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Cost-Risk Analysis of the ERCOT Region Using Modern Portfolio Theory

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COST-RISK ANALYSIS OF THE ERCOT REGION

USING MODERN PORTFOLIO THEORY

by

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ABSTRACT

In this work, we study the use of modern portfolio theory in a cost-risk analysis of the Electric Reliability Council of Texas (ERCOT). Based upon the risk-return concepts of modern portfolio theory, we develop an n-asset minimization problem to create a risk-cost frontier of portfolios of technologies within the ERCOT electricity region. The levelized cost of electricity for each technology in the region is a step in evaluating the expected cost of the portfolio, and the historical data of cost factors estimate the variance of cost for each technology. In addition, there are several constraints in our minimization problem to account for real-world limitations. Using certain scenario data given by the National Renewable Energy Laboratory (NREL) and ERCOT, we analyze the efficient frontier of technology mixes for risk reduction of cost.

ACKNOWLEDGMENTS

To complete such a project as outlined in this thesis, one requires dedication, encouragement, and a long list of unwavering support from faculty and advisors. I am no exception, and the professionals and faculty of the University of Southern Mississippi have been boundlessly supportive and have guided me in my academic career as I complete my graduate degree.

I would like to start with a special thank you to my thesis advisor, Dr. Haiyan Tian, in her support of my academic work and progress. The initial idea of this work came from a spark in her differential equations class back in my first semester as a graduate student. She has an eye for piquing students' interests in topics of mathematics that they would find intriguing and fulfilling. It is not easy presenting an idea while also learning about the conceptual theories behind the idea, and without her mentoring and guidance, this thesis would not exist. Critiquing and evaluating my work for the past two years requires a level of commitment that many would disregard, and I am for ever grateful of the dedication and knowledge that Dr. Tian has offered to me. I would also like to acknowledge Dr. John Harris and Dr. Huiqing Zhu for their commitment to my masters thesis committee. They took the time to offer input on my work, and I appreciate the reflections and ideas presented for future research.

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Lastly, my family and partner deserve a thank you in their love and support of my goals and accomplishments. Completing this thesis would not be possible without the optimism outside of work and school, and the encouragement to keep going when I felt like quitting has considerably shaped my path in life.

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LIST OF ABBREVIATIONS

ACEEE	-	American Council for an Energy-Efficient Economy
ATB	-	Annual Technology Baseline
BEP	-	Brookfield Renewable Partners LP. Stock
CAPEX	-	Capital Expenditures
DSM	-	Demand Side Management
EERS	-	Energy Efficiency Resource Standards
EIA	-	Energy Information Administration
ERCOT	-	Electric Reliability Council of Texas
EU-BAU	-	European Union Business-as-Usual
FSLR	-	First Solar Inc. Stock
kWh	-	Kilowatt hours
MAXN	-	Maxeon Solar Stock
MMBtu	-	Million British Thermal Units
MPT	-	Modern Portfolio Theory
MVP	-	Minimum Variance Portfolio
MWh	-	Megawatt hours
NREL	-	National Renewable Energy Laboratory
NYSE	-	New York Stock Exchange
OEDI	-	Open Energy Data Initiative
OPEX	-	Operation Expenditures
PV	-	Photovoltaic
S	-	SentinelOne Inc. Stock
sLCOE	-	Simple Levelized Cost of Energy (Electricity)

NOTATION AND GLOSSARY

$E(x)$: expected value

r_i: return of ith asset

c_i: levelized cost of energy for ith technology

$E(r_i)$: expected return of ith asset

$E(c_i)$: expected levelized cost of ith technology

$E(r_p)$: expected return of portfolio p

$E(c_p)$: expected levelized cost of portfolio p

σ_i^2: Variance of return or cost of ith asset or technology

σ_p^2: Variance of portfolio p

σ_p: Risk of the portfolio; also the standard deviation

σ_{ij}: Covariance between ith and jth asset or technology

$[\sigma, E(r)]$: Risk-return space

$[\sigma, E(c)]$: Risk-cost space

w_i: Weight of the ith asset; for technology, this describes the sharing capacity

ρ_{ij}: correlation coefficient/pearson correlation coefficient between ith and jth asset or technology

W: Shapiro-Wilks normality test statistic

$\max\{S_i\} = s_i$: maximum sharing capacity s_i in the set S_i of sharing capacities for an ith technology

L: Lagrangian objective function

λ, γ, ϕ: Lagrangian constraint multipliers

r_f: return of a risk-free asset

Chapter 1

INTRODUCTION

In finance and business, minimizing cost is a constant dilemma [4]. This dilemma is no stranger to the sector of energy, where electrical grids that give power to consumers must factor demand based on the capacity of generating power. Electricity is a necessary part of our modern lives, and the plants that distribute this power do so based on the different electrical generating technologies in the region, such as the availability of wind, gas, or nuclear power. In fact, many regions share the electricity produced in one area through transmission lines. An electricity grid pools power from a variety of resources into a power plant and distributes that power to consumers. Every electricity grid has a maximum capacity that the system can deliver reliably. This capacity can differ year to year based on the reliability and capacity of its mix of generating technologies. These technologies have operating costs, and much like many business operations, to minimize cost is an optimization problem of generating technology mixes.

Common methods to optimize generating technologies in an electrical grid come from the methods of demand side management (DSM) [5]. Demand side management methods are used in electrical systems to manage demand of electricity by incentivizing consumers and creating more energy efficiency. For example, when demand for electricity is low, management will conserve that energy generated at that time for when demand for energy is higher. In addition, some technologies are more costly and have more associated risk than other technologies. Natural gas and other fossil fuel prices fluctuate, and the risk associated with this fluctuation can affect the cost of producing electricity from those resources. In addition to minimizing costs, many countries have opted to progress toward greener energies by implementing green energy initiatives to increase the reliability of renewable resource on the electrical grid. With green energy in mind, the shift in the reliance of fossil fuels in the grid capacity is forecasted to favor renewables. Reactively, reducing CO_2 emissions is now a major goal in the energy sector of many countries and adds constraints to minimize the emissions caused by generating technologies that rely on fossil fuels and other non-renewable resources.

Optimization techniques used in demand side management are varied and numerous. Deterministic, stochastic, and hybrid techniques are umbrella terms for a plethora of optimization techniques, their use dependent upon factors such as maximization or minimization of a particular factor, cost factors to observe, and technologies to emphasize in analysis.

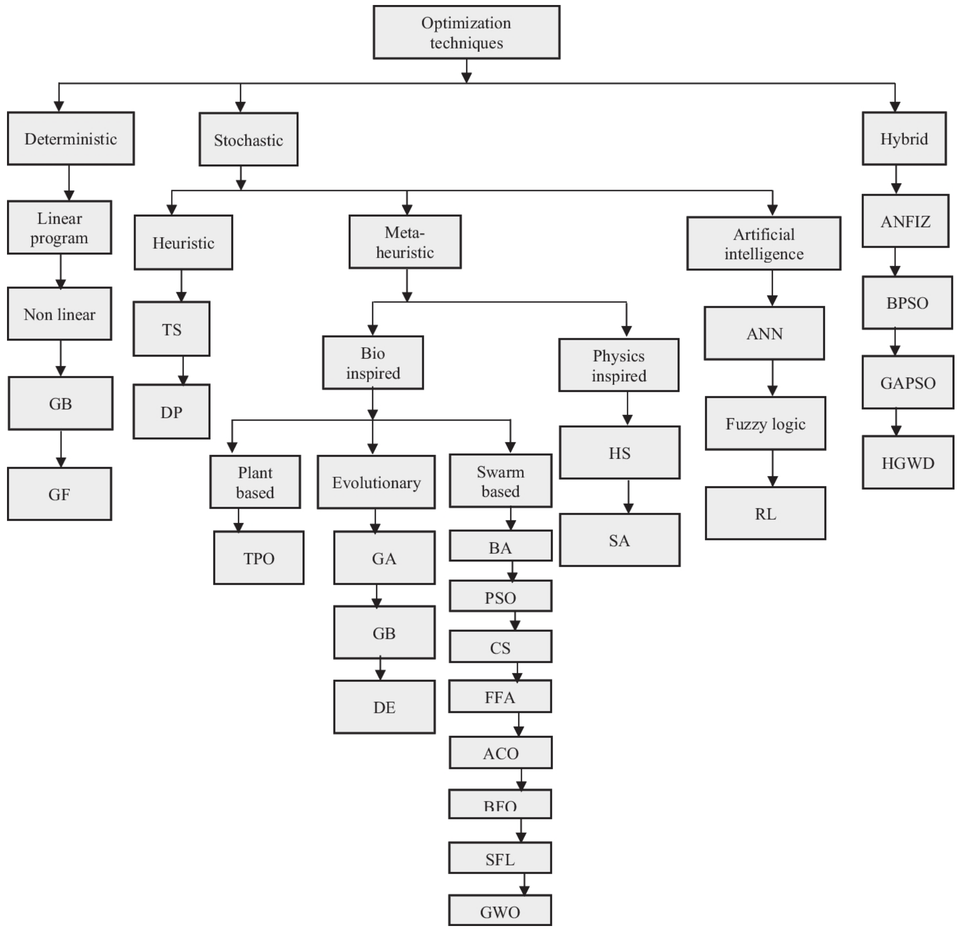


Figure 1.1: Optimization Techniques in DSM Tree Diagram [5]

Deterministic strategies deliver a clear option for the optimization of a system, and many rely on linear or nonlinear programming. However, deterministic models do not factor in the uncertainty that can arise in real-world problems like stochastic models. Stochastic models allow for uncertainty and unpredictability that produces a variety of solutions. Stochastic models can make use of fuzzy parameters and heuristic models that use artificial intelligence. Hybrid techniques combine characteristics of both deterministic and stochastic models that include improvements to the system, depending on the problem at hand. A hybrid technique can account for the uncertainty in real-world factors while giving a definitive solution, resulting in a method that is often more adaptable and efficient than deterministic or stochastic alone. A list of deterministic, stochastic, and hybrid techniques are described and compared comprehensively in 2023 by Bakare, Abdulkarim, Zeeshan, and Shuaibu in their article "A comprehensive overview on demand side energy management towards smart grids: challenges, solutions, and future direction" [5]. Figure 1.1 shows the tree diagram of different optimization techniques compared in the article.

In this paper, we will take an entirely different approach to optimizing the cost of generating technologies within an electrical grid. Instead of looking at only one solution or observing only one particular set of optimal technology mixes, we want to create a collection of feasible solutions that emphasize the relationships between a technology's cost and the risk of cost associated with operating that technology within the electrical grid. In other words, we are using a form of optimization through mean-variance analysis. A particular method of mean-variance analysis is often applied in finance when trading different stocks with different risks and returns. Modern portfolio theory (MPT), developed by Harry Markowitz in 1952, is a framework by which we will study in order to create this collection of feasible solutions.

1.1 A Brief Introduction to Modern Portfolio Theory

Also known as Markowitz portfolio theory or the Markowitz model, modern portfolio theory is a model that produces a collection of possible efficient portfolios that are categorized by their expected return at a certain level of risk. Let us begin with a few definitions. In finance, a portfolio is a collection of investments, such as stocks in the New York Stock Exchange (NYSE) or bonds and mutual funds. Returns refer to the value of the investment at a particular time. In trading stocks, we observe the difference in closing prices of a particular time period to calculate the return on the investment. Risk is the chance of losing money on an investment. An investor will buy only a certain amount of a particular stock to create their portfolio, and by using modern portfolio theory, the investor wishes to create an optimal mix of investments to reduce risk but maximize return on their investments. These optimal portfolios create what is called the efficient frontier, from which an investor may choose a particular portfolio based on their aversion to risk. Likewise, one may use the concept of MPT to analyze the cost-risk relationship of a collection of assets. In this paper, those assets are our generating technologies in an electrical grid, where we analyze the cost and the risk associated with the cost.

1.2 Related Works in Optimization

Many methods have been used to minimize costs, but applications of MPT have begun to emerge outside of investment problems. One such novel application was determined by a former University of Southern Mississippi student David Patterson, under the supervision of his advisor Dr. Haiyan Tian, in his master's thesis work titled *Using Modern Portfolio Theory to Analyze Virgil's Aeneid (or Any Other Poem)*. In his work, he analyzes literature using modern portfolio theory to find an optimum writing structure [14]. While Patterson's work represents one of many potential novel ways to use MPT outside of finance, we will continue our research with applications in electricity grids.

As one of the first to use modern portfolio theory in this nature in 2007, Awerbuch and Yang apply MPT to produce an evaluation of the projected 2020 European Union-Business-as-Usual (EU-BAU) electricity generating mix [3]. In their paper, the goal was to reduce cost and market risk as well as CO_2 emissions by identifying alternative generating portfolios and strategies. In addition, Awerbuch and Yang emphasized that technologies contributing to an electrical grid must be analyzed in terms of overall portfolio cost instead of individual cost. Modern portfolio theory helped them identify the contribution of overall cost relative to the contribution of the overall risk in a portfolio of generating technologies. Further, technologies are characterized by cost streams, meaning each technology cost will have cost factors or inputs. From their study, they found that the operating cost of a generating system with more wind will fluctuate less from year to year than a system with no wind. In addition, they detailed how to use cost factors, the standard deviations of components, and the correlation of components in their optimization model. As the pioneers of using MPT to optimize electricity grids, Awerbuch and Yang helped determine another feasible method of cost analysis.

In 2021, Arévalo, Paz, and Gomez propose the use of modern portfolio theory to analyze the electricity grid in Germany [2]. In their paper, they describe the characteristics of the German electricity mix and predict the evolution of the grid for Germany's current transition to cleaner energy. In forecasting for the years 2030, 2040, and 2050, they emphasize public health, pollution, and the cost of electricity to the consumer. From their analysis, they determined that the proportion of green energy to the proportion of fossil fuels will change significantly, with 90% of the grid capacity generated from green energy by 2050. The use of green energy is expected to increase while the use of conventional energy sources such as fossil fuels are expected to decrease. This paper describes how MPT was used to create a sustainable production of energy, minimize the dependence on external sources, reduce air pollution, diversify production of energy with wind and solar, and provide energy security to Germany consumers and suppliers.

As another group of researchers to use modern portfolio theory in 2021, Castro, Regner, et. al. analyze generating electricity portfolios for energy economics in Brazil [6]. In their paper, they propose a number of improvements to the basic Markowitz optimization model applied to optimizing an electrical grid with only variable renewable energy sources. These changes include: changing the measure of risk by incorporating a constraint to obtain portfolios that maintain a minimum generation level at a given risk; minimizing the standard deviation of electricity balance instead of electricity generation; incorporating load into the model as a power plant with negative production. These improvements made significant differences when compared to the basic model. Portfolios from the improved model resulted in lower or equal costs than the basic model and different costs for all levels of risk. In addition, adding demand to the model increased the sharing capacity of photovoltaic solar power. Lastly, they emphasize the importance of diversification in their model to help reduce risk of costs. This paper is a study on obtaining an optimal portfolio on only renewable resources, excluding non-renewable resources such as fossil fuels. Therefore, the experiment is not indicative of the entire electricity grid of Brazil; instead, it emphasizes an analysis on a particular resource to determine the costs and risks if included into the overall grid capacity.

In this paper, we study the application of mean-variance analysis to a cost-risk analysis of technologies that supply electricity to the electrical grid of a particular servicing agency. Since the state of Texas has an independent electrical grid to the United States, the region of this service area provides for a small region of interest to conduct our study while still maintaining much of the same available country technology data of the US. The Electric Reliability Council of Texas (ERCOT) provides electricity to over 90% of the Texas state region [10]. The objective is to minimize the risk of costs for a mixture of different generating technologies that provide electricity to the region: natural gas, coal, wind, nuclear, and solar. From year to year, each of these technologies share different capacities of the electrical grid based on demand for power. Minimizing risks associated with the cost of each technology helps manage costs for both the electricity provider and the customers who rely on them for such services over the years.

The next chapters of this paper lay out our study of modern portfolio theory to the application to finance and the application to the ERCOT electricity grid. Chapter 2 begins with a short study of MPT for different finance applications using stocks from the NYSE, where we define the basic definitions and theoretical analysis of the Markowitz model and end with the application of the model to two different sets of stocks. Chapter 3 lays the ground work for our application of finding efficient portfolios of technology mixes by defining the model in the n-asset case. Chapter 4 is our cost-risk analysis of the ERCOT region using MPT. With redefining equations to cost and risk instead of return and risk, we develop an efficient frontier of feasible technology portfolio mixes using available energy data. Results and conclusions follow the analysis and calculations.

Chapter 2

PORTFOLIO ANALYSIS OF TWO ASSETS

2.1 Definitions and Theoretical Analysis

An efficient portfolio is one that has either more return than any other portfolio with the same risk, or is one that has less risk than any other portfolio with the same return. We will observe efficient portfolios in the risk and return space, denoted $[\sigma, E(r)]$. In this chapter, we define the return, risk, covariance,and the correlation coefficient between only two assets.

In our portfolio analysis, we seek to find the optimum balance between risk and return in an investment portfolio.

Definition 2.1.1. The expected rate of return on a two-asset portfolio is defined as the expected value on a return of a two-asset portfolio, where the return is a weighted average of the return on the first and second portfolio.

The expected return of the portfolio is denoted as

$$E(r_p) = w_1 E(r_1) + w_2 E(r_2), \tag{2.1}$$

where the weights $w_1, w_2 \in [0, 1]$ and $w_1 + w_2 = 1$.

Definition 2.1.2. The risk, $\sqrt{\sigma_p^2}$, of a two-asset portfolio is the variability of the return, r_p of the portfolio and is measured by the standard deviation of the returns. We can define the risk of the return r_i for any asset $i \in \mathbb{N}$ in our list of assets by first defining the variance of returns, where S is the number of states that occur.

$$\sigma_i^2 = E[r_i - E(r_i)]^2 = \sum_{s=1}^{S} w_s [r_s - E(r_i)]^2 \tag{2.2}$$

$$\begin{aligned}
\sigma_i^2 &= E[(r_i - E[r_i])(r_i - E[r_i])] \\
&= E[r_i^2 - 2r_i E[r_i] + (E[r_i])^2] \\
&= E[r_i^2] - 2(E[r_i])2 + (E[r_i])^2 \\
\sigma_i^2 &= E[r_i^2] - (E[r_i])^2
\end{aligned}$$

If we consider a two-security portfolio, then we have $(w_1 r_1 + w_2 r_2) = r_p$, and by substituting this into Equation (2.2), we can derive the risk of portfolio p

$$\begin{aligned}
\sigma_p^2 &= E[r_p - E(r_p)]^2 = E\{w_1 r_1 + w_2 r_2 - [w_1 E(r_1) + w_2 E(r_2)]\}^2 \\
&= E\{w_1[r_1 - E(r_1)] + w_2[r_2 - E(r_2)]\}^2 \\
&= E\{w_1^2[r_1 - E(r_1)]^2 + w_2^2[r_2 - E(r_2)]^2 + 2w_1 w_2[r_1 - E(r_1)][r_2 - E(r_2)]\} \\
&= w_1^2 E[r_1 - E(r_1)]^2 + w_2^2 E[r_2 - E(r_2)]^2 + 2w_1 w_2 E\{[r_1 - E(r_1)][r_2 - E(r_2)]\}
\end{aligned}$$

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1 w_2 \sigma_{12}. \tag{2.3}$$

The risk for a portfolio p is now denoted as

$$\sqrt{\sigma_p^2} = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1 w_2 \sigma_{12}}, \tag{2.4}$$

where σ_{12} is the covariance.

Definition 2.1.3. The covariance is defined as a statistical measure of the association between two random variables and is applied when the price movement of one asset is associated with another asset.

$$\mathrm{Cov}(r_i, r_j) \text{ or } \sigma_{ij} := E[r_i - E(r_i)][r_j - E(r_j)] \tag{2.5}$$

Definition 2.1.4. Using the definition of covariance, we can define the correlation coefficient. Let X be the vector of return data for asset 1 and let Y be the vector of return data for asset 2, i.e. $X = \{x_i | i = 1, ..., n \text{ and } n \in \mathbb{N}\}$ and $Y = \{y_i | i = 1, ..., n \text{ and } n \in \mathbb{N}\}$.

Pearson's Correlation Coefficient

The Pearson's correlation coefficient can be written in the form

$$\rho_{12} = \frac{\mathrm{cov}(XY)}{\sigma_1 \sigma_2}, \tag{2.6}$$

where ρ_{12} is the correlation between asset 1 data and asset 2 data, σ_1 is the standard deviation of asset 1 data, and σ_2 is the standard deviation of asset 2 data. By rewriting the correlation coefficient using the derivation of covariance from Equation (2.5), σ_1, and σ_2, we obtain

$$\rho_{12} = \frac{E[XY] - E[X]E[Y]}{\sqrt{E[X^2] - [E[X]]^2}\sqrt{E[Y^2] - [E[Y]]^2}}$$

Rewriting the Pearson's correlation coefficient for data sets $\{x_i\}, \{y_i\}$ from $i = 1, ..., n$ where n is the sample size and $\bar{x}, \bar{y}$ are the sample mean for variables x and y respectively is shown in the derivation of Equation (2.7).

$$\begin{aligned}\rho_{12} &= \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}} \\ &= \frac{\sum_{i=1}^{n}(x_i y_i - x_i\bar{y} - \bar{x}y_i + \bar{x}\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - 2x_i\bar{x} + \bar{x}^2)}\sqrt{\sum_{i=1}^{n}(y_i - 2y_i\bar{y} + \bar{y}^2)}}\end{aligned}$$

$$\rho_{12} = \frac{\sum_{i=1}^{n}(x_i y_i) - n\bar{x}\bar{y}}{\sqrt{(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2)\sqrt{(\sum_{i=1}^{n} y_i^2 - n\bar{y}^2)}}} \tag{2.7}$$

In addition, the covariance can be defined in terms of the correlation coefficient and standard deviations of each asset, i.e. $\sigma_{12} = \rho_{12}\sigma_1\sigma_2$. When observing the efficient frontier of portfolios in the $[\sigma, E(r)]$ space, we expect to see concavity due to the covariance effect. The covariance effect occurs due to the typical correlation between two portfolios being between -1 and 1, i.e. $-1 < \rho_{12} < 1$.

To find the optimum balance between risk and return, we utilize all statistics defined above and minimize the variance of the portfolio. The weights that minimize a portfolio's variance can be found through standard calculus by taking the derivative of Equation (2.3) with respect to each weight w_1 and w_2:

$$\begin{aligned}\frac{\partial \sigma_p^2}{\partial w_1} &= \frac{\partial}{\partial w_1}[w_1^2\sigma_1^2 + (1 - w_1)^2\sigma_2^2 + 2w_1(1 - w_1)\sigma_{12}] \\ &= 2w_1\sigma_1^2 - 2(1 - w_1)\sigma_2^2 + 2(1 - 2w_1)\sigma_{12} \\ w_1 &= \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}} \\ w_2 &= 1 - w_1 = \frac{\sigma_1^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}\end{aligned}$$

2.2 Case 1: Mean-Variance Analysis of Two Clean Energy Stocks

In our first mean-variance analysis of two assets, we wish to understand the mechanisms of modern portfolio theory by analyzing a pair of assets that would likely have a fairly positive correlation. Now we will consider two clean energy stocks traded under the New York Stock Exchange (NYSE). Founded in 2019, Maxeon Solar Technologies, Ltd. is a company that designs, manufactures, and sells solar technology. As one of the leading solar farms, Maxeon has locations across the globe to help expand solar energy. Maxeon stock is abbreviated to

MAXN, as will henceforth be referenced. Founded in 2011, Brookfield Renewable Partners, L.P. is a partnership that owns and operates renewable resource assets. Brookfield stock is abbreviated to BEP, as will henceforth be referenced.

Figure 2.1: MAXN and BEP Closing Share Prices 2020-2023

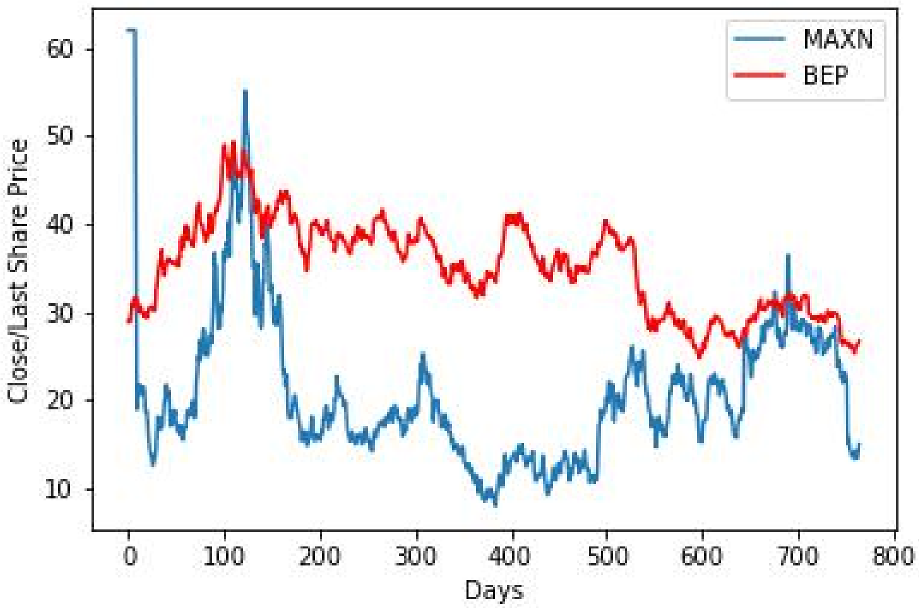


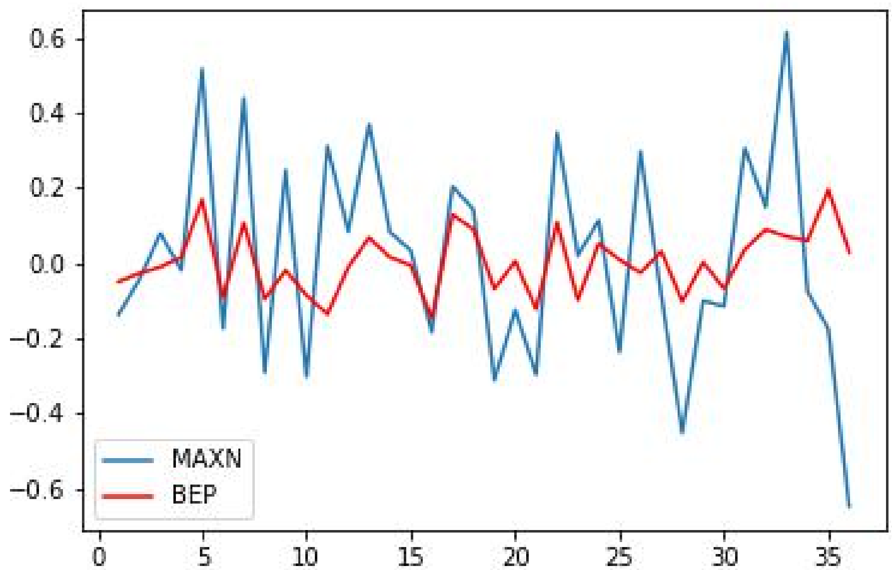
Figure 2.1 shows the daily closing share price of each stock from August 14, 2020 to August 29, 2023. The data in Figure 2.1 is downloaded directly into Excel from the NASDAQ website. Figure 2.2 shows the monthly returns for both MAXN and BEP stocks up to thirty-six months from August 2020 to August 2023. The first closing price of each month was extracted and used to calculate the monthly returns for each stock. Only the most recent three years were collected because of the short existence of Maxeon Solar Technologies.

To determine the distribution of each monthly return stock data, we conduct both a graphical analysis as well as a normality test. From Figures 2.3 and 2.4, we see that both monthly returns for each stock appear to have a normal Gaussian distribution. To confirm our conclusions of the graphical analysis, we use the Shapiro-Wilks normality test in the programming language Python. In the Shapiro-Wilks normality test, we can test if a data set is normally distributed by calculating the equation

$$W = \frac{\left(\sum_{i=1}^{n} a_i (x_{n+1-i} - x_i)\right)^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2}, \tag{2.8}$$

where W is the significant value we compare with the chosen alpha. Weights a_i are specified based on the pre-defined Shapiro-Wilks weight table, and $\{x_i | i = 1, ..., n \text{ and } n \in \mathbb{N}\}$ are

Figure 2.2: MAXN and BEP Monthly Returns 2020-2023



the n number of data points of the variable we are testing, sorted from least to greatest in value. Our null hypothesis is that the data set is normally distributed. Table 2.1 shows the statistics calculated with the Shapiro-Wilks normality test in Python. With the alpha level set to 0.05, we compare the significant value of each data set. MAXN stock has a significant p value of 0.986, which is greater than our alpha value of 0.05. Similarly, BEP stock has a significant p value of 0.796, which is greater than our chosen alpha value of 0.05. Therefore, we fail to reject the null hypotheses that each of the data sets are normally distributed.

Table 2.1: Shapiro-Wilks Normality Test Statistics Table

	Statistic	Degree of Freedom	Significant Value
MAXN	0.990	35	p=0.986
BEP	0.982	35	p=0.796

Let X be the vector $(x_1, ..., x_n)$ of share prices for the MAXN common stock and Y be the vector $(y_1, ..., y_n)$ share prices for the BEP common stock, where $n = 36$. We can calculate the correlation of these stocks by the Pearson Correlation Coefficient:

$$\rho_{12} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2} \tag{2.9}$$

Then the correlation between common stocks MAXN and BEP is $\rho_{12} = 0.481$. Since this value is relatively close to 0.5, we can say that the two stocks share a significant level of

Figure 2.3: Histogram of MAXN Monthly Return

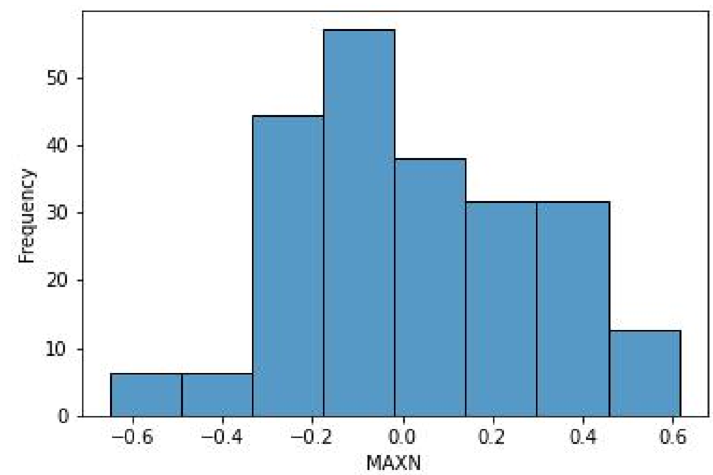
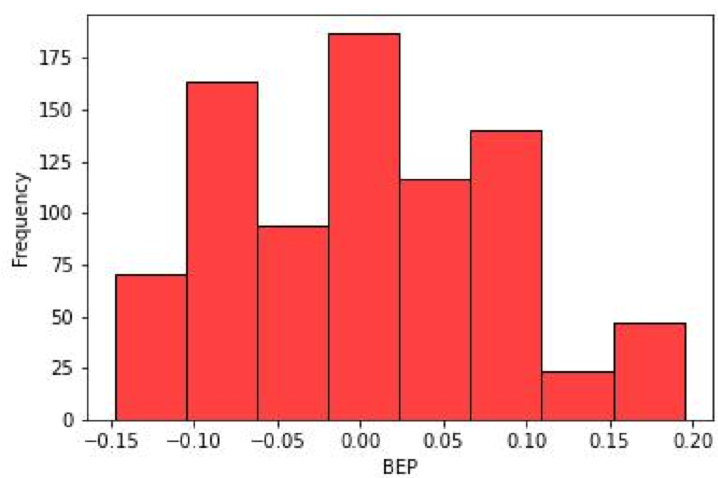


Figure 2.4: Histogram of BEP Monthly Return



positive correlation, meaning that they move similarly in the market. Figure 2.5 graphs the monthly returns of each stock as points to show the correlation between each stock. Note how the linear regression line illustrates that the stocks have a positive correlation.

Using Excel Spreadsheet, we calculated the average monthly returns, monthly variance, average yearly returns, and yearly variances for each stock observed. We calculate returns for each month by $r = \frac{(P_1 - P_0)}{P_0}$ where r is the monthly return, P_0 is the starting purchase price and P_1 is the purchase price of the next month. Then $(P_1 - P_0)$ is the price change in a month. Then we find the average return for each month over 3 years, adding the return of

Figure 2.5: Correlation between MAXN and BEP Monthly Returns

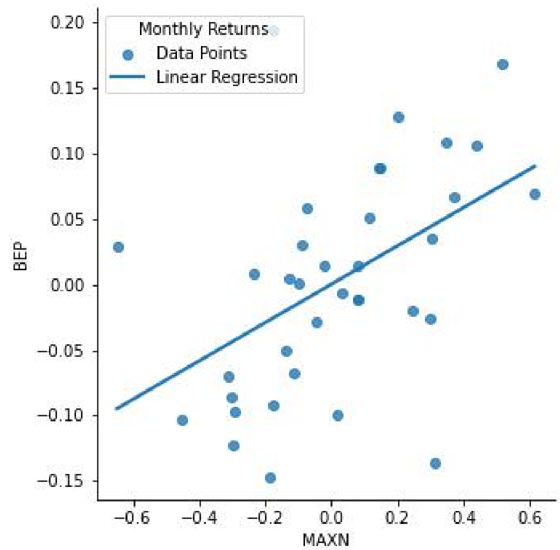


Table 2.2: Returns and Variances from Monthly Share Prices

	MAXN	BEP
Average Monthly Return	1.5566%	0.2548%
Monthly Variance	7.8595%	0.7300%
Average Yearly Return	18.6791%	3.0578%
Yearly Variance	94.3173%	8.7597%

each month and dividing by the number of months. Then we calculate the returns of each month as follows:

$$R_1 = (r_1, ..., r_{n-1}) \text{ where } r_i = \frac{x_i - x_{i-1}}{x_{i-1}}, i = 1, ...n-1 \text{ are the monthly returns of MAXN}$$

$$R_2 = (\tilde{r}_1, ..., \tilde{r}_{n-1}) \text{ where } \tilde{r}_i = \frac{y_i - y_{i-1}}{y_{i-1}}, i = 1, ...n-1 \text{ are the monthly returns of BEP}$$

The average yearly return is the average monthly return multiplied by 12 (the number of months in a year), i.e. $r_1 = \bar{R}_1 \cdot 12$ and $r_2 = \bar{R}_2 \cdot 12$ .The results of these calculations are observed in Table 2.2.

Let μ_1 be the mean of monthly returns of R_1, and μ_2 the mean of monthly returns of R_2. The variance of each common stock is found through the formulas:

$$\sigma_1^2 = \left(\sum_{i=1}^{n-1}(r_i - \mu_1)^2\right)/(n-1), \qquad \sigma_2^2 = \left(\sum_{i=1}^{n-1}(\tilde{r}_i - \mu_2)^2\right)/(n-1) \tag{2.10}$$

Through calculating the variances from the data, we obtain $\sigma_1^2 = 0.9299$ and $\sigma_2^2 = 0.08636$. From Definition 2.1.3, we define the covariance in terms of the correlation coefficient and standard deviations of each asset:

$$\sigma_{12} = \rho_{12}\sigma_1\sigma_2 = 0.481(\sqrt{0.9299})(\sqrt{0.08636}) = 0.1343$$

The two-asset portfolio variance σ_p^2, as denoted in Equation (2.3), can be written as a matrix:

$$\begin{pmatrix} +w_1w_1\sigma_{11} & +w_1w_2\sigma_{12} \\ +w_2w_1\sigma_{21} & +w_2w_2\sigma_{22} \end{pmatrix}$$

Table 2.3 is the variance-covariance matrix, calculated by using the average yearly returns of 18.6791% and 3.0578% for MAXN and BEP stock respectively. By looking at the covariance between MAXN and BEP, we see that the returns move in the same directions, i.e. both stocks appear to increase over time. The matrix also confirms that the covariance of a variable with itself is positive.

Table 2.3: Variance-Covariance Matrix

	MAXN	BEP
MAXN	0.9299	0.1343
BEP	0.1343	0.08636

For the optimal weighted risky portfolio, we minimize the variance by solving for the weights. We take the derivative with respect to w_1 using Equation (2.3):

$$\begin{aligned} \frac{\partial \sigma_p^2}{\partial w_1} &= \frac{\partial}{\partial w_1}[w_1^2\sigma_1^2 + (1-w_1)^2\sigma_2^2 + 2w_1(1-w_1)\sigma_{12}] \\ &= 2w_1\sigma_1^2 - 2(1-w_1)\sigma_2^2 + 2(1-2w_1)\sigma_{12} \end{aligned}$$

Solving for w_1,

$$w_1 = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}} \tag{2.11}$$

$$= \frac{(0.08636) - (0.1343)}{(0.9299) + (0.08636) - 2(0.1343)}$$

Solving for w_2,

$$w_2 = \frac{\sigma_1^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}} \tag{2.12}$$

$$= \frac{(0.9299) - (0.1343)}{(0.9299) + (0.08636) - 2(0.1343)}$$

The minimum variance portfolio weights for the risky portfolio is

$$w_1 = -0.064 \qquad w_2 = 1.064$$

Using Equation (2.1) with these weights, $E(r_p) = -0.064(0.186791) + 1.064(0.030578) = 0.0205 = 2.05\%$.

Calculating the standard deviation:

$$\sigma_p = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{12}}$$

$$\sigma_p = \sqrt{-0.064^2(0.9299) + 1.064^2(0.08636) + 2(-0.064)(1.064)(0.1343)}$$

$$\sigma_p = 0.2889 = 28.89\%$$

Figure 2.6: Efficient Frontier 1 for MAXN and BEP

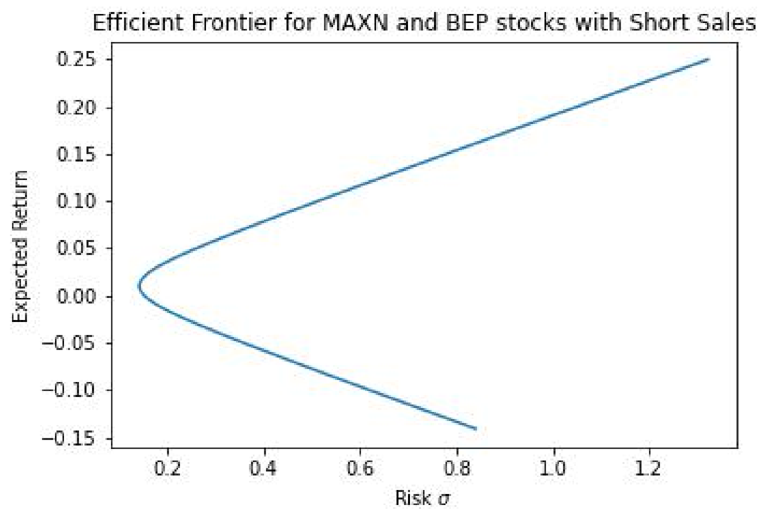


Figure 2.6 displays the opportunity curve of efficient and inefficient portfolios in the risk and return space $[\sigma, E(r)]$. The minimum variance portfolio occurs at the point $(0.02, 0.28)$ for which we know has a negative weight. For a negative weight to be present, short sales must be allowed. Short sales occur when an investor borrows shares of an asset and sells them in anticipation that the price will fall in order to buy back the share for cheaper to make a profit and pay back what was borrowed. Although short selling stock shares is legal in the U.S., there have been times when short selling was temporarily banned, such as in the 2008 financial crisis. If short selling is not allowed, then the weights must be constrained to $w_1, w_2 > 0$. In this case, we must only look at portfolios within the efficient frontier that come from nonnegative weights. To observe when a negative weight may occur, we see from the numerator in Equation (2.11) of minimizing w_1 of the minimum variance portfolio that

$$\begin{aligned}\sigma_2^2 - \sigma_{12} &= \sigma_2^2 - \sigma_2\rho_{12}\sigma_1 \\ &= \sigma_2^2\left(1 - \rho_{12}\frac{\sigma_1}{\sigma_2}\right)\end{aligned}$$

To determine when a negative weight would occur, we observe that

$$\left(1 - \rho_{12}\frac{\sigma_1}{\sigma_2}\right) < 0 \Rightarrow \rho_{12} > \frac{\sigma_2}{\sigma_1} \tag{2.13}$$

We observe that a negative weight occurs when the correlation between the returns for each asset is sufficiently positive. Using the variances of MAXN and BEP,

$$\sigma_1 = \sqrt{0.9299} = 0.9643, \qquad \sigma_2 = \sqrt{0.08636} = 0.2939$$
$$\frac{\sigma_2}{\sigma_1} = \frac{0.2939}{0.9643} = 0.305 < \rho_{12} = 0.481$$

Figure 2.7: Efficient Frontier 2 for MAXN and BEP

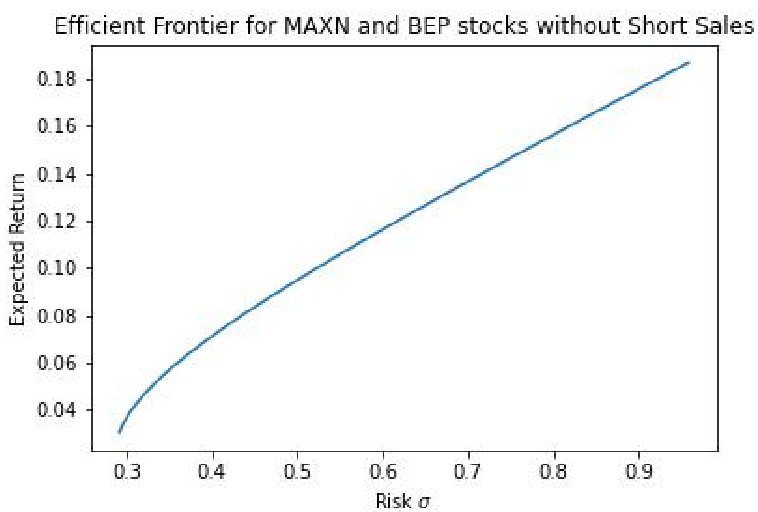


Figure 2.7 displays the efficient frontier of possible portfolios for MAXN and BEP stocks with weights constrained to be nonnegative. If we constrained the weights to the nearest nonnegatives, we would use

$$w_1 = 0 \qquad \text{and} \qquad w_2 = 1.$$

Using these weights, we again use Equation (2.1) to find the expected return:

$$E(r_p) = 0(0.186791) + 1(0.030578) = 0.030578$$

Similarly, we calculate the standard deviation:

$$\sigma_p = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{12}}$$

$$\sigma_p = \sqrt{0^2(0.9299) + 1^2(0.08636) + 2(0)(1)(0.1343)}$$

$$\sigma_p = 29.39\%$$

Then we have invested into the BEP stock entirely and our return is the average yearly return of BEP at 3.06% and our risk is the variance of the BEP monthly returns at approximately 29.39%. Different weights give different risks and returns, and the efficient frontier shows how each pairs of weights affect the optimal portfolio. From our observations of the daily share price for each stock in Figure 2.1, we see that it is reasonable to assume investment into only BEP would give us the minimum risk with max return. With respect to the volume of each stock, the daily share price of BEP stays above MAXN share prices for most of the time. Despite both stocks being clean energy stocks, there may be other factors not considered in this study that affect MAXN monthly returns. This conclusion brings to light one of the distinct caveats of using Modern Portfolio Theory to minimize risk in an investment.

2.3 Case 2: Mean-Variance Analysis of Two Opposing Stocks

Next, we will consider two stocks of different origin. Founded in 1999, First Solar Energy is a sustainable energy stock whose company is an American-based leading manufacturer of solar panels. In contrast, Sentinel One stock is an artificial intelligence stock founded in 2013 whose company delivers cybersecurity methods run by their patented AI modeling. In the NYSE, First Solar Energy is abbreviated by FSLR and Sentinel One is abbreviated by S, by which they will henceforth be referenced. Collecting stock data from the official NASDAQ website, each stock has a different length of data based on how much previous data from the past 5 years were collected from the NYSE. The stock for FSLR begins in September of 2018, while the stock for S starts in July of 2021. Therefore, we concatenate FSLR stock to July of 2021 to have the same length of data.

Figure 2.8 illustrates the closing share price in USD of each stock from July 31, 2021 until September 5, 2023. The green line represents FSLR stock and shows a drastic decrease in closing price before steadily increasing again later. S stock is shown by the yellow line and describes how the stock settled with low closing prices relative to FSLR before increasing price near 2023. The data is downloaded directly into Excel, where the monthly returns for both FSLR and S stocks are calculated from July 2021 to September 2023. We calculate returns for each month by $r = \frac{(P_1 - P_0)}{P_0}$ where r is the monthly return, P_0 is the starting purchase price and P_1 is the purchase price of the next month. Then $(P_1 - P_0)$ is the price

Figure 2.8: FSLR and S Closing Share Prices 2021-2023

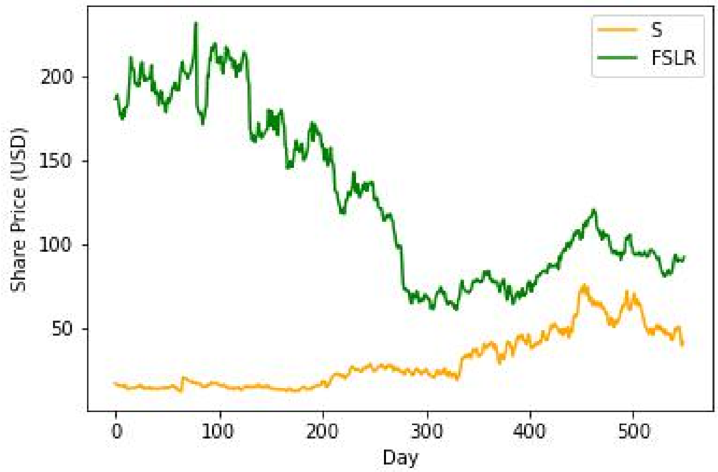
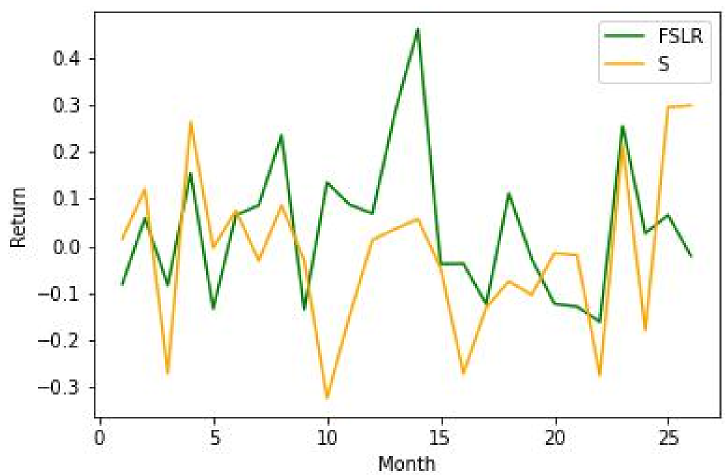


Figure 2.9: FSLR and S Monthly Returns 2021-2023



change in a month. Then we find the average return for each month over 2 years, adding the return of each month and dividing by the number of months. Then we calculate the returns of each month as follows:

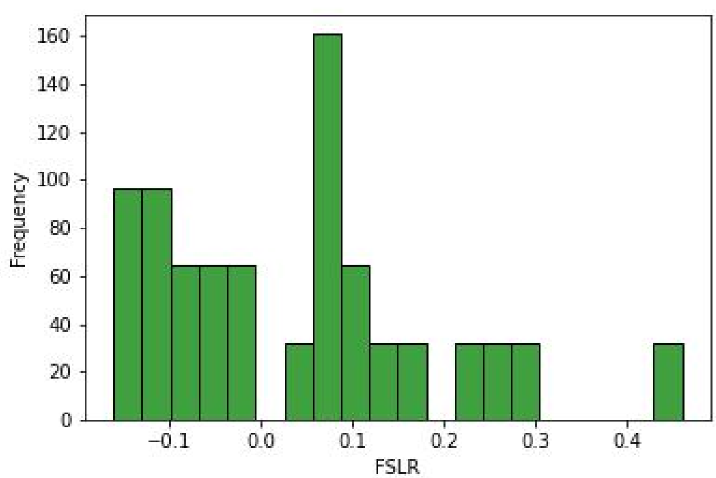
$$R_1 = (r_1, ..., r_{n-1}) \text{ where } r_i = \frac{x_i - x_{i-1}}{x_{i-1}}, i = 1, ...n-1 \text{ are the monthly returns of FSLR}$$

$$R_2 = (\tilde{r}_1, ..., \tilde{r}_{n-1}) \text{ where } \tilde{r}_i = \frac{y_i - y_{i-1}}{y_{i-1}}, i = 1, ...n-1 \text{ are the monthly returns of S}$$

The average yearly return for FSLR, r_1, and the average yearly return for S, r_2, is the average monthly return multiplied by 12, i.e. $r_1 = \bar{R}_1 \cdot 12$ and $r_2 = \bar{R}_2 \cdot 12$ respectively. The results of these calculations are observed in Table 2.5. Figure 2.9 shows the monthly returns for 26 months; FSLR is again in green while S is in yellow.

To determine the distribution of each monthly return stock data, we again conduct both a graphical analysis as well as a normality test. From Figures 2.10 and 2.11, we see that stock S monthly returns appear to have a normal Gaussian distribution; however, it is difficult to determine whether stock FSLR distribution is normal. To confirm our conclusions from the graphical analysis, we use the Shapiro-Wilks normality test in Python. Using the same equation as Equation (2.8) in the two clean energy stocks case, we can test if a data set is normally distributed by calculating the equation with our new opposing stock data with 26 data points where W is the test statistic we compare with the chosen alpha. Our null hypothesis is that the data set is normally distributed. Table 2.4 shows the statistics calculated with the Shapiro-Wilks normality test in Python. With the alpha level at 0.05, we compare each statistical value to determine whether the data shows normality. Table 2.4 displays the statistical values calculated and the p value compared to the alpha. As seen from Table 2.4, the p-value of FSLR data is just barely above our chosen alpha of 0.05, but we still fail to reject the null hypthesis. Then the monthly return data for FSLR is normally distributed. The p-value of S data is 0.442, which is well above our alpha 0.05. Then we can safely reject the null hypothesis and declare that S monthly return data is also normally distributed.

Figure 2.10: Histogram of FSLR Monthly Return



Let X be the vector $(x_1, ..., x_n)$ of monthly returns for the FSLR common stock and Y be the vector $(y_1, ..., y_n)$ of monthly returns for the S common stock, where $n = 26$. We can calculate the correlation of these returns by the same Pearson Correlation Coefficient

Figure 2.11: Histogram of S Monthly Returns

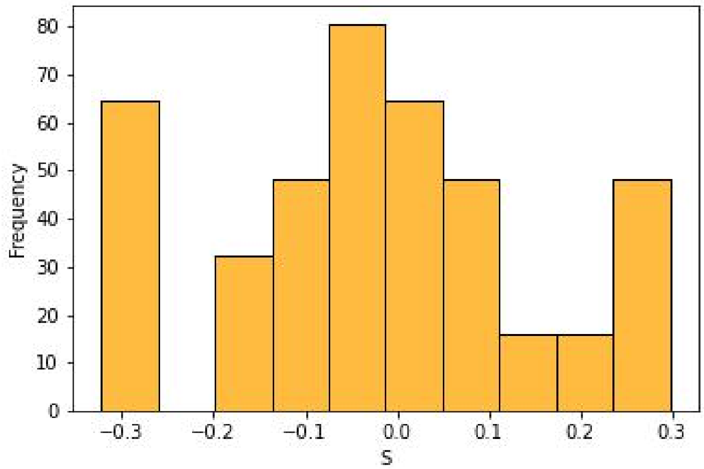


Table 2.4: Shapiro-Wilks Normality Test Statistics Table FSLR & S

	Statistic	Degree of Freedom	Significant Value
FSLR	0.928	26	p=0.069
S	0.962	26	p=0.442

defined in Equation (2.7). Then the correlation between common stocks FSLR and S is $\rho_{12} = 0.3474$. Since this value is close to 0, we cannot say that these two stocks have a high positive correlation. Figure 2.12 plots the monthly returns of FSLR and S stocks as points. The linear regression fit shows how little correlation occurs between these two stocks.

Figure 2.12: Correlation between FSLR and S Monthly Returns

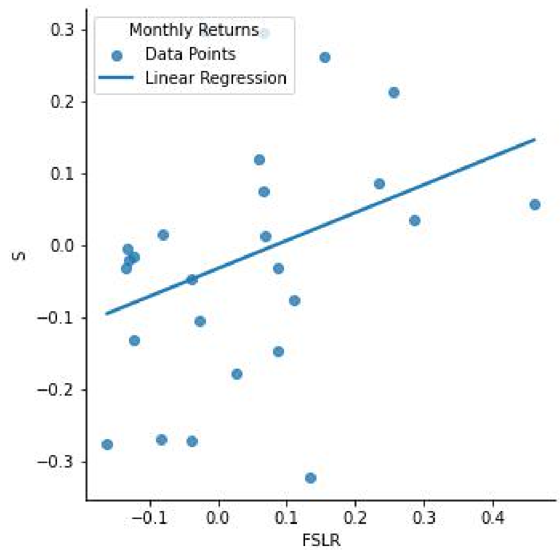


Table 2.5: Returns and Variances from Monthly Share Prices

	FSLR	S
Average Monthly Return	3.86%	-1.74%
Monthly Variance	2.34%	2.93%
Average Yearly Return	46.27%	-20.88%
Yearly Variance	28.14%	35.12%

Let μ_1 be the mean of monthly returns of FSLR, and μ_2 the mean of monthly returns of S. The variance is calculated using the same formula from Equation (2.3). By calculating the variances from the data, we obtain $\sigma_1^2 = 0.2705$ and $\sigma_2^2 = 0.3377$. From Definition 3.1.2, we define covariance in terms of the correlation coefficient and standard deviations of each asset:

$$\sigma_{12} = \rho_{12}\sigma_1\sigma_2 = 0.3474\sqrt{0.2705}\sqrt{0.3377} = 0.105$$

Table 2.6 is the variance-covariance matrix for FSLR and S stock, calculated by using the average yearly returns of for FSLR and S stock respectively. By looking at the covariance between each stock, we see that the returns move in the same directions, i.e. both stocks appear to increase over time.

Table 2.6: Variance-Covariance Matrix

	FSLR	S
FSLR	0.2705	0.105
S	0.105	0.3377

For the optimal weighted risky portfolio, we first minimize the variance to find the minimum variance portfolio (MVP). We solve for the weights and take the derivative with respect to weight 1 (w_1) again using Equation (2.3).
Solving for w_1, we obtain:

$$w_1 = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}} = \frac{(0.3377) - (0.105)}{(0.2705) + (0.3377) - 2(0.105)}$$

Solving for w_2, we obtain:

$$w_2 = \frac{\sigma_1^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}} = \frac{(0.2705) - (0.105)}{(0.2705) + (0.3377) - 2(0.105)}$$

The approximate minimum variance weights for the minimum variance portfolio are calculated to be:

$$w_1 = 0.5843, \qquad w_2 = 0.4156$$

Next, using the expected return Equation (2.1) with these weights we obtain:

$$E(r_p) = 0.5843(0.4627) + 0.4156(-0.2088) = 0.18357$$

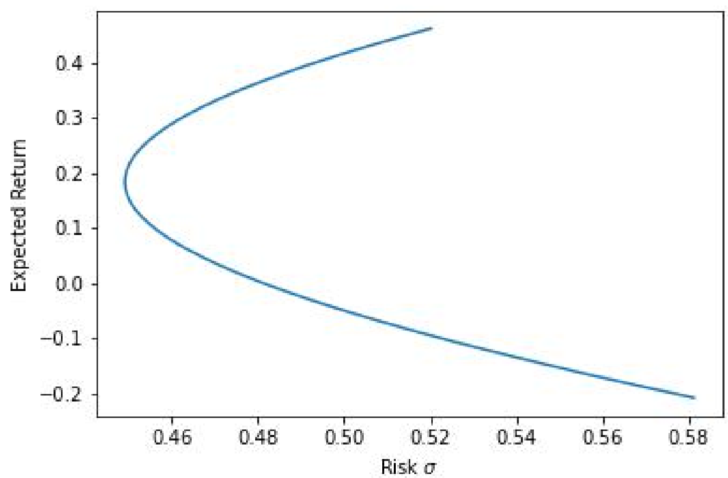
In calculating the standard deviation, we obtain:

$$\sigma_p = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{12}}$$

$$\sigma_p = \sqrt{0.5843^2(0.2705) + 0.4156^2(0.3377) + 2(0.5843)(0.4156)(0.105)}$$

$$\sigma_p = 0.44912$$

Figure 2.13: Efficient Frontier for FSLR and S stocks



The efficient frontier shown in Figure 2.13 displays the elliptical nature of risk and return of both assets, known as the opportunity curve. The MVP is the dividing point between inefficient and efficient portfolios, and in Figure 2.13, this point is at $(0.18, 0.45)$ within the risk and return space $[\sigma, E(r)]$. Those points above the MVP on the elliptical line are efficient and is known as the efficient frontier, while those below are inefficient.

2.4 Comparing Results Between the Two-Asset Cases

In Case 1, we observed a positively correlated pair of assets in the clean energy sector. Through creating an efficient frontier, we found that the minimum variance portolio between these two assets indicated that short selling our first asset, MAXN, would create the minimum risk for our level of return. Without allowing short-selling an asset, we determined that putting all weight into our second asset, BEP, will give us our minimum risk portfolio. In

Case 2, we observed a pair of less correlated assets in two completely different sectors, solar energy asset FSLR and artificial intelligence asset S. With these two assets we created an efficient frontier with a minimum variance portfolio weighted almost equally between our two assets. In our two cases, we see the effects of correlation and diversification played into our efficient portfolios. Table 2.7 displays the summary of results from our two-asset cases.

Table 2.7: Results of Two-Asset Cases 1 and 2

	MAXN & BEP	FSLR & S
Correlation Coefficient	0.481	0.3474
MVP Weights w_1, w_2	0,1	0.5843, 0.4156
MVP Return $E(r)$	0.0306	0.1836
MVP Risk σ_p	0.2939	0.44912

Table 2.8: Returns and Variances from Monthly Share Prices: Case 1

	MAXN	BEP
Average Yearly Return	18.6791%	3.0578%
Yearly Variance	94.3173%	8.7597%

As one of the main conceptual ideas of modern portfolio theory, diversification helps us minimize risk for a given level of return. The correlation coefficient helps us determine how much diversification we are able to achieve between our two assets. If our two assets are highly positively correlated, we will not achieve much diversification because the assets move so similarly in the market. This means that we must choose the asset with the least risk for its level of yearly return. Table 2.8 displays the average yearly return and variance of MAXN and BEP. Looking back at the annual returns and variances of MAXN and BEP, we see that although MAXN has a higher yearly return than BEP, it also has a much higher yearly variance for its yearly return. Our asset BEP has a much smaller yearly return compared to MAXN, but also a much smaller yearly variance for its return. We can interpret this as meaning BEP has less risk associated with its returns than MAXN, despite the yearly return being negative. Hence, the weight of the minimum variance portfolio between MAXN and BEP all goes to BEP.

Table 2.9: Returns and Variances from Monthly Share Prices: Case 2

	FSLR	S
Average Yearly Return	46.27%	-20.88%
Yearly Variance	28.14%	35.12%

If our two assets have a lower positive correlation, or close to no correlation such as our assets FSLR and S, then we can increase our level of diversification and minimize risk by putting equal weight into each asset. Table 2.9 restates the yearly return and variance for FSLR and S stocks. Here, FSLR has less yearly variance than S, meaning there is less risk involved with investing yearly in that particular stock. In addition, with a higher yearly return of FSLR, the diversification of a negative yearly return from S reduces our overall portfolio risk. Hence, we invest only a little more into FSLR than S in our minimum variance portfolio. The importance of finding less positively correlated assets is now clear: diversification helps reduce risk, and diversification is found in less positively correlated assets. We can now expand our lesson of diversification to an application with more than two assets, applying our knowledge of this mean-variance analysis to n-asset applications to reduce risk within a portfolio.

Chapter 3

PORTFOLIO ANALYSIS OF N-ASSETS

3.1 Definitions for N-Asset Case

Suppose now that we want to observe a portfolio with more than two assets. We can redefine the expected return and risk of a portfolio with n assets. We begin with the basic modern portfolio assumptions that our data must first have a normal distribution. Again, we can ensure this assumption with any data set either graphically with a histogram or by using a normality test. The Shapiro-Wilks normality test as defined in the previous chapter can be restated for the n-asset case.

Definition 3.1.1. Given a set of data $X = \{x_i | i = 1, ..., n \text{ and } n \in \mathbb{N}\}$, we can test if the set is normally distributed by calculating the significant value W:

$$W = \frac{(\sum_{i=1}^{n} a_i(x_{n+1-i} - x_i))^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2} \tag{2.8}$$

and compare with our chosen alpha value, usually 0.05. Weights a_i are predetermined by a Shapiro-Wilks weight table, and data in X is sorted from least to greatest in value. If our significant p-value is greater than our alpha value, then our null hypothesis that the data set is normally distributed fails to be rejected.

Definition 3.1.2. The expected return of an n-asset portfolio is defined as the expected value on a return of an n-asset portfolio, where the return is a weighted average of the return on each portfolio. The expected return of the portfolio is denoted as

$$E(r_p) = \sum_{i=1}^{n} w_i E(r_i) \tag{3.1}$$

Definition 3.1.3. The risk, σ_p, of an n-asset portfolio is defined as the variability of the return, r_p, of the portfolio, measured by the standard deviation.

With the return of an n-asset portfolio defined as $r_p = \sum_{i=1}^{n} w_i r_i$, we define the variation:

$$\begin{aligned}
\sigma_p^2 = E[r_p - E(r_p)]^2 &= E\left\{\sum_{i=1}^{n} w_i r_i - \left[\sum_{i=1}^{n} w_i E(r_i)\right]\right\}^2 \\
&= E\left\{\sum_{i=1}^{n} (w_i r_i - w_i E(r_i))\right\}^2 \\
&= E\left\{\sum_{i=1}^{n} w_i (r_i - E(r_i))\right\}^2 \\
&= E\left\{\sum_{i=1}^{n} w_i^2 (r_i - E(r_i))^2 + 2\sum_{i<j}^{n} w_i w_j (r_i - E(r_i))(r_j - E(r_j))\right\} \\
&= \sum_{i=1}^{n} w_i^2 E(r_i - E(r_i))^2 + 2\sum_{i<j}^{n} w_i w_j E\left[(r_i - E(r_i))(r_j - E(r_j))\right] \\
&= \sum_{i=1}^{n} w_i^2 \sigma_i^2 + 2\sum_{i<j}^{n} w_i w_j \sigma_{ij} \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij}
\end{aligned}$$

Then the variation of an n-asset portfolio can be denoted as

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij} = \begin{pmatrix} w_1 w_1 \sigma_{11} & \dots & w_1 w_n \sigma_{1n} \\ \vdots & \ddots & \vdots \\ w_n w_1 \sigma_{n1} & \dots & w_n w_n \sigma_{nn} \end{pmatrix} \tag{3.2}$$

The risk is the standard deviation: $\sqrt{\sigma_p^2} = \sigma_p$. For the n-asset portfolio, we still use the same definition of covariance as found in Definition 2.1.3 and Equation (2.5).

3.2 Generalized Analysis of N-Asset Cases

We wish to create the efficient frontier for this n-asset portfolio where the variance is minimized. The problem now becomes one of minimizing the weights in our objective function: the variance Equation (3.2). The Equation (3.2) is subject to two Lagrangian constraints:

I. We wish to achieve a desired expected level of return

$$\sum_{i=1}^{n} w_i E(r_i) = E(r_p) \tag{3.3}$$

$$-\left[E(r_p) - \sum_{i=1}^{n} w_i E(r_i)\right] = 0$$

II. The weights must sum to 1

$$\sum_{i=1}^{n} w_i = 1 \tag{3.4}$$

$$-\left(1 - \sum_{i=1}^{n} w_i\right) = 0$$

With these Lagrangian constraints, we obtain our Lagrangian objective function of the form $L = f(x) - \lambda g(x) - \gamma h(x)$, where $f(x)$ is our objective function and the desired expected return constraint $g(x)$ and weight constraint $h(x)$ are multiplied by the Lagrangian multipliers λ and γ, respectively.

$$L = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij} + \lambda\left[E(r_p) - \sum_{i=1}^{n} w_i E(r_i)\right] + \gamma\left(1 - \sum_{i=1}^{n} w_i\right) \tag{3.5}$$

To minimize the Lagrangian objective function in Equation (3.5), we set all partial derivatives from $i = 1, ..., n$ respective to w_i, and the partial derivatives with respect to λ and γ equal to 0. This results in a system of $n + 2$ partial derivative equations:

$$\begin{aligned}
\frac{\partial L}{\partial w_1} &= w_1\sigma_{11} + \ldots + w_n\sigma_{1n} - \lambda E(r_1) - \gamma = 0 \\
&\vdots \\
\frac{\partial L}{\partial w_n} &= w_1\sigma_{n1} + \ldots + w_n\sigma_{nn} - \lambda E(r_n) - \gamma = 0 \\
\frac{\partial L}{\partial \lambda} &= w_1 E(r_1) + \ldots + w_n E(r_n) - E(r_p) = 0 \\
\frac{\partial L}{\partial \gamma} &= w_1 + \ldots + w_n - 1 = 0
\end{aligned}$$

These partial derivatives can be written as the Jacobian matrix equation

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \ldots & \sigma_{1n} & E(r_1) & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \ldots & \sigma_{nn} & E(r_n) & 1 \\ E(r_1) & E(r_2) & \ldots & E(r_n) & 0 & 0 \\ 1 & 1 & \ldots & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_n \\ -\lambda \\ -\gamma \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ E(r_p) \\ 1 \end{bmatrix} \tag{3.6}$$

To rewrite the matrix equation in matrix notation, let A be the $(n+2)$ by $(n+2)$ Jacobian matrix, $\vec{x}$ the $(n+2)$ by 1 matrix of weights to be minimized and the Lagrangian multipliers, and $\vec{b}$ the $(n+2)$ by 1 matrix of constants. We can solve for $\vec{x}$ by the equation

$$\begin{aligned} A\vec{x} &= \vec{b} \\ \vec{x} &= A^{-1}\vec{b} \end{aligned}$$

Definition 3.2.1. Portfolios with a risk free asset are portfolios in which the risk σ_i^2 of asset i means $\sigma_i^2 = 0$. If one risk-free asset is held in conjunction with a portfolio of risky assets, the expected return of the entire portfolio is defined as

$$E(r_p) = (1 - \sum_{i=1}^{n} w_i) r_f + \sum_{i=1}^{n} w_i E(r_i), \tag{3.7}$$

where n is the number of risky asset returns r_i from $i = 1, ..., n$, and r_f is the return of the risk-free asset.

While the analysis of an n-asset case can vary based on the listed constraints needed for a particular optimization, the generalization as described in Chapter 3 lays the foundation by which Chapter 4 is created. Through our study of mean-variance analysis in modern portfolio theory, we now understand the mechanisms and equations needed for an application outside of strictly finance. Our next chapter details how we convert our study of MPT to a different application related to analyzing cost and risk in the service field of electricity providers.

Chapter 4

COST-RISK ANALYSIS

In our application of using Modern Portfolio Theory to the real world, we will take an examination of the electricity grid of the US state of Texas. In a report from the American Council for an Energy-Efficient Economy (ACEEE), Texas was cited to have Energy Efficient Resource Standards (EERS) that result in energy efficient program investments and savings below the national average [1]. However, instead of analyzing the return, it is more realistic to observe the cost to determine the least-cost portfolio. Hence, we will utilize modern portfolio theory to analyze the cost and risk of electricity operation based on the different technologies that produce electricity for the state of Texas. It is important to note that the primary source of electricity does not serve all of the state of Texas, although the servicer covers a majority of the state. The Electric Reliability Council of Texas (ERCOT) covers about 75% of the land area in Texas and carries 90% of Texas electricity load [10]. Figure 4 shows the service area of ERCOT.

In our analysis of cost-risk for efficient electricity generating portfolio, we measure the generating cost (USD per kWh) as the inverse of a return (kWh per USD).

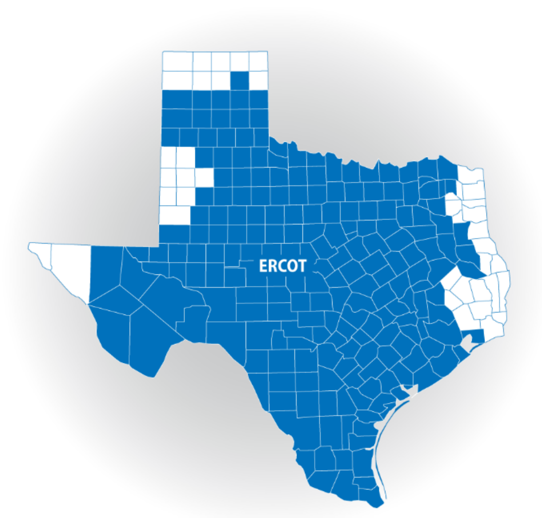


Figure 4.1: ERCOT Service Area [10]

Definition 4.0.1. Expected Portfolio Cost of Electricity:

$$E(C_p) = \sum_{i=1}^{n} w_i E(C_i) \tag{4.1}$$

where C_i is the generating cost of the ith of n number of technologies, known as the levelized cost of electricity (LCOE), and w_i is the capacity share of the ith technology.

Definition 4.0.2. Expected Portfolio Risk: (expected year-to-year variation in generating costs)
We can define the expected portfolio risk in terms of the variation in generating costs.

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij} \tag{4.2}$$

where σ_{ij} is the covariance of the LCOE for each technology. Then the portfolio risk can be defined as the standard deviation:

$$\sigma_p = \sqrt{\sigma_p^2} \tag{4.3}$$

To create the cost-risk efficient frontier, we want to minimize the weights in our objective function, the variance σ_p^2, subject to some reasonable constraints relative to an electricity grid.

I. The weights must sum to 0.97:

$$\sum_{i=1}^{n} w_i = 0.97 : \forall i = 1, 2, ..., n \tag{4.4}$$

Observing the historical data of the grid sharing capacity of our major technologies, we choose a sum of 0.97 to account for the 3% of "other" non-major contributors to the electrical grid which include biothermal, hydro-electric power, and other technologies.

II. Each weight must be positive:

$$w_i \geq 0 : \quad \forall i = 1, 2, ..., n \tag{4.5}$$

It is impossible for a technology to share a negative capacity in an electrical grid. There is no way for a technology to "give back" its electrical generation through its resources. However, as noted in one of the literatures reviewed, negative weights could be used to denote the load of electrical generation as a power plant with negative generation. This improvement will not be used in our analysis in this paper.

III. Each weight must not be greater than the maximum fractional sharing capacity of its technology, denoted as $\max\{S_i\}$, where S is the sharing capacity:

$$w_i \leq \max\{S_i\} : \forall i = 1, 2, ..., n \tag{4.6}$$

In other words, each technology produces a varying percentage of the overall electrical grid that supplies power to consumers of the region, and it would be unrealistic to assume a technology can produce more than its capable amount of power for the grid.

We formulate our minimization problem as follows:

$$\begin{aligned}
\textbf{min} \qquad & f(w_1, ..., w_n) = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij} \\
\textbf{subject to} \qquad & \sum_{i=1}^{n} w_i = 0.97 \\
& w_i \geq 0 : \forall i = 1, 2, ..., n \\
& w_i \leq \max\{S_i\} : \forall i = 1, 2, ..., n
\end{aligned}$$

where $\max\{S_i\} = s_i$ is the maximum sharing capacity of an ith technology. Our Lagrangian objective function becomes

$$L = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij} + \lambda\left(0.97 - \sum_{i=1}^{n} w_i\right) + \sum_{i=1}^{n}\gamma_i(q^2 - w_i) + \sum_{i=1}^{n}\phi_i(s_t - w_i - p^2) \tag{4.7}$$

where λ, γ_i, and ϕ_i are the Lagrangian multipliers, and q^2, p^2 are the slack variables of the inequality constraints. We solve the optimization problem through linear programming in MATLAB using cost and capacity factor data extracted from the ERCOT, the NREL, and the Energy Information Administration (EIA) [15] [11] [7]. Additionally, this optimization problem can be directly solved in Excel, where the data is stored.

The main data we need to complete our cost-risk analysis includes data for each generating power source: the predicted levelized cost of electricity, historical levelized cost of electricity, the variances of cost, and the covariances of cost between each generating power source. Instead of analyzing returns of each asset as done in our previous analyses, we calculate the levelized cost of electricity (LCOE) for each power source to analyze the expected cost of each power source. Some assumptions must also be made about our data. First, the historical LCOE for each generating technology is assumed to be normally distributed in order to conduct our analysis in modern portfolio theory. Our first step in our cost-risk analysis is to find the LCOE for each generating technology, both predicted values and historical data.

4.1 Levelized Cost of Electricity

The need for a levelized cost of electricity comes from calculating the cost of a generating technology that encompasses more than just the surface cost of producing power. The levelized cost of electricity is a metric that evaluates all costs of a generating technology over its entire lifespan. This assessment includes operation and maintenance costs, fuel costs, capital costs, and capacity factors. Since this cost is determined for the technological lifetime, we also have to include the interest rate, also known as the discount rate, by which the investment changes over time due to annuity factors and inflation.

The levelized cost of electricity (LCOE) is determined by calculating all of the expenditures of a generating technology. These expenditures include:

capital construction costs (CAPEX),

operation and maintenance costs (OPEX),

fuel costs,

and carbon costs.

These costs, except carbon costs, can be found for each technology in the 2022 Annual Technology Baseline (ATB) Workbook provided by the U.S. Department of Energy's National Renewable Energy Laboratory. As part of the Office of Energy Efficiency and Renewable Energy, this workbook provides capital expenditures (CAPEX), operation expenditures (OPEX), net capacity, energy production, and all other factors needed to calculate the LCOE for generating technologies in the United States. For convenience, the NREL, partnered with the Open Energy Data Initiative (OEDI), has provided the workbook in an AWS CLI data lake for easy programming and research. We use the Python programming language to extract this data for easier filtering and calculations, while using Excel to calculate the LCOE and analyze the data.

The simple levelized cost of electricity (sLCOE) is calculated from the ATB data using the following formula defined by the NREL:

$$c_i = \frac{C_c * C_r + F_{om}}{C_f * 8760} + (F_c * H_r) + V_{om} + (C_{o2} * C_{cc}), \tag{4.8}$$

where

c_i is the levelized cost of electricity for the ith technology,

C_c is capital costs (\$/kW),

C_r is the capital recovery factor and $C_r \in [0, 1]$,

F_{om} is the fixed operation and maintenance costs (\$/kW-yr),

C_f is the capacity factor, a percentage of the annual production of energy of a system, multiplied by the number of hours in a year (8760),

F_c is fuel cost (\$/MMBtu),

H_r is the heat rate (Btu/kWh),

C_{cc} is carbon clearing cost,

and V_{om} is the variable operation and maintenance cost (\$/kWh).

To calculate specifically for the ERCOT region, we take the capacity factor of each of the 5 technologies listed from the ERCOT 2022 Long-Term System Assessment for the years 2023, 2027, 2032, and 2037. Fuel cost, heat rate, and carbon clearing costs are optional since these variables are not applicable to renewable energies. Texas does not have carbon emissions pricing policies, nor do they have a statewide emissions reduction goal. Therefore, we take the U.S. standard carbon clearing price of \$47.10 per ton of CO_2.

The data included in the ATB is categorized by certain scenarios for each technology, namely advanced, moderate, and conservative. These scenarios dictate the different values of factors for each year based on a level of advancement from innovations in the marketplace [12]. For this study, we chose the moderate scenario to reflect the expected costs for a base year of 2022. There are a total of 18 generating technologies outlined in the ATB that displays each generating technology's total electricity generation from the years 2022 to 2050. From the ATB, we collected the capital expenditures, operation and maintenance expenditures, and fuel cost for the years 2023, 2027, 2032, and 2037, since the capacity factors for Texas were only found for these years. We keep the base assumptions within the ATB to collect the calculated costs for each technology. The chosen financial case is Market, which presents market and policy changes over time. The second assumption is the capital recovery period, defined as the time in years it takes to recover from an initial investment in a technology. This capital recovery period is 30 years.

Although the ATB lists 18 different technologies, we will only collect data for the 5 major technologies listed in the capacity of the Texas grid. The first technology is natural gas, and more specifically, the natural gas-combined cycle energy source. As a fossil energy, this technology will have a fuel cost designated by the yearly fuel cost per MWh of fuel and a carbon clearing cost. The second technology is coal, another fossil fuel that will have a fuel cost per MWh and carbon clearing cost for each observed year. For specific coal technology, we chose the common verson of coal resources for Texas: integrated gasification combined cycle (IGCC). Third, we have solar energy, by which we specify the utility-scale solar photovoltaic. We use the utility-scale solar data only to simplify our equation and levelized cost of energy, and according to the NREL, utility-scale solar creates the majority of solar energy in the United States. In addition, the utility-scale solar energy is also split into 10 classes, each with a difference level of average yearly solar irradiance. Figure 4.2 shows the levels of solar irradiance for all parts of the United States. Observing the levels in Texas, we see all irradiance levels. Therefore, in our calculation of LCOE for solar data, we chose the average of the United States, Class 5 Solar PV. Next, we have wind power. For calculating the LCOE of wind power for Texas, we use the class 5 land-based wind data, since this type of wind power produces more electricity than the off-shore wind power in Texas. Lastly, we have nuclear power, which does not have a specified technology. This is because all nuclear power in the United States is created by nuclear fission; in addition, nuclear power plants produce electricity at a consistent level under nuclear fission, based on the size of the nuclear plant. Texas has two nuclear power plants that each have two reactors. These plants each have an installed capacity of 5,000 MW [9].

Figure 4.2: NREL Global Solar Irrandiance for the United States [16]

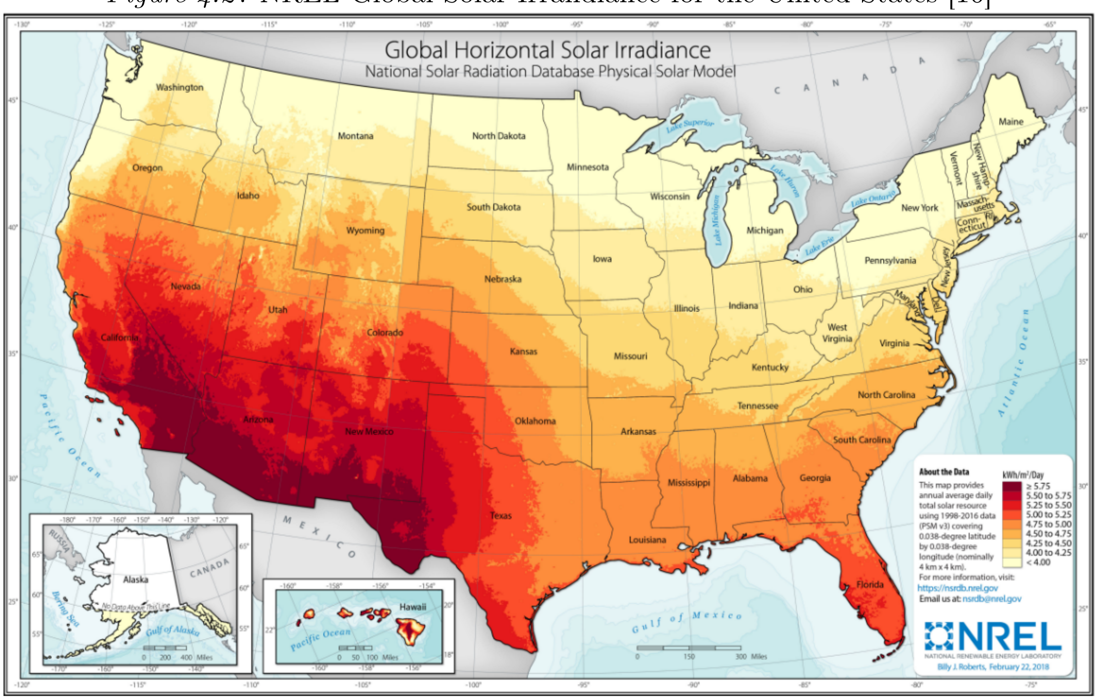


Table 4.1 lists the capacity factors of each technology as calculated in the ERCOT Long-Term System Assessment. The capacity factors describe the proportion of electricity

Table 4.1: ERCOT Capacity Factors of each generating technology

Capacity Factors (%)				
Technology	2023	2027	2032	2037
Natural Gas	54.6	42.97	39.37	38.53
Coal	54	54	54	54
Wind	55.2	55.2	55.2	55.2
Solar	25.3	25.3	25.3	25.3
Nuclear	95.4	95.4	95.4	95.4

Table 4.2: EIA Fuel Costs

Fuel Costs ($/MMBtu)				
Technology	2023	2027	2032	2037
Natural Gas	3.10	3.44	4.39	5.04
Coal	1.85	2.21	2.48	2.58
Nuclear	0.69	0.69	0.70	0.71

each technology provides to the overall grid. Table 4.2 lists the fuel costs for natural gas, coal, and nuclear resources. Although not listed in the ATB, these prices can be found within the data of the Energy Information Administration. Since wind and solar are renewable resources, they do not incur a fuel cost. Table 4.3 lists the heat rate from technologies that incur a fuel cost, listed in the ATB. Heat rate describes the rate of converting fuel into electricity and the heat generated from this conversion. Heat rate is directly correlated to carbon emissions, as the conversion from fuel to electricity also produces CO_2.

To calculate the levelized cost of energy, we use data in Tables 4.1-4.3 as well as additional cost factor data for each technology located in the ATB to input into our LCOE Equation (4.7). The resulting LCOE for each technology in each year is listed in Table 4.4.

4.2 Results of Cost-Risk Analysis

Now that we have the levelized cost of electricity of each technology, the efficient frontier can be created. First, we solve our minimization problem by minimizing Equation (4.7) using data from Table 4.1 to create the minimum variance portfolio (MVP). The MVP is the portfolio with the minimum amount of variance for an expected cost. The weights solved for

Table 4.3: ATB Technology Heat Rates

Heat Rate (MMBtu/MWh)				
Technology	2023	2027	2032	2037
Natural Gas	6.36	6.36	6.36	6.36
Coal	8.01	7.60	7.20	7.20
Nuclear	10.44	10.44	10.44	10.44

Table 4.4: Levelized Cost of Electricity of each technology for selected years

Levelized Cost of Energy						
Technology	2023	2027	2032	2037	Average Cost	Variance
Natural Gas	2.879	2.863	2.859	2.858	2.86	0.0001
Coal	5.055	4.91	4.856	4.836	4.91	0.01
Wind	3.15	2.665	2.257	2.142	2.55	0.21
Solar	5.634	4.675	3.888	3.712	4.48	0.77
Nuclear	4.639	4.543	4.455	4.307	4.49	0.02

Table 4.5: Pearson Correlation Coefficients for 5 Major Technologies

	Gas	Coal	Wind	Solar	Nuclear
Gas	1	0.998	0.969	0.973	0.854
Coal	0.998	1	0.982	0.984	0.887
Wind	0.969	0.982	1	0.999	0.934
Solar	0.973	0.984	0.999	1	0.925
Nuclear	0.854	0.887	0.934	0.925	1

our minimization problem describes the sharing capacity of each technology. These weights are listed in Table 4.7. As seen from the correlation coefficient Table 4.5, all technologies are highly positively correlated. Nuclear has the least correlation with each technology. Wind and solar and coal and gas have high positive correlations with one another. This may be due to the fact that those pairs of resources come from much of the same places. The variance-covariance matrix between each of the generating technologies is listed in Table 4.6.

Using Equations (4.1) and (4.2), we find the expected cost and risk for the MVP. The expected cost of the minimum variance portfolio of the listed technologies is \$3.07/kWh while the risk of cost is 15.72%

$$E(c_p) = 3.07$$

$$\sigma_p = \sqrt{0.027} = 0.1572$$

The efficient frontier of possible portfolios is displayed in Figure 4.3. The efficient frontier, created in Excel, uses 10,000 possible combinations of sharing capacities of the major

Table 4.6: Variance-Covariance Matrix for 5 Major Technologies

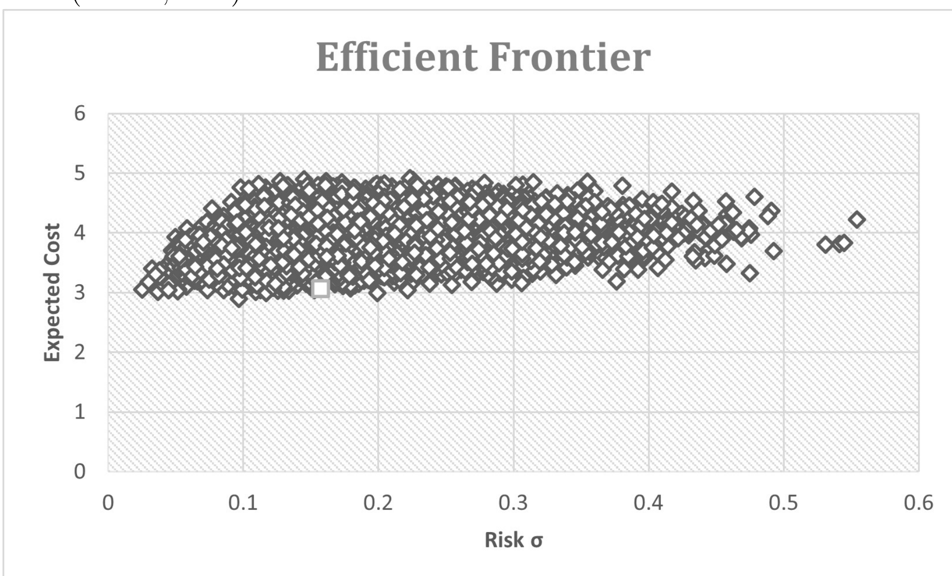
	Gas	Coal	Wind	Solar	Nuclear
Gas	7.88×10^{-5}	0.0008	0.0034	0.0066	0.0009
Coal	0.0008	0.007	0.0333	0.064	0.0093
Wind	0.0034	0.0333	0.1563	0.3003	0.045
Solar	0.0066	0.064	0.3003	0.5773	0.0858
Nuclear	0.0009	0.0093	0.0451	0.0858	0.0149

Table 4.7: Weights calculated for the Minimum Variance Portfolio (MVP)

Natural Gas	Coal	Wind	Solar	Nuclear
$w_1 = 0.416$	$w_2 = 0.119$	w_3 =0.334	$w_4 = 3.547 \times 10^{-}8$	$w_5 = 0.099$

technologies. Each combination of differently weighted technologies is shown as a diamond, while the MVP is shown as a square. The efficient frontier is a result of our constraints as well as our specific data chosen for our minimization problem.

Figure 4.3: Efficient Frontier for ERCOT region. The square among the diamonds represents the MVP at (0.1572, 3.07)



4.3 Conclusions

In our risk-cost analysis we observe a calculated minimum variance portfolio among our efficient frontier in the risk-cost space $[\sigma, E(c)]$, denoted by the square amidst the diamonds at point (0.157, 3.07). The MVP denotes that the minimum risk of cost is at approximately 16% for the cost of \$3.07 per kilowatt hour produced by the combination of all technologies. The efficient frontier displayed in Figure 4.3 is a study of mean-variance analysis applied to a small amount of data available from the ERCOT website and the Annual Technology Baseline. The minimum variance portfolio tells us that the options along the bottom points in the space are caused by our constraints and optimization of the weights for our variance function. Any points like the ones displayed in Figure 4.3 are viable options for decision-makers with different degrees of risk-taking. These are forecasted solutions because we used forecasted data as outlined in the Annual Technology Baseline workbook. We also observe that the expected cost of the minimum variance portfolio is near the average of the total cost of each listed technology. Observing the sharing capacities obtained from the MVP, natural gas still has the highest sharing capacity, with solar having the least amount of sharing capacity, nearly 0. In addition, since the solar costs have the most risk associated with its cost, the minimum variance portfolio gave solar technology the smallest share of total electricity grid capacity. With sharing capacities for generating technologies weighted as 41.6% natural gas, 11.9% coal, 33.4% wind, and 9.9% nuclear, we are left with a 3.2% sharing capacity for other technologies in the electricity grid mix of ERCOT. Therefore, we conclude from this minimum variance portfolio that ERCOT should minimize usage of solar technology due to the increased risk in solar costs.

Some setbacks of our analysis include not having as much data available to us as the NYSE stock finance minimization problems. This is due to finding the capacity factor data for each technology specific to the state of Texas. While the EIA does list a few years of capacity factors for every state, the data is lacking in the years observed. Our conclusions here are not meant to be the final discussion with this application.

4.4 Further Research

Continued and current research includes adding more historical data to the levelized cost of electricity for each generating technology. We could also add more terms and weights to include all technologies listed in the sharing capacity of the ERCOT service grid, given the correct data for all technologies can be found. In addition, there are more constraints that could be added to our lagrangian objective function to increase specifications. Numerous other scenarios can be analyzed, including scenarios that increase the sharing capacity of different weights based on forecast sharing capacity data. Lastly, the ATB lists factors for technologies based upon different levels of advancement. From advanced, moderate, and conservative scenarios, the ATB allows for different analyses to be conducted based upon the observers assumptions and expectations.

Appendix A

DATA

A.1 Chapter 2 Stock Data MAXN and BEP

Table A.1: MAXN and BEP Monthly Returns Data from August 2020 to August 2023

MAXN	BEP		MAXN (Contd.)	BEP (Contd.)
-0.1376	-0.0499		-0.1256	0.0042
-0.0437	-0.0282		-0.2974	-0.1231
0.0784	-0.0113		0.3471	0.1087
-0.0200	0.0148		0.0193	-0.0991
0.5160	0.1685		0.1127	0.0511
-0.1741	-0.0917		-0.2367	0.0081
0.4377	0.1061		0.2962	-0.0266
-0.2919	-0.0965		-0.0874	0.0300
0.2471	-0.0198		-0.4524	-0.1029
-0.3021	-0.0861		-0.1010	0.0014
0.3107	-0.1366		-0.1150	-0.0681
0.0825	-0.0117		0.3051	0.0355
0.3686	0.0668		0.1489	0.0885
0.0814	0.0147		0.6139	0.0698
0.0324	-0.0069		-0.0762	0.0591
-0.1843	-0.1474		-0.1761	0.1946
0.2027	0.1283		-0.6482	0.0285
0.1423	0.0888			
-0.3129	-0.0697			

A.2 Chapter 2 Stock Data FSLR and S

Table A.3: FSLR and S Monthly Return Data from July 2021 to September 2023

FSLR	S
-0.0805	0.0153
0.0590	0.1203
-0.0831	-0.2698
0.1542	0.2626
-0.1326	-0.0036
0.0655	0.0751
0.0862	-0.0316
0.2344	0.0858
-0.1341	-0.0306
0.1344	-0.3230
0.0875	-0.1471
0.0687	0.0128
0.2850	0.0359
0.4599	0.0566
-0.0384	-0.0471
-0.0376	-0.2708
-0.1234	-0.1309
0.1109	-0.0747
-0.0268	-0.1039
-0.1228	-0.0153
-0.1285	-0.0196
-0.1610	-0.2747
0.2544	0.2123
0.0267	-0.1787
0.0656	0.2942
-0.0211	0.2980

A.3 2022 Annual Technology Baseline Data for Major Technologies

Note: 2022 Annual Baseline Technology Data is used in the "Market" scenario and the "Moderate" scenario for all technologies.

Table A.4: ATB and EIA Factors and Units

C_c	F_{om}	V_{om}	F_c	C_f	C_r	H_r	C_{o2}	C_{cc}
\$/kW	\$/kW-yr	\$/MWh	\$/MMBtu	(%)	N/A	MMBtu/MWh	lbs/MMBtu	\$/t

Table A.5: Natural Gas ATB and EIA Data

	C_c	F_{om}	V_{om}	F_c	C_f	C_r	H_r	C_{o2}	C_{cc}
2023	1026.098	27.94	1.78	3.1	54.6	0.274841	6.36	118.62	47.1
2027	1005.624	27.94	1.78	3.44	42.97	0.146693	6.36	118.62	47.1
2032	981.0515	27.94	1.78	4.39	39.37	0.095843	6.36	118.62	47.1
2037	957.4631	27.94	1.78	5.04	38.534	0.077799	6.36	118.62	47.1

Table A.6: Coal ATB and EIA Data

	C_c	F_{om}	V_{om}	F_c	C_f	C_r	H_r	C_{o2}	C_{cc}
2023	5184.772	140.54	13.86	1.85	54	0.274841	8.01	199.4	47.1
2027	4990.793	140.54	13.86	2.21	54	0.146693	7.6	199.4	47.1
2032	4859.078	140.54	13.86	2.48	54	0.095843	7.2	199.4	47.1
2037	4744.127	140.54	13.86	2.58	54	0.077799	7.2	199.4	47.1

Table A.7: Wind ATB and EIA Data

	C_c	F_{om}	V_{om}	F_c	C_f	C_r
2023	1308.4	41.785	0	0	55.2	11.61
2027	1106.348	40.165	0	0	55.2	11.61
2032	936.8009	38.365	0	0	55.2	11.61
2037	889.0049	36.90513	0	0	55.2	11.61

Table A.8: Solar ATB and EIA Data

	C_c	F_{om}	V_{om}	F_c	C_f	C_r
2023	1073.784	19.35443	0	0	25.3	11.61
2027	890.9969	16.98315	0	0	25.3	11.61
2032	740.952	15.02043	0	0	25.3	11.61
2037	707.3404	14.52195	0	0	25.3	11.61

Table A.9: Nuclear ATB and EIA Data

	C_c	F_{om}	V_{om}	F_c	C_f	C_r	H_r
2023	7302.005	145.96	2.84	7.179558	95.4	5.2	10.44
2027	7011.51	145.96	2.84	7.242345	95.4	5.3	10.44
2032	6872.195	145.96	2.84	7.326079	95.4	5.3	10.44
2037	6637.247	145.96	2.84	7.409802	95.4	5.3	10.44

BIBLIOGRAPHY

[1] American Council for an Energy Efficient Economy. *State and Local Policy Database.* Texas Utilities. Last Updated: July 2017. Accessed September 2023. https://database.aceee.org/state/texas

[2] Arévalo, J. E. A., Paz, F., and Gomez, G.I. Analysis and Application of Modern Portfolio Theory to the German Electricity Mix. *Papels de Energia*, 15, 2021.

[3] Awerbuch, S. and Yang, S. Efficient electricity generating portfolios for Europe: Maximising energy security and climate change mitigation. *EIB Papers*, 12(2):8-37, 2007. European Investment Bank, Luxembourg. ISSN: 0257-7755

[4] Bakar, N. A. and Rosbi, S. Robust Statistical Portfolio Investment in Modern Portfolio Theory: A Case Study of Two Stocks Combination in Kuala Lumpur Stock Exchange. *IJEAT*, 8(5c), 2019. ISSN:2249-8958

[5] Bakare, M.S., Abdulkarim, A., Zeeshan, M., & Shuaibu, A. A comprehensive overview on demand side energy management towards smart grids: challenges, solutions, and future direction. Energy Inform 6, 4 (2023). https://doi.org/10.1186/s42162-023-00262-7

[6] Castro, G. M., Klöckl, C., Regner P., Schmidt, J., & Olimpio Pereira Jr., A. Improvements to Modern Portfolio Theory based models applied to electricity systems. *Energy Economics*, Vol. 111, 2022.

[7] EIA. *State Profiles and Energy Estimates.* U.S. Energy Information Administration. eia.gov

[8] Francis, J. C. and Kim, D. *Modern Portfolio Theory: Foundation, Analysis, and New Developments.* John Wiley & Sons, Inc, Hoboken, New Jersey, 2013.

[9] Hegar, G. *Texas Comptroller Energy Tour: NUCLEAR OVERVIEW* Texas Comptroller. "Economy". 2023. https://comptroller.texas.gov/economy/economic-data/energy/2023/nuclear.php

[10] Minton, L. *Texas' Electricity Resources: Where Power Comes From- And How it Gets to You.* FiscalNotes: A Review of the Texas Economy, 2020. comptroller.texas.gov/economy/fiscal-notes/2020/august/ercot

[11] NREL. *Annual Technology Baseline Workbook 2022 v2.* National Renewable Energy Laboratory. U.S. Department of Energy. Office of Energy Efficiency and Renewable Energy. 2022 atb.nrel.gov

[12] NREL. *2022 Annual Technology Baseline Definitions.* National Renewable Energy Laboratory. U.S. Department of Energy. https://atb.nrel.gov/electricity/2022/definitions

[13] NREL. *Simple Levelized Cost of Energy Calculator Documentation.* Energy Analysis. National Renewable Energy Laboratory. U.S. Department of Energy. Office of Energy Efficiency and Renewable Energy. nrel.gov/analysis/tech-lcoe-documentation.html

[14] Patterson, D. "Using Modern Portfolio Theory to Analyze Virgil's Aeneid (or Any Other Poem)." *Master's Theses.* 737, 2020. https://aquila.usm.edu/masters_theses/737

[15] Rhodes, J. D. ERCOT 2040: A Roadmap for Modernizing Texas' electricity infrastructure. Electricity Reliability Council of Texas. Advanced Energy United. IdeaSmiths, LLC. 2023.

[16] Sengupta, M., Xie, Y., Lopez, A., Habte, A., Maclaurin, G., Shelby, J., & Gueymard, C. "The National Solar Radiation Data Base (NSRDB)." Renewable and Sustainable Energy Reviews 89 (June): 51-60. 2018.