependent variable knowledge, and then the factors M.S. degree (structure coefficient = -0.20) and age above 45 (structure coefficient = -0.20) followed. According to the Pearson r indexes, the variables U.S. (r = -0.55), M.S. degree (r = -0.13), age above 45 (r = -0.12), years 6-10 (r = -0.08), age 26-30 (r = -0.05), years 21-25 (r = -0.04), age 20-25 (r = -0.03), and age 36-40 (r = -0.02) had negative correlations with the dependent variable knowledge. All of the other variables high levels (r = 0.26), IntegratedFra (r = 0.21), years 11-15 (r = 0.20), male (r = 0.15), LearnedFra (r = 0.09), age 41-45 (r = 0.05), years above 45 (r = 0.04), and years 16-20 (r = 0.01) had positive correlations with the dependent variable knowledge. According to the squared semipartial correlation indexes, my hypothesis of Ha3 was not fully supported. The factors U.S. (Sr^2 = 0.17), high levels (Sr^2 = 0.08), age 36-40 (Sr^2 = 0.01), age 41-45 (Sr^2 = 0.01), and age above 45 ($Sr^2 = 0.01$) accounted for more variance to the model of knowledge than the other variables. The factors high levels (grade levels), age 36-40, age 41-45, and age above 45 (age) were not in the proposed list of accounting more variance in Ha3, while LearnedFra (experience of learning fractals), IntegratedFra (experience of integrating fractals), and M.S. degree (or degree) were in the proposed list.

Conclusion 6

According to the coefficient values of the regression equation of awareness, we would know how much of a unit change in Y will occur for a unit increase (positive coefficient)/ decrease (negative coefficient) in a particular X predictor variable, given that the other variables are held constant. The predictor years 21-25 had the largest absolute coefficient value (0.66) in the regression equation of awareness. With the years of teaching math represented 0 for years 0-5, 1 for years 21-25, if we held the other predictors constant, then for years 21-25 we would expect a 0.66 percent increase in the average of awareness score. The regression equation for the model of awareness was: Awareness (predicted) = 1.92 – (0.01) (U.S.) + (0.11) (Male) + (0.11) (age 20-25) +(0.21) (age 26-30) – (0.10) (age 36-40) – (0.22)(age 41-45) + (0.07) (age above 45) + (0.12) (M.S. degree) + (0.15) (years 6-10) + (0.38) (years 11-15) + (0.53) (years 16-20) + (0.66) (years 21-25) + (0.33) (years above 25) + (0.07) (high levels) + (0.36) (LearnedFra) + (0.32) (IntegratedFra). Like the interpretation of the coefficient for the predictor variable years 21-25, the other predictor variables' coefficients could be interpreted in the same way.

According to the coefficient values of the regression equation of interest, the predictor age 36-40 had the largest absolute coefficient value (-0.47) in the regression equation of interest. With the years of teaching math represented 0 for years 0-5, 1 for years 11-15, if we held the other predictors constant, then for years 11-15 we would expect a 0.47 percent decrease in the average of interest score. The regression equation for the model of interest was: Interested (predicted) = 2.27 + (0.35) (U.S.) + (0.02) (Male) – (0.10) (age 20-25) – (0.12) (age 26-30) + (0.44) (age 36-40) + (0.19)(age 41-45) + (0.38) (age above 45) + (0.20) (M.S. degree) + (0.09) (years 6-10) – (0.47) (years 11-15) - (0.07) (years 16-20) – (0.16) (years 21-25) – (0.36) (years above 25) – (0.08) (high levels) + (0.20) (LearnedFra) + (0.37) (IntegratedFra). The other predictor variables' coefficients could be interpreted in the same way as the variable years 11-15.

According to the coefficient values of the regression equation of knowledge, the predictor U.S. had the largest absolute coefficient value (-20.02) in the regression equation of interest. With nationality represented 0 for the Chinese group and 1 for the U.S. group, if we held the other predictors constant, then for the U.S. group we would expect a 20.02 percent decrease in the average of knowledge score. The regression equation for the model of knowledge was Knowledge (predicted) = 60.28 – (20.02) (U.S.) – (2.02) (Male) – (0.17) (age 20-25) – (1.87) (age 26-30) – (7.19) (age 36-40) – (6.78)(age 41-45) – (7.29) (age above 45) – (0.03) (M.S. degree) – (2.65) (years 6-10) + (3.29) (years 11-15) – (2.66) (years 16-20) – (2.68) (years 21-25) + (5.04) (years above 26) + (12.13) (high levels) – (0.27) (LearnedFra) + (2.47) (IntegratedFra). The other predictor variables' coefficients could be interpreted in the same way as the variable U.S.

Discussions

The results from this study revealed the levels of the participating secondary teachers' awareness of, interest in, and knowledge of fractals. The average awareness scores of the fractal concepts (which are not demanded in the math curriculum) were very low. This can be explained by the fact that only 56 out of 166 (33.73%) Chinese and U.S. participating secondary math teachers had ever learned about fractals and only 38 out of 166 (22.90%) Chinese and U.S. teachers had integrated fractals into their teaching. These two variables, experience of learning fractals and experience of integrating fractals, accounted for the most and the second most variance on the average of awareness scores according to the squared semipartial correlation indexes. Therefore, to make math teachers more aware of fractal concepts, we should offer educational opportunities in fractals to in-service and pre-service teachers. This will also be necessary in order for math teachers to integrate fractals into their teaching.

With the years of teaching math, math teachers who had taught longer were more aware of fractals with the exception of over 25 years. This situation seems reasonable because math teachers with more experience have more opportunities to learn about fractals.

No typical patterns were shown on the variable age. The factors nationality, gender, age, degree, and grade levels did not account for much variance in the awareness scores, which seems reasonable since fractal geometry has not been included in the teacher education, neither in the U.S. nor in China. Most secondary math teachers have not had the chance to learn about fractals, neither male nor female, neither young nor old, neither bachelor's nor master's degree, and neither low grade levels nor high grade levels. To increase math teachers' levels of awareness of fractal geometry, professional development and practice in fractals are the key elements.

Compared to the average level of awareness of fractals in this study, the average level of math teachers' interest in fractals was higher. This indicated that most math teachers would like to learn about fractals and its connections with the secondary math curriculum. U.S. math teachers showed more interest in how fractal geometry supports learning across disciplines, how fractal geometry supports teaching mathematics from very basic concepts to the most advanced concepts, how fractal geometry as a supplementary material influences the way that students learn and think about geometry, and how fractal geometry inspires student motivation, interest, and curiosity in teaching and learning mathematics. Chinese math teachers showed more interest in participating in professional development on fractals if opportunities were provided, integrating fractal geometry into the core curriculum if the knowledge and ability of teaching fractals was available, how inquiry study/active learning can be used when integrating fractal geometry for students' learning in the class, and how fractal geometry inspires students' motivation, interest, and curiosity. This is interesting and we might think about some cultural differences in both education systems. From the researcher's perspective, educators in the U.S. have more open views on the curriculum than the educators in China. As evidence is the fact that the U.S. does not have a national curriculum. Math teachers in both countries showed the same rank (the fourth rank) in interests in how fractal geometry inspires students' motivation, interest, and curiosity among all items. This indicated that math teachers in both countries hold the view that the value of inspiring students' motivation, interest, and curiosity is very important in teaching math.

We would mention that U.S. math teachers had a higher average score of each interest item than Chinese math teachers. This is understandable because Chinese education system emphasizes the materials required in the exam system. Most math teachers want to focus on the content required to be tested in the exam because students' performance on exam is the key element in evaluating their teaching performance. Although some fractal topics are addressed in the Chinese national curriculum as optional studies, most teachers choose to ignore it because it is optional and not part of the test.

Math teachers between the ages of 36 and 40 showed more interest in fractals than the other age groups. Teachers who had taught for 16 to 20 years showed less interest in fractals than the other groups. Math teachers who had experience in integrating or learning fractals also showed more interest in fractals than the group of math teachers who had not. It indicated that the experience of learning and integrating fractals makes math teachers more intrigued by its value. The group of teachers who had a masters' degree showed a little more interest in fractals than those who had a bachelor's degree, while gender and grade levels did not account for much variance in the average interest score.

The average score of knowledge was low for both U.S. and Chinese teachers, matching the average levels of awareness scores for both. But the nationality did make differences on the average score of knowledge. The differences were found on problems 31, 32, and 34 - 42, thus demonstrating significant differences in problem-solving and related knowledge and skills needed such as the concepts of ratios, infinity, limits, geometric sequences, trigonometry, how to find the area of a triangle and the sum of geometric sequences, etc. Ma (1999) pointed out that Chinese elementary math teachers have a deeper knowledge of mathematics than U.S. elementary math teachers. This phenomenon might exist in the secondary schools. Additionally, math teachers who taught high grade levels (10-12) had higher scores than those who taught low grade levels (6-9). One reason is because some knowledge and skills for solving problems are frequently used when teaching high grade levels, especially the knowledge of geometric sequences and limits.

The results of this study revealed that experience of learning fractals and experience of integrating fractals did not account for much difference in the average scores of knowledge. This was not an expected finding, especially for the factor experience of learning fractals having negative correlation with the dependent variable knowledge. As proposed, we would expect that those who had learned about fractals would have had a significant and positive effect on the average score of knowledge.

Limitations

1. This study was limited to the secondary math teachers who taught math in the greater metropolitan area of Hattiesburg, MS, and the greater metropolitan area of Shanghai, China. The data sources might not represent some other areas.
2. This study was limited to the truthfulness of what the math teachers reported their awareness and interest in fractals. Therefore, the answers were biased to how the teachers reported them and may not reflect the truth.
3. This study was limited to the attitude of how the math teachers treated the test. Therefore the test scores were biased to whether the teachers seriously took the test or not.

Recommendations for Future Research

1. Perform this study again and the test instrument can be shortened by removing the last two problems 43 and 44 because both were not used in data analysis. The shortened test makes participants feel more comfortable when doing the test.
2. Perform this study again and the best way of collecting data is using hard copies. Leave some spaces between any two problems for participants to work the problems. The researcher can use it to see how teachers work out the problem and why U.S and Chinese secondary teachers make differences on some problems.
3. This study showed that the math teachers who had experience learning and integrating fractals produced positive effects on the average score of awareness and interest. A future study can focus on whether a fractal workshop can make a significant difference in math teachers' levels of awareness and interest in fractals.
4. Although this study showed that the factors experience of learning and integrating fractals did not have much effect on their test scores, future research can investigate whether a fractal workshop can make a significant difference on the pre- and post- test.
5. This study focused on in-service teachers. A future study might change the subjects to pre-service teachers or secondary students. In this case, the researcher needs to adjust the instruments or create new instruments.

Recommendations for Policy and Practice

1. Based on the indicated low levels of secondary math teachers' awareness in fractals, the researcher recommends that fractal geometry should be considered in

math teachers' educational system. As a math teacher of the modern era, their geometry knowledge cannot be limited to the Euclidean geometry.

2. For in-service teachers, providing fractal workshops is an excellent way to increase their awareness levels of fractals. Learning and practicing will greatly help math teachers' increase their interests in fractals.
3. Curriculum designers might think about integrating some basic fractal knowledge into the traditional geometry curriculum. This will inspire both teachers and students to discover the novelty and build the connections between traditional geometry and modern geometry.
4. The Chinese education system should launch a deeper reform in their exam system in order to encourage teachers to not only focus on the exam materials. Math teachers should keep an open mind and embrace the challenges from the newly emerged knowledge with the development of the society.
5. The U.S. math teachers' education system should strengthen teachers' problem-solving skills, as well as their conceptual understanding with mathematical concepts, based on the results of this study through the test of fractals.
6. Based on the results from the test of fractals, the teachers who taught from grade 6 through 9 had poorer problem-solving skills and abilities than those who taught in higher grades. Therefore, more attention to professional development should be paid to teachers who teach in these grade levels.

APPENDIX A

THE SURVEY OF AWARENESS AND INTEREST

The following survey is designed to measure your awareness and interest of fractal geometry that you have had in your previous learning or teaching experience. Awareness as it relates to this survey is the certainty that you know some basic concepts of fractal geometry (items 13-20). Interest as it relates to this survey is the certainty that you would like to do something (items 21-28). Demographics are asked in items 1-12 to investigate the relationship between teachers' background and the awareness, interest, and knowledge of fractal geometry.

This questionnaire should take between 5-10 minutes. Your participation is completely voluntary and you may discontinue participation without penalty or prejudice against you. You may choose to not answer any questions that make you uncomfortable. By completing this survey, you are choosing to participate in the study. Questions regarding research should be directed to Mrs. Chen (601-620-9528).

This project has been reviewed by the Human Subjects Protection Review Committee, which ensures that research projects involving human subjects follow federal regulations. Any questions or concerns about rights as a research subject should be directed to the chair of the Institutional Review Board, The University of Southern Mississippi, 118 College Drive #5147, Hattiesburg, MS, 39406-0001, (601) 266-6820.

Please answer the following questions to the best of your ability. Your responses to this survey will remain anonymous.

1) Please select your gender.
   a.) Male b.) Female

2) Please select your age range.
   a.) 20-25 b.) 26-30 c.) 31-35 d.) 36-40
   e.) 41-45 f.) 46 or above

3) Please select the highest degree you have obtained.
   a.) Bachelor degree b.) Master degree c.) Specialist degree
   d.)Doctoral degree e.) Other types:________________________.

4) Please select the years that you have taught math.
   a.) 0-5 b.) 6-10 c.)11-15 d.) 16-20
   e.) 21-25 f.) 26 or above

5) Please select the current grade that you are teaching math.
   a.) 6th grade b.) 7th grade c.) 8th grade d.) 9th grade
   e.) 10th grade f.) 11th grade g.) 12th grade

6) Have you ever taught geometry in any of your math courses?
   a.) Yes b.) No

7) Have you ever learned fractals in your educational experience?
   a.) Yes b.) No

8) Have you ever integrated fractal geometry in your math curriculum?
   a.) Yes b.) No

9) How many geometric fractal examples do you know?
   a.) None b.) Only one c.) More than one

10) Have you ever used inquiry study/active learning for students' learning in your class?
   a.) Yes b.) No

11) Do you teach in a rural, suburban, and city school?
   a.) Rural b.) Suburban c.) City

12) In what state do you teach? ____

For items 13-20, please rate your level of awareness of fractal knowledge on a scale of 1-4.

1 = completely unknowing
2 = somewhat unknowing
3 = somewhat knowing
4 = completely knowing

13) I know the concept of geometric transformations （translation, reflection, rotation, etc.）
   1 2 3 4

14) I know the concept of geometric sequences.
   1 2 3 4

15) I know the concept of similar figures.
   1 2 3 4

16) I know the concept of geometric iteration.

1 2 3 4

17) I know the concept of self-similarity.
1 2 3 4

18) I know the concept of magnification factor of fractals.
1 2 3 4

19) I know the concept of fractal dimension.
1 2 3 4

20) I know how to create a geometric fractal.
1 2 3 4

Please answer the following questions to the best of your ability after you read the short paragraph below.

The research has demonstrated that fractal geometry connects many mathematics concepts and theorems in the secondary math core curriculum and to other subject areas. It can be well used for the applications of algebra and traditional geometry in secondary mathematics learning, and it can also be used for motivating students to conduct inquiry study/active learning, inspiring students to discover novelty, and increasing interest in learning.

For items 21-28, please rate your level of interest on a scale of 1-4.
1 = completely disagree
2 = somewhat disagree
3 = somewhat agree
4 = completely agree

21) I would like to know how Common Core supports teaching fractal geometry at the secondary school level.
1 2 3 4

22) I would like to know how fractal geometry as a supplementary material influences the way that students learn and think about geometry.
1 2 3 4

23) I would like to know how fractal geometry supports teaching mathematics from very basic concepts to the most advanced concepts.
1 2 3 4

24) I would like to know how fractal geometry supports learning across disciplines.
1 2 3 4

25) I would like to know how fractal geometry inspires students' motivation, interest, and curiosity in teaching and learning mathematics.

1 2 3 4

26) I would like to know how inquiry study/active learning can be used when integrating fractal geometry for students' math learning in my class.

1 2 3 4

27) If I had the opportunity to learn fractal geometry in a professional development program, I would like to participate in the program.

1 2 3 4

28) If I had the knowledge and ability to integrate fractal geometry into the core curriculum, I would like to integrate fractals in my curriculum.

1 2 3 4

下面所设计的问卷调查是用来测量你在过去的学习经历和工作经历中对分形几何的认知，教学兴趣。在这个问卷中所指的认知：是指你在多大程度上知道那些基本的分形几何概念 （问题 13-20）。教学兴趣：是指你将在多大程度上对整合分形几何进入数学课程教学的兴趣 （问题 21-28）。第 1 到第 12 题是用来收集你的个人信息和经历背景，目的是用来探究个人信息和经历背景与教师对分形几何的认知，教学兴趣和知识的理解和运用的关系。

完成问卷大概一共需要花 5 到 10 分钟的时间。你的参与完全是自愿的，你也可以随时终止参与，这将不会对你产生任何影响和伤害。如果某个问题让你产生不舒服的感觉你可以对此不做解答。如果你完成并上传了问卷，说明你本人已经同意为本研究提供数据。你的数据将最后汇总给陈算荣女士（01-601-620-9528）用于博士论文研究.

这个研究项目已经获得人权保护中心的审批，也就是确保了这个研究项目所涉及的人权是遵守联邦法规的。如有任何关于人权的问题担忧可以联系南密西西比大学的机构审查委员会。地址和电话联系方式是： 118 College Drive #5147, Hattiesburg, MS, 39406-0001, (601) 266-6820.

第一部分：问卷　　　　　　代码：

本问卷将不会使用你的真实姓名，请尽你最大的能力回答下面的问题。

1) 请选择你的性别.
   b.) 男　　　　b.) 女

2) 请选择你的年龄段.

   a.) 20-25　　b.) 26-30　　c.) 31-35　　d.) 36-40

   e.) 41-45　　f.) 46 or 以上

3) 请选择你获得的最高学历.

   a.) 学士学位　　b.) 硕士学历　　c.) 专家学历

   d.)博士学位　　e.) 其它: ______________________.

4) 请选择你教学数学的年限.
   b.) 0-5　　b.) 6-10　　c.)11-15　　d.) 16-20
   e.) 21-25　　f.) 26 或以上

5) 请选择你现在任教的年级.
   b.) 6 年级　　b.) 7 年级　　c.) 8 年级　　d.) 9 年级
   e.) 10 年级　　f.) 11 年级　　g.) 12 年级

6) 在你的数学教学中，你有过几何教学的经历吗？
   b.) 有　　b.) 没有

7) 在你的教育经历中（包括培训），你有学习过分形知识吗？
   b.) 有　　b.) 没有

8) 在你的数学教学中，你有过整合分形几何到你的数学课程教学中的经历吗？
   a.) 有　　b.) 没有

9) 你所知道的几何分形的例子
   a.) 一个也没有　　b.) 只有一个　　c.) 一个以上

10) 你曾经在你的教学中运用过研究性学习（或探究性，或积极性学习）教学策略吗？
   a.) 有　　b.) 没有

11）你所在的学校是在乡村，城郊，还是城市？
   a.) 乡村　　b.) 城郊　　c.) 城市

12）请填写你学校所在的省份或直辖市：____________________。

在第 13 到第 20 的题目选项里，请你按照下面给出的四分制指标评估你对分形几何概念的认知程度.

1 = 完全不知道
2 = 多少有点不知道
3 = 多少有点知道
4 = 完全知道

13) 我知道几何变换的概念（平移，旋转，翻折，放缩）。
   1　　2　　3　　4

14) 我知道等比数列的概念。
   1　　2　　3　　4

15) 我知道相似图形的概念。
   1　　2　　3　　4

16) 我知道几何迭代的概念。
1 2 3 4

17) 我知道自相似的概念。
1 2 3 4

18) 我知道分形中放大因数的概念.
1 2 3 4

19) 我知道分形维数的概念。
1 2 3 4

20) 我知道如何构造一个几何分形的图形。
1 2 3 4

读完下面的短文后，请尽你最大的能力回答下面的问题。

经研究论证，分形几何与中学数学课程中许多的数学概念和原理以及和其它课程能够建立紧密联系，它还能用于促进学生进行研究性或积极性学习，鼓励学生探究新事物，增加学习兴趣。

在下面的第 21 到 28 题中，请你根据给出的四分制评价指标评估你对分形几何的教学兴趣.

1 = 完全不同意
2 = 多少有点不同意
3 = 多少有点同意
4 = 完全同意

21) 我愿意知道中学课程标准是怎样支持分形几何教学的。
1 2 3 4

22) 我愿意知道分形几何作为教学的补充材料是怎样影响学生学习和思考几何的。
1 2 3 4

23) 我愿意知道分形几何怎样和最基础的数学概念直到高等数学概念建立联系的。
1 2 3 4

24) 我愿意知道分析几何是怎样和跨学科建立联系的。
1 2 3 4

25) 我愿意知道分形几何在数学教学中是如何激发学生的学习动机，兴趣和好奇心的。

1 2 3 4

26) 我愿意知道怎样运用研究性或积极性教学策略整合分形几何进入学生的数学课程学习中。

1 2 3 4

27) 如果我有机会参加分形几何教学的教师专业发展培训项目的话，我愿意获得这个学习的机会。

1 2 3 4

28) 如果我具备了整合分形几何进入数学课程教学的知识和能力，我愿意整合分形几何进入学生的数学课程学习中。

1 2 3 4

APPENDIX B

KNOWLEDGE OF FRACTAL GEOMETRY

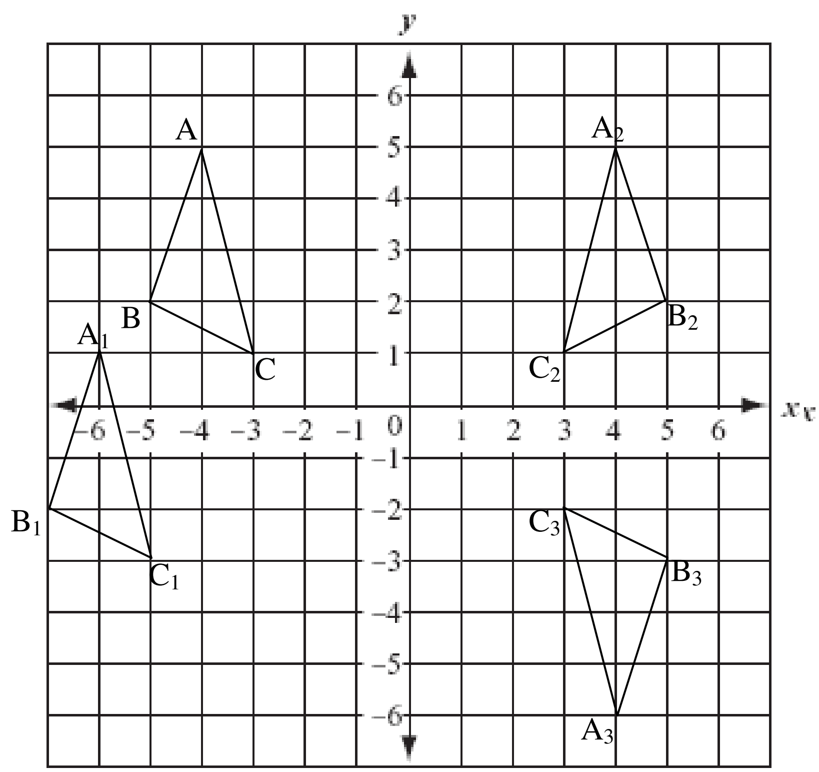
The following test is designed to measure knowledge of fractal geometry that you have had in your previous learning and teaching experience. Knowledge as it relates to the test is some basic fractal geometry knowledge that you have being asked in items 29-44. This is a research study with the goal of statistically analyzing the awareness, interest, and knowledge of fractal geometry among secondary math teachers.

This test should take about 30 minutes. Your participation is completely voluntary and you may discontinue participation without penalty or prejudice against you. You may choose to not answer any questions that make you uncomfortable. By completing this survey, you are choosing to participate in the study. Questions regarding research should be directed to Mrs. Chen (601-620-9528).

This project has been reviewed by the Human Subjects Protection Review Committee, which ensures that research projects involving human subjects follow federal regulations. Any questions or concerns about rights as a research subject should be directed to the chair of the Institutional Review Board, The University of Southern Mississippi, 118 College Drive #5147, Hattiesburg, MS, 39406-0001, (601) 266-6820.

Please solve the following problems (29-44) to the best of your ability.

Triangles ABC, $A_1B_1C_1$, $A_2B_2C_2$, and $A_3B_3C_3$ are given on a coordinate grid. Answer the items from 29-31.



Please use the coordinate grid and select the exact geometric transformation description for the items 29-31.

____29) Describe the geometric transformations from Triangle ABC to Triangle $A_1B_1C_1$

a.) Reflect over a line
b.) Rotate around a point
c.) Translate down 4 units, then left 2 units
d.) Translate up 4 units, then right 2 units
e.) None of these answers

____30) Describe the geometric transformations from Triangle ABC to Triangle $A_2B_2C_2$:

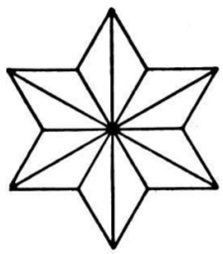
a.) Translate right 4 units
b.) Rotate around a point
c.) Reflect over x-axis
d.) Reflect over y-axis
e.) None of these answers

____31) Describe the geometric transformations from Triangle ABC to Triangle $A_3B_3C_3$:

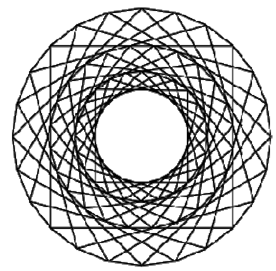
a.) Reflect over a line
b.) Rotate180 degrees around the origin point
c.) Rotate 90 degrees around the origin point
d.) Reflect and then translate
e.) None of these answers

____ 32) Circle the art pieces which are designed by geometric fractals.

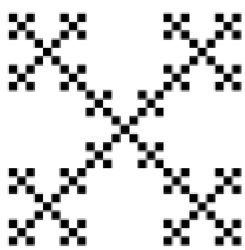
a.)



b.)



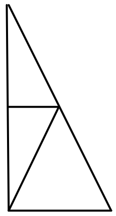
c.)



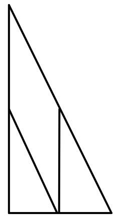
d.)

____ 33) Among the following figures, which figure possesses the property of self-similarity?

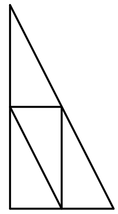
a.)



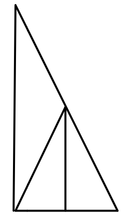
b.)



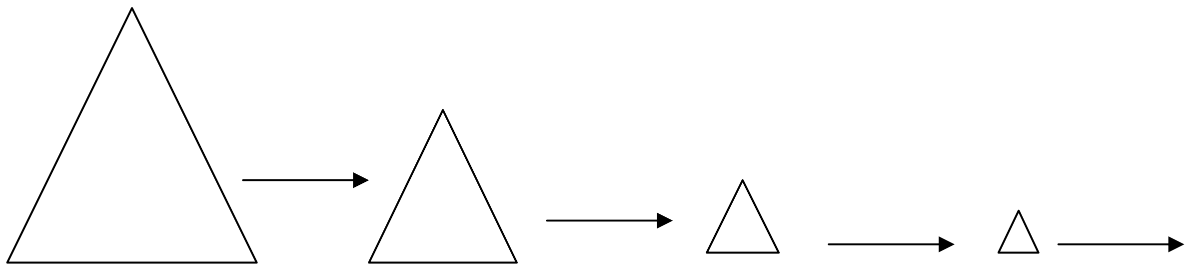
c.)



d.)



____34) Assume the length of a side of the original equilateral triangle (called the seed) is 1. The geometric iteration rule is given as follows: shrink the equilateral triangle so that each side is half of the current one. After the nth geometric iteration, find the length of each side (using the algebraic expression of n).



T_0, the seed T_1, iteration 1 T_2, iteration 2 T_3, iteration 3

a.) $\left(\frac{1}{2}\right)^{n+1}$ b.) $\left(\frac{1}{2}\right)^{n}$ c.) $\left(\frac{1}{2}\right)^{n-1}$ d.) $3\left(\frac{1}{2}\right)^{n}$

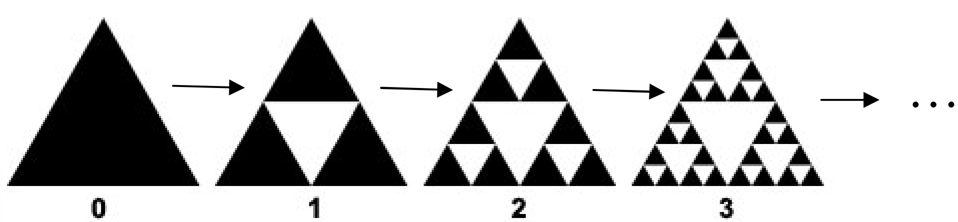
e.) None of these answers

____35) Based on Problem 34), After the nth geometric iteration, find the area of the triangle(using the algebraic expression of n):

a.) $\left(\frac{1}{4}\right)^{n+1}$ b.) $\left(\frac{1}{4}\right)^{n}$ c.) $\sqrt{3}\left(\frac{1}{4}\right)^{n+1}$ d.) $\sqrt{3}\left(\frac{1}{4}\right)^{n}$

e.) None of these answers

____36) The Sierpiński triangle is formed by geometric iterations. The seed is an equilateral triangle and its interior. The geometric iteration rule is: Remove the triangle formed from the middle points of each side of the original triangle, so that each side is one-half of the original one and three other congruent triangles remain. The number of total removed triangles after the nth geometric iteration is:



a.) $\frac{3^{n+1}-1}{2}$ b.) $\frac{3^{n}}{2}$ c.) $\frac{3^{n-1}-1}{2}$ d.) $\frac{3^{n}-1}{2}$

e.) None of these answers

____37) Based on Problem 36), if the area of the first removed triangle after the first step of the geometric iteration is x, write an algebraic expression to represent the total area of the removed triangles after the 10^{th} geometric iteration:

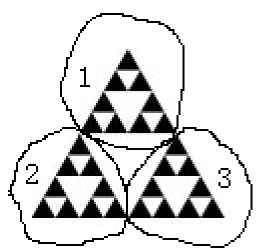
a.) $4\left[1-\left(\dfrac{3}{4}\right)^{9}\right]x$ b.) $4\left[1-\left(\dfrac{3}{4}\right)^{10}\right]x$ c.) $\left[4-\left(\dfrac{3}{4}\right)^{9}\right]x$

d.) $\left[4-\left(\dfrac{3}{4}\right)^{10}\right]x$ e.) None of these answers

____38) Based on Problem 37), if the area of the first removed triangle after the first step of geometric iteration is 1, find the total area of the removed triangles when the geometric iteration goes to infinity:

a.) Infinity b.) 3.75 c.) 3.9 d.) 4
e.) None of these answers

____39) The following Sierpiński triangle can be broken into three circled pieces. Find the magnification factor of these three pieces to yield the entire figure:

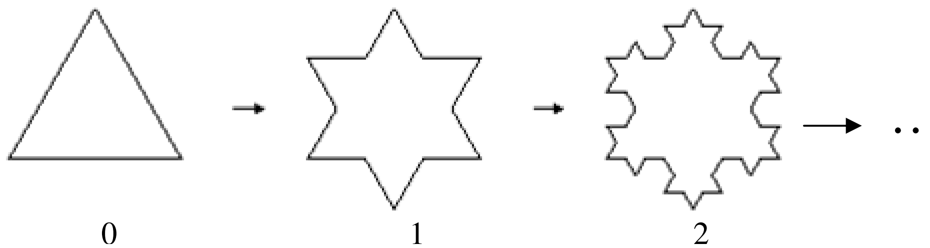


a.) 1 b.) 2 c.) 3 d.) 4
e.) None of these answers

____40) Accurately describe the geometric transformation from the circled piece 1 to the circled piece 2 in the above Sierpiński triangle (Assume the length of the side of the biggest equilateral triangle in the figure is 1).

a.) Rotate around the common point of piece 1 and piece 2
b.) Rotate 120 degrees around the common point of piece 1 and piece 2
c.) Rotate 120 degrees by clockwise around the common point of piece 1 and piece 2
d.) Translate left 0.5 units and then down 0.5 units
e.) None of these answers

____41) The following is the process to form the Koch snowflake fractal. The seed is an equilateral triangle which has sides of length 1. The geometric iteration rule is: on each edge of the figure, add a new equilateral triangle with sides of length 1/3 of the edge and remove the middle length1/3 of the edge. What is the perimeter of the figure after the infinite geometric iteration?

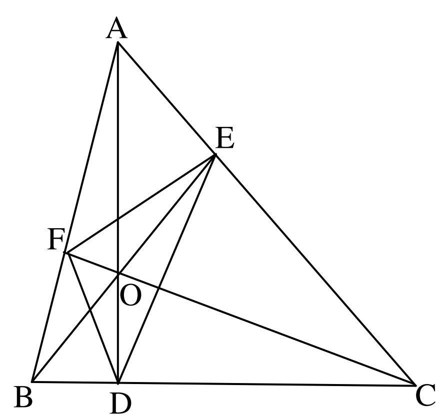


a.) 4 b.) $\frac{64}{3}$ c.) $\frac{4^{2n-1}}{3^{n-1}}$ d.) infinity
e.) None of these answers

____42) Based on Problem 41), what is the area of the above figure after the infinite geometric iteration?

a.) $\frac{2\sqrt{3}}{5}$ b.) $\frac{3\sqrt{3}}{5}$ c.) $\frac{8}{5}$ d.) infinity
e.) None of these answers

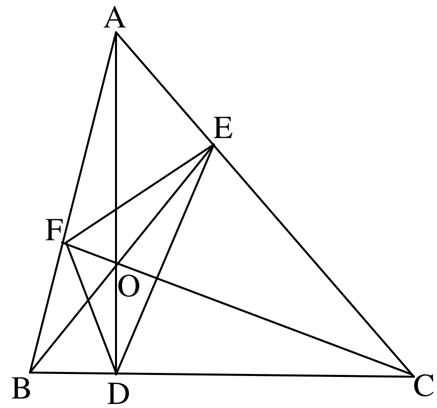
____43) In triangle ABC, AD is perpendicular to BC; BE is perpendicular to AC; and CF is perpendicular to AB. Find the triangles which are similar to the triangle ABC:



a.) Triangle AEF b.) Triangle DBA c.) Triangle DEC d.) Triangle DEF
e.) Triangle AEF , Triangle DBA, and Triangle DEC.

____44) In Problem 43), triangle EFD is called pedal triangle. If $\angle BAC = 60^\circ$ and BC=4, find the length of EF.

a.) $\sqrt{3}$ b.) 2 c.) $2\sqrt{3}$ d.) 3
e.) None of these answers



下面所设计的测试是用来测量你在过去的学习经历和工作经历中对分形几何基本知识的理解和运用。你将运用一些基本的分形几何知识和数学技能去解答问题 29 到 44。

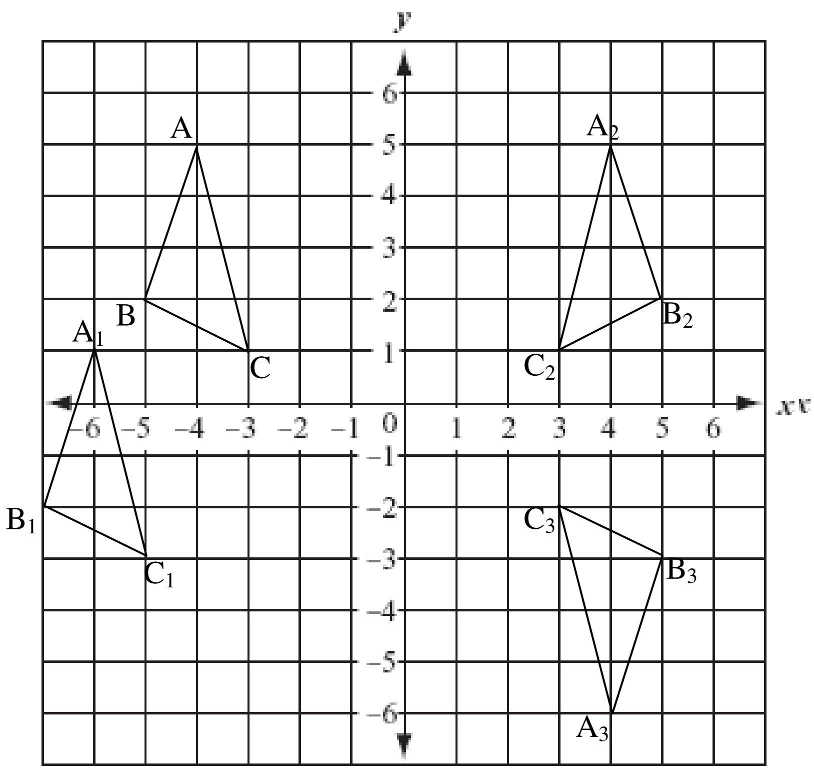
完成该测试大概一共需要花 30 分钟到 50 分钟的时间。你的参与完全是自愿的，你也可以随时终止参与，这将不会对你有任何的影响和伤害。如果某个问题让你产生感觉不舒服，你可以该问题不做解答。如果你完成并上交了问卷和测试，说明你本人已经同意为本研究提供数据。你的数据将最后汇总给陈女士（01-601-620-9528）用于博士论文研究.

这个研究项目已经获得人权保护中心的审批，也就是确保了这个研究项目所涉及的人权是遵守联邦法规的。如有任何关于人权的问题担忧可以联系南密西西比大学的机构审查委员会。地址和电话联系方式是： 118 College Drive #5147, Hattiesburg, MS, 39406-0001, (601) 266-6820.

基本分形知识测试

请尽你最大的能力解答下面第 29 到第 44 题，并请把答案填在相应的括号内。

在网格坐标平面上给出Δ ABC, $\Delta A_1B_1C_1$, $\Delta A_2B_2C_2$, 以及 $\Delta A_3B_3C_3$ ，请回答第 29 到第 31 题。



请利用网格坐标系和准确的几何变换术语（平移，翻折，旋转等）回答问题 29-31。

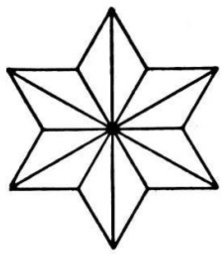
29) 描述Δ ABC 是经历怎样的几何变换可以得到 $\Delta A_1B_1C_1$ ………………………( )
a.) 沿着某一条线翻折
b.) 绕着某个点旋转
c.) 向下平移 4 个单位再向左平移两个单位
d.) 向上平移四个单位再向右平移两个单位
e.) 以上答案都不是

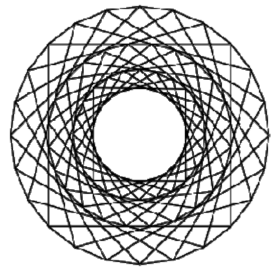
30) 描述Δ ABC 是经历怎样的几何变换可以得到 $\Delta A_2B_2C_2$……………………….( )
f.) 向右平移 4 个单位
g.) 绕着某个点旋转
h.) 沿着 x 轴翻折
i.) 沿着 y 轴翻折
j.) 以上答案都不是

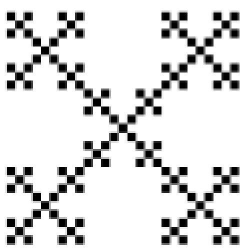
31) 描述Δ ABC 是经历怎样的几何变换可以得到 $\Delta A_3B_3C_3$…………………( )

f.) 沿着某条直线翻折
g.) 绕着原点旋转 180 度
h.) 绕着原点旋转 90 度
i.) 翻折然后平移
j.) 以上答案都不是

32) 在下面的艺术图案设计中，圈出其中运用了几何分形设计的图案…… (   )

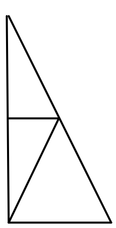


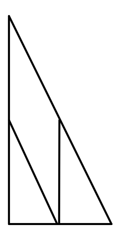


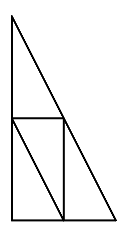


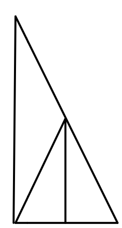
33) 在下面的图形中，选出包含自相似特征的图形……………………………..(   )

b.) b.) c.) d.)

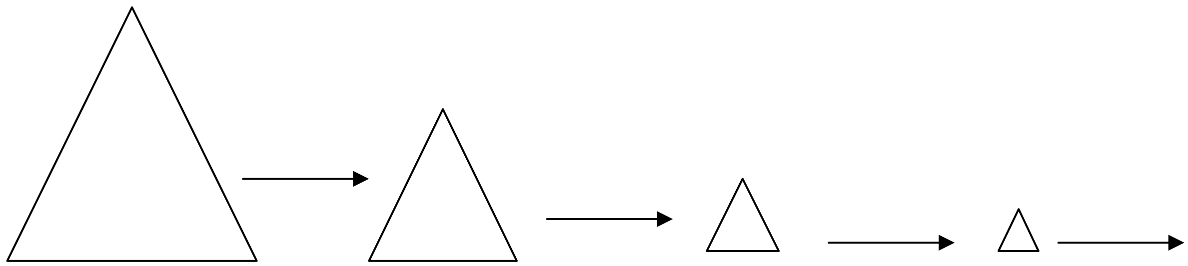








34) 假设给出边长为 1 的一个初始等边三角形（叫做种子）。给出如下的几何迭代规则：缩小当前的等边三角形使每边的长度变为当前三角形每边长的一半。经过第 n 次迭代后，用含 n 的代数式表示每边的长度……………………………………( )



T_0, 种子　　T_1, 第 1 次迭代　　T_2, 第 2 次迭代　　T_3, 第 3 次迭代

b.) $\left(\frac{1}{2}\right)^{n+1}$　　b.) $\left(\frac{1}{2}\right)^{n}$　　c.) $\left(\frac{1}{2}\right)^{n-1}$　　d.) $3\left(\frac{1}{2}\right)^{n}$

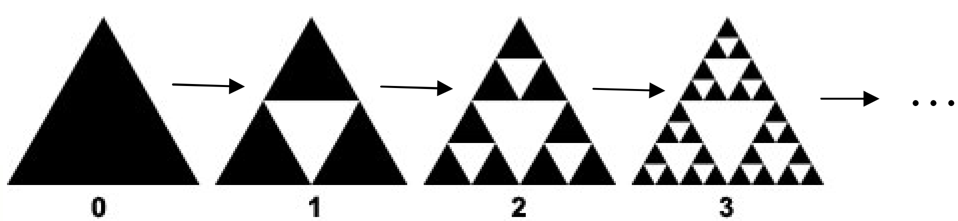
e.) 以上答案都不是

35) 基于问题 34), 经过第 n 次迭代后, 用含 n 的代数式表示这个三角形的面积……………………………………………………………………………………..( )

b.) $\left(\frac{1}{4}\right)^{n+1}$　　b.) $\left(\frac{1}{4}\right)^{n}$　　c.) $\sqrt{3}\left(\frac{1}{4}\right)^{n+1}$　　d.) $\sqrt{3}\left(\frac{1}{4}\right)^{n}$

e.) 以上答案都不是

36) 谢尔宾斯基三角形是用几何迭代的方法形成的。种子是三角形及其内部。迭代规则是：去掉中间那个由每边的中点连接而成的三角形，以便每边长是初始三角形边长的一半，留下三个互相全等的三角形。经过第 n 次迭代后，去掉的中间三角形的总数量是多少？ ……………………………………………………………………( )



b.) $\frac{3^{n+1}-1}{2}$　　b.) $\frac{3^{n}}{2}$　　c.) $\frac{3^{n-1}-1}{2}$　　d.) $\frac{3^{n}-1}{2}$

e.) 以上答案都不是

37) 基于问题 36), 如果第一次去掉的中间三角形的面积是 x, 请用代数式表达第 10 次迭代后去掉的所有中间三角形的总面积之和………………………………..(　)

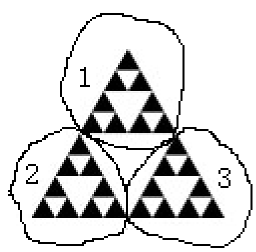
b.) $4\left[1-\left(\frac{3}{4}\right)^{9}\right]x$　　b.) $4\left[1-\left(\frac{3}{4}\right)^{10}\right]x$　　c.) $\left[4-\left(\frac{3}{4}\right)^{9}\right]x$

d.) $\left[4-\left(\frac{3}{4}\right)^{10}\right]x$　　e.) 以上答案都不是

38) 基于问题 37), 如果第一个在第一次去掉的中间三角形的面积是 1, 那么当迭代次数趋近无限时，所有去掉的中间三角形的总面积之和是……………………..(　)

a.) 无穷大　　b.) 3.75　　c.) 3.9　　d.) 4
e.) 以上答案都不是

39) 下面的谢尔宾斯基三角形被分成三个小部分并被圈出。这三个部分若每部分成为整个大图，需要的放大因子是多少？……………………………………..(　)

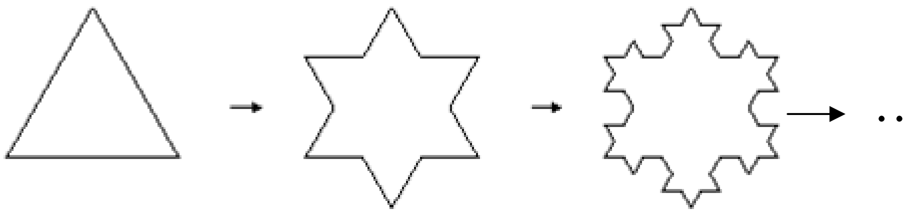


a.) 1　　b.) 2　　c.) 3　　d.) 4
e.) 以上答案都不是

40) 在上图的谢尔宾斯基三角形中，描述从圈出的第 1 部分图形通过怎样的几何变换可以得到圈出的第 2 部分图形？ (假设最大的那个等边三角形的边长为 1) …………………………………………………………………………(　)

a.) 绕着两个图形的公共点旋转
b.) 绕着两个图形的公共点顺时针旋转 120 度
c.) 绕着两个图形的公共点旋转 120 度
d.) 先左平移 0.5 个单位再下平移 0.5 个单位
e.) 以上答案都不是

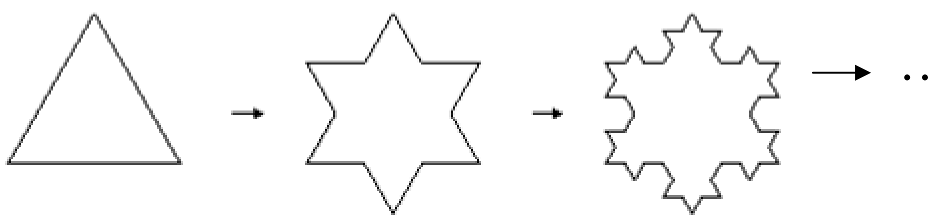
41) 下面是形成科赫雪花分形的过程。种子是一个边长为 1 的等边三角形。几何迭代的规则是：图形的每边三等分，在中间增加一个新的等边三角形并去掉这个中间的三分之一边长。经过无限次迭代后，图形的周长是多少？……………….. (   )



0　　1　　2

a.) 4　　b.) $\frac{64}{3}$　　c.) $\frac{4^{2n-1}}{3^{n-1}}$　　d.) 无穷大
e.) 以上答案都不是

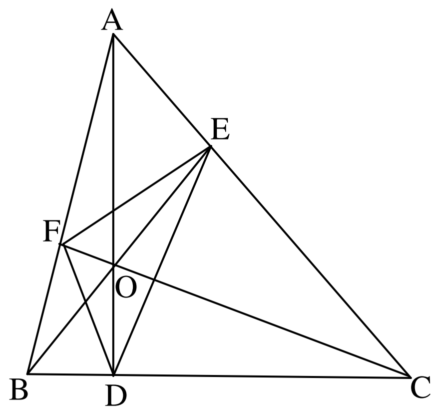
42) 基于问题 41), 经过无限次迭代后，科赫雪花分形的图形面积是多少？……………………………………………………………………………(   )



0　　1　　2

a.) $\frac{2\sqrt{3}}{5}$　　b.) $\frac{3\sqrt{3}}{5}$　　c.) $\frac{8}{5}$　　d.) 无穷大
e.) 以上答案都不是

43) 在ΔABC, AD 垂直于 BC; BE 垂直于 AC; CF 垂直于 AB. 找出所有和ΔABC 相似的三角形：…………………………………………………………………………..(　)



b.) 三角形 AEF　 b.) 三角形 DBF　 c.) 三角形 DEC　 d.) 三角形 DEF
e.) 三角形 AEF , 三角形 DBF, 以及三角形 DEC.

44) 在问题 43)中, ΔEFD 叫做垂足三角形, 如果 $\angle BAC = 60^{\circ}$, BC=4, 那么 EF=。 ………………………………………………………………………….……. ……. ……. ( )

a.) $\sqrt{3}$　　b.) 2　　c.) $2\sqrt{3}$　　d.) 3
e.) 以上答案都不是

APPENDIX C

INSTITUTIONAL REVIEW BOARD NOTICE OF COMMITTEE ACTION

THE UNIVERSITY OF SOUTHERN MISSISSIPPI

INSTITUTIONAL REVIEW BOARD
118 College Drive #5147 | Hattiesburg, MS 39406-0001
Phone: 601.266.5997 | Fax: 601.266.4377 | www.usm.edu/research/institutional.review.board

NOTICE OF COMMITTEE ACTION

The project has been reviewed by The University of Southern Mississippi Institutional Review Board in accordance with Federal Drug Administration regulations (21 CFR 26, 111), Department of Health and Human Services (45 CFR Part 46), and university guidelines to ensure adherence to the following criteria:

- The risks to subjects are minimized.
- The risks to subjects are reasonable in relation to the anticipated benefits.
- The selection of subjects is equitable.
- Informed consent is adequate and appropriately documented.
- Where appropriate, the research plan makes adequate provisions for monitoring the data collected to ensure the safety of the subjects.
- Where appropriate, there are adequate provisions to protect the privacy of subjects and to maintain the confidentiality of all data.
- Appropriate additional safeguards have been included to protect vulnerable subjects.
- Any unanticipated, serious, or continuing problems encountered regarding risks to subjects must be reported immediately, but not later than 10 days following the event. This should be reported to the IRB Office via the "Adverse Effect Report Form".
- If approved, the maximum period of approval is limited to twelve months.
  Projects that exceed this period must submit an application for renewal or continuation.

PROTOCOL NUMBER: 14112002
PROJECT TITLE: Assessing Awareness, Interest, and Knowledge of Fractal Geometry among Secondary Mathematics Teachers in China and the U.S.
PROJECT TYPE: New Project
RESEARCHER(S): Suanrong Chen
COLLEGE/DIVISION: College of Science and Technology
DEPARTMENT: Center for Science and Math Education
FUNDING AGENCY/SPONSOR: N/A
IRB COMMITTEE ACTION: Expedited Review Approval
PERIOD OF APPROVAL: 12/18/2014 to 12/17/2015
Lawrence A. Hosman, Ph.D.
Institutional Review Board

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