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COINCIDENCE THEORY: SEEKING A PERCEPTUAL PREFERENCE FOR JUST INTONATION, EQUAL TEMPERAMENT, AND PYTHAGOREAN INTONATION IN EXCERPTS FOR WIND INSTRUMENTS

by

Derle Ray Long

Abstract of a Dissertation
Submitted to the Graduate Studies Office
of The University of Southern Mississippi
in Partial Fulfillment of the Requirements
for the Degree of Doctor of Philosophy

December 2008

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2008

The University of Southern Mississippi

AN INVESTIGATION OF COINCIDENCE THEORY FOR HARMONY CONSONANCE IN EXCERPTS FOR WIND INSTRUMENTS

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Derle Ray Long

A Dissertation
Submitted to the Graduate Studies Office
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ABSTRACT

COINCIDENCE THEORY: SEEKING A PERCEPTUAL PREFERENCE FOR JUST INTONATION, EQUAL TEMPERAMENT, AND PYTHAGOREAN INTONATION IN EXCERPTS FOR WIND INSTRUMENTS

by Derle Ray Long

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Coincidence theory states that when the components of harmony are in enhanced alignment the sound will be more consonant to the human auditory system. An objective method of examining the components of harmony is by investigating alignment of the mathematics of a particular sound or harmony. The study examined preference responses to excerpts tuned in just intonation, Pythagorean intonation, and equal temperament. Musical excerpts were presented in pairs and study subjects simply picked one version from the pair that they perceived as the most consonant. Results of the study revealed an overall preference for equal temperament in contradiction to coincidence theory. Several additional areas for research are suggested to further investigate the results of this study.

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TABLE OF CONTENTS

LIST OF TABLES

Table

LIST OF ILLUSTRATIONS

Figure

CHAPTER I

INTRODUCTION

Conductors of wind instrument ensembles must possess the ability to audibly detect intonation problems in harmony produced by an ensemble. This skill provides the initial effort in a four step intonation process that combines detection, analysis of the problem, selection of a remedy, and synthesis of the remedy into rehearsal procedures and performance practice. Conductors utilize this process to provide information and training for an ensemble with the goal of developing proficiency by each performer in each of the four steps. A combined effort by conductor and ensemble members is required to create and maintain consonant intonation in wind instrument ensembles.

One problem that wind ensemble conductors and performers encounter in daily rehearsal is the juxtaposition of multiple intonation systems and the task of choosing one that will produce the most consonant sound. Brass instruments have the capacity to play in just intonation due to the fact that the valves derive notes from the natural overtone series. Remedies for intonation problems in brass instruments generally consist of small changes in the embouchure, changes in the length of slides, and alternate fingerings. Keyboards, melodic percussion, and woodwinds are constructed to produce a scale that primarily conforms to equal temperament. Remedies for intonation problems in woodwinds generally consist of small embouchure changes and alternate fingerings. All remedies are contingent upon correct embouchure formation, adequate air support, characteristic tone quality, and the ability to detect discordant beats that are aurally perceived when frequencies do not coincide.

Norden (1936) wrote, "As soon as we sing sharp or flat of the true intervals, beats arise…" (p. 219). Wilkinson (1988) offered an acoustic explanation of beats that described alternating reinforcing and canceling effects between vibrating frequencies as they move in and out of synchronism (p. 29). Benade (1990) stated that two simultaneously sounding frequencies which move out of synchronism five times each second will produce five beats of audible interference per second (p. 239).

A psychoacoustic explanation of the beating phenomenon describes perception of beats as a product of neural processing in the cochlea. Plomp (1967) referred to this phenomenon as a nonlinear, or distorted, perceptual response by the human auditory system (p. 462). A large portion of the literature on the reception and processing of sounds by the human nervous system identified beats as nonlinear responses due to the fact that they are not components of the original stimulus tones.

Beats have the potential to create distortion in a harmony from the moment they are detected to a point where the frequencies are separated sufficiently so that two distinct tones are perceived. The research indentifies the point at which the human auditory system is capable of detecting a change in pitch as the just noticeable difference (jnd). A review of the literature revealed that research on the jnd largely involved presentation of tones in a sequential, or melodic pattern, and not the simultaneous sounding of tones that comprise a harmony.

Helmholtz (1877/1954) wrote that beats become unpleasant at a rate of six per second and reach maximum discord at around thirty-three per second (pp. 164-172). Wilkinson identified the point of separation where two distinct sounds are perceived as the "limit of discrimination" (p. 31). The literature did not establish a correlation between the limit of discrimination and the maximum discord beat rate established by

Helmholtz. It is the region of audible perception between frequency coincidence and the limit of discrimination where conductors of wind instrument ensembles are able to detect beats. It is this region that was a focus of the current study.

Terhardt (1974) considered frequency distance as a decisive parameter of consonance between two tones (p.1061). Frequency distance may be expressed by the formula $f_1 - f_2$ which is also identified as the beat frequency. This is also the formula used to calculate the psychoacoustic phenomenon identified as difference tones. *The Oxford Companion to Music* referred to beats as a type of difference tone (p. 9). Distinction between the two is that frequencies associated with the generation of beats have critical bands that overlap.

The critical band is identified as a small range of frequencies that are processed along the same portion of the basilar membrane. Two sounds with perfectly synchronized frequencies reinforce each other along this area of the membrane and produce what Truax (1999) identified as an amplitude modulation (p. 1). Plomp and Levelt (1965) determined maximum consonance at the unison of identical frequencies, also referred to as a "perfect unison" in *The Oxford Companion to Music* (p. 9). This offered an argument, in opposition to an early theory by Rameau (1722/1971), for inclusion of the unison as an interval by which harmony consonance may be perceived (p. 8).

As a unison moves away from synchronism, the alternating reinforcing and canceling effect is perceived as beats within the sound. Wilkinson identified these types of beats as "first order beats" and wrote that they are created when "the waves alternately reinforce each other and cancel each other out" (p. 29). As the frequency separation approaches the limit of discrimination the beating sensation becomes increasingly unpleasant. At the limit of discrimination, the perception is no longer what the *Oxford Companion to Music* referred to as a "near unison" and instead is perceived as two separate tones (p. 114). Jourdain (1997) wrote that this is the area where critical bands of the tones no longer overlap (p. 101).

Two non-coincidental frequencies whose critical bands overlap will produce a sound that is rough. As the frequencies separate to the point where their critical bands no longer overlap, the roughness caused by beating begins to diminish. Jourdain identified that point of separation as approximately the interval of a minor third (p. 101). Truax identified the critical bandwidth in complex tones as the smallest frequency difference between two partials such that each can be heard as a separate tone (p. 1).

Terhardt (1974) wrote that when beats occur rapidly, roughness is perceived in the sound that is strongly correlated with dissonance (p. 1061). Denckla (1997) added that "the rate of beating is proportional to the amount of dissonance" (p. 1). Terhardt and Denckla described dissonance on a broader scale than was required by the current study. They described dissonant sounds that are the product of overlap between critical bands. Their roughness theory explains why the interval of a minor second is perceived as more dissonant than a perfect fifth. The critical bands of frequencies that comprise a minor second overlap and beats generated by their difference add roughness to the composite sound. The critical bands of frequencies that comprise a perfect fifth do not overlap.

Seashore (1967) wrote, "Consonance depends fundamentally on the degree of coincidence of sound waves" (p. 126). Whitcomb (2005) discussed what is identified in the literature as *Coincidence Theory* and attributed the first use of that term to H. F. Cohen in 1984 (p. 69). He wrote, "The degree of consonance of a group of sounds is determined by, and is proportional to, the rate at which the wave patterns of those sounds coincide" (p. 70). Whitcomb (1999) had earlier traced coincidence theory back to the writings of Benedetti, Galileo, and Mersenne (pp. 12-16).

The current study focused on the presence of beats within a narrow window, where one boundary was frequency coincidence and a second boundary, the limit of discrimination. This is the area where beats can be audibly detected by wind instrument ensemble conductors. To facilitate examination of this area, the study focused on similar frequencies among primary stimulus tones, partials of complex tones, and difference tones. The goal was to examine harmonies that are perceived as consonant to the human ear and provide information as to why those sounds are more agreeable than others.

Need for Study

A review of the available research pointed to areas where information is lacking. The available literature was deficient in studies that:

1. Addressed perceptual preference for harmony consonance utilizing one intonation system or another.
2. Examined harmony intonation within a musical context.
3. Utilized complex tones as auditory stimuli in an investigation of intonation.
4. Examined coincidence theory as an indicator of harmony consonance.

Investigations by Johnson (1962) and Bisel (1987) addressed some of these issues. These studies investigated perceptual preference for one intonation system over others. There is need for additional study in this area, particularly for a study that combined musical context and complex tones in an investigation of perceptual preference for harmony intonation consonance.

Statement of the Problem

The research problem was to apply an auditory perception test in which study participants chose a musical excerpt that they perceived as the most consonant for harmony intonation.

Sub-problems

1. Musical excerpts were chosen as examples of harmony intonation problems encountered by wind instrument ensemble conductors.
2. The excerpts were converted to MUS and MIDI files using Finale 2008 music notation software.
3. Excerpts were reproduced using a computer capable of synthesizing complex tones similar to those produced by wind instruments. This computer was equipped with Justonic Pitch Palette software which allows playback of the excerpts in selected intonation systems. The computer was also equipped with Roland Virtual Sound Canvas 3.2 software that enhanced the computer generated sounds.
4. A compact disc was produced that contains the excerpts in a format that allowed collection of data.
5. A data collection document was developed that allowed an expressed choice for excerpt tuning among study participants.
6. The compact disc was distributed to study participants and a preference response

solicited from said group using the data collection document.

7. The data was analyzed and reported.

Purpose of the Study

The purpose of this study was to investigate whether enhanced mathematical coincidence of harmony components would influence the expressed preference of study subjects for musical excerpts played in equal temperament, just intonation, or Pythagorean intonation.

Scope and Delimitations

The number of musical excerpts used in the study was limited to three. These excerpts were selected from the *MLR Instrumental Score Reading Series* published by G.I.A. Publications. Consideration of compositional style or technical difficulties in each excerpt did not serve as determining factors for inclusion in this study. Techniques utilized to create this music are considered to be representative of accepted compositional practice. The determination of excerpts used in this study was accomplished through a series of pre-tests among student musicians at the University of Louisiana at Monroe.

The presentation of musical examples was arranged so that study subjects spent no more than thirty minutes receiving instructions, listening to the excerpts, and responding to the data collection document. The intonation systems utilized in this study were Pythagorean intonation, equal temperament, and just intonation. just intonation served as the system representative of maximum alignment of harmony components as described by coincidence theory.

Definition of Terms

Beats are acoustical disturbances created when two or more frequencies do not vibrate coincidentally.

Beat frequency refers to the rate of beating interference each second.

Cents refers to a unit of tuning measurement equal to $1/100^{th}$ of a semitone.

Chord root refers to the note on which a harmony is structured.

Commas are minute intervals encountered in intonation systems. The most common are the *Pythagorean comma* (also referred to as the *Ditonic comma*) with ratio 531441/524228, the *Syntonic comma* (also called the *Comma of Didymus*) with ratio 81/80, and the *Septimal comma* with ratio 64/63.

Consonance refers to a combination of sounds that are perceived by the listener as being at rest. Consonance has also been referred to as a sound that is agreeable.

Difference tones are additional sounds created by the interaction of two frequencies. The frequency of a difference tone can be determined by the formula; $f_2 - f_1$ = difference tone. Difference tones are optimally generated when pure intervals are utilized. Difference tones are useful in the determination of optimum frequency coincidence within a harmony.

Dissonance refers to a combination of sounds that are not perceived by the listener as being at rest. Dissonance has also been referred to as a sound that is disagreeable.

Equal temperament is an intonation system based on successive powers of the twelfth root of two. This irrational quantity may be expressed as $\sqrt[12]{2}$.

Extended Reference is a term used by Boomsliter and Creel in 1961 to describe a system of intonation that identifies a tuning root for each harmony, which may or may not be the chord root.

Frequency is defined as the number of vibrations per second of a tone. It is commonly expressed in cycles-per-second, abbreviated cps. The acoustic term for frequency is hertz.

Harmony is defined as a combination of sounds presented simultaneously. Tonal harmony incorporates one key at a time within its structure. A polytonal harmony incorporates more than one key in its structure. In terms of intonation, a polytonal harmony will have a tuning root for each key present, which may be identifiable as the chord root or key tonic, but is not limited to it. An atonal harmony has no discernible key within its structure. While a chord root may not be identifiable in an atonal structure, a tuning root is possible with the intended outcome of creating optimum frequency coincidence within the harmony structure.

Just intonation is based on ratios of simple whole numbers found in the natural overtone series.

Key Tonic refers to a tone on which the scale is based. For intonation systems other than extended reference, all intervals have a direct mathematical reference to the key tonic.

Meantone Temperament is also referred to as *one-quarter meantone*. It is based on pure major thirds (5:4). The fifth in mean-tone intonation is not pure because each fifth is tempered by one-fourth of the syntonic comma.

Pitchbend refers to the ability of some MIDI keyboards to bend the pitch of notes above or below a given pitch, commonly in relation to equal temperament.

Pitchwheel refers to the capability of some computer notation programs to adjust the pitch of individuals notes above or below a relative pitch, which is commonly in relation to equal temperament.

Pure Harmony is comprised of pure intervals.

Pure Interval refers to an interval prescribed by a pure ratio.

Pure Ratio is one prescribed by simple, whole numbers such as found in the natural overtone series.

Pythagorean intonation is based on a cycle of pure fifths. The concept of Pythagorean intonation is derived from the projection of twelve intervals of a fifth. The enharmonic note derived from this projection is not a mathematically correct frequency multiple of the primary tone.

Ratio refers to the mathematical relationship that determines the size of an interval. It is a proportional quantity expressed in a common comparative format. For example, the ratio associated with a perfect fifth is expressed as 3:2. The higher tone of the perfect fifth has three units of frequency and the lower tone has two units. Other common ratios are 2:1 (octave), 4:3 (perfect fourth), 5:4 (major third), 6:5 (minor third), and 9:8 (whole tone).

Sine Tone is a sound produced without upper harmonics. Only the fundamental frequency is present. This is in contrast to a complex tone in which many harmonics may be present above the fundamental tone. All wind and string instruments create complex tones to some degree. The presence and strength of various harmonics in the composite sounds comprise the unique tone quality of each instrument. Sine tones are also referred to in the literature as sinusoidal or pure tones.

Tonic is the note on which a scale or harmony is based. In a *direct reference* intonation system, all intervals have a direct relationship to the tonic. This is in contrast to *extended* reference in which a flexible tonal center may not be the tonic.

Tuning Root refers to a note on which the harmony is tuned. The tuning root is not necessarily the chord root and allows the largest number of intervals within the harmony to be prescribed by simple whole number ratios.

Statement of the Hypothesis

Small numbers of beats among similar frequencies distort harmony consonance. Elimination of beats will increase the perception of consonance in the sound. The null hypothesis for this study may be expressed as H_u: $f_o = f_e$. The alternate hypothesis may be expressed as H_u: $f_o \neq f_e$.

CHAPTER II

REVIEW OF RELATED LITERATURE

Simple Number Ratios

Discovery of a connection between simple number ratios and consonant intervals is generally attributed to Greek philosopher and mathematician Pythagoras (c. 500 B.C.) In a discussion of the theories of Pythagoras, Isacoff (2001) offered:

> Pythagoras' discovery was that the most "agreeable" harmonies-those whose tones seem to be "in sync" with each other, like marchers lockstepped to the beat of the same drum-are formed by the simplest kind of mathematical relationships. If the vibrations of one tone are twice as fast as the vibrations of another's, for example, the two will blend so smoothly the result will sound almost like a single entity. (p. 34)

The cornerstones of Pythagoras' contribution to consonance theory connect the interval of an octave to the numerical ratio 2:1, the interval of a pure fifth to the ratio 3:2, and the interval of a pure fourth to the ratio 5:4.

Pythagoras believed that a mathematical series of twelve pure fifths would produce the same note as a series of seven octaves. In actuality, a series of twelve fifths is sharp of the series of seven octaves by the distance of the Pythagorean comma, which has the numerical ratio 531441:524228. Weyler and Gannon wrote, "This unwieldy fraction, which we now know as the Pythagorean comma, is about a quarter of a semitone, a little interval with huge implications" (p. 26). The Pythagorean comma is one of three commas identified in the review of literature for this study. The other commas are the *Syntonic comma* with ratio 81:80 and the *Septimal comma* with ratio

64:63.

Archytas (c. 450 B.C.) was a member of a group of Pythagorean followers identified in the literature as the "Harmonists" (Weyler and Gannon, p. 30). The term Harmonists has also been used to describe a modern group of researchers, writers, and composers who advocate use of just intonation for harmony consonance. (Weyler and Gannon, pp. 87 - 98). Archytas is credited with discovering that singers intuitively sing a pure third with a numerical ratio of 5:4 rather than the sharp Pythagorean third with ratio 81:64. Weyler and Gannon described his as follows:

> Archytas himself must have had an extraordinary ear. He literally picked these pure harmonic tones out of the air without any tradition or aid to guide him, and some of his enharmonic tetrachords suggest a keen ear that was able to discriminate among a variety of tiny intervals. (p. 34)

Archytas' discovery was verified in the thirteenth century by William Odington and in the sixteenth century by Gioseff Zarlino. According to Weyler and Gannon, Odington discovered "that singers in the *fauxbourdon* vocal tradition intuitively used the pure ratio intervals and not the Pythagorean intervals" (p. 55). Use of the term fauxbourdon to describe Odington's discovery is confusing due to the fact that the vocal technique associated with that term is generally associated with the fifteenth century Franco-Burgundian tradition. *The Oxford Companion to Music* acknowledges ambiguity between the terms fauxbourdon and faburden, the later used to describe "a type of improvised polyphony, chiefly in parallel motion, in 6-3 chords with 8-5 chords at the beginnings and ends of phrases, popular in England from the 15th century to the Reformation" (p. 439). Despite confusion regarding the vocal tradition Odington utilized for his observations, the fact remains that he observed singers intuitively singing pure thirds.

Weyler and Gannon described Zarlino as a "choir master with a keen ear, and although he investigated all the options, he clearly favored the pure harmonies that the voices intuitively and naturally found" (p. 63). An interesting aspect of this observation is that consonance was initially determined through innate sensitivity of the human ear for a pure major third. It was subsequently determined that this interval is prescribed by a simple number ratio. These types of simple ratio intervals are also referred to as pure intervals.

Helmholtz (1877/1954) observed that intervals prescribed by simple number ratios were more consonant than others:

> The justly-intoned chords, in favorable positions, notwithstanding the rather piercing quality of the tone of the vibrators, possess a full and as it were saturated harmoniousness; they flow on, with a full stream, calm and smooth, without tremor or beat. Equally tempered or Pythagorean chords sound beside them rough, dull, trembling, restless. The difference is so marked that every one, whether he is musically cultivated or not, observes it at once. (p. 319)

Helmholtz's statement is important for three reasons. First, his theory of consonance was based on frequency coincidence of partials above the primary tones. Pure intervals exhibit a high degree of coincidence among upper partials. Secondly, Helmholtz considered difference tones as integral to harmony consonance. The literature indicated that pure intervals exhibit high capacity for the generation of difference tones. Lastly, the statement reinforced the concept that the human auditory system possesses innate sensitivity for pure intervals.

Hindemith (1942) acknowledged innate sensitivity for pure intervals:

> The ear...is the one sense organ that is unerring in its sense of measurement and proportion. The eye is like a mirror that reports faithfully and disinterestedly on what is before it. But the ear is like a fabulous sieve, that not only sorts what it receives into large and small, but measures it exactly. It hears simple ratios as beautiful and correct sounds, and it recognizes perfectly that the purity of the octave, the fifth, or the fourth is clouded when the proportions of length or vibration frequency are not in the ratios of 1:2, 2:3, or 3:4. (p. 23)

This concept was supported by Révész (1954) who wrote, "The intonation does not follow from mathematico-physical speculations--that is, it is not based on any numerical calculation or on physical computations--but proceeds from the musical ear, which seems innately inclined to the intonation of pure intervals" (p. 22).

Terhardt (1973) echoed Hindemith and Revesz's description of innate sensitivity for pure intervals:

> The kind of music which is called tonal appears to prove that the human auditory system possesses a sense for certain special frequency intervals of tones. These particular intervals usually are called musical or harmonic intervals. They are described by frequency ratios of small integers as 1:2 (octave), 2:3 (fifth), 3:4 (fourth). (p. 1061)

Terhardt's statement is important in that it describes sensitivity for pure intervals in regards to traditional tonal music, or what is referred to in the literature as common practice. The statement does not describe the use of pure intervals in non-traditional, polytonal, or atonal music.

Averitt (1973) added to the discussion:

> Hence, to be in tune would mean to play intervals with ratios corresponding to interval ratios found in the harmonic series and in particular the following ratios: octave 2:1, perfect fifth 3:2, perfect fourth 4:3, major third 5:4, minor third 6:5, twelfth 3:1, major tenth 5:2, major sixth 5:3, and minor sixth 8:5. (p. 3)

Averitt's statement is interesting for two reasons. First, it includes more intervals in the consonant category than other writers on the subject. Secondly, inclusion of the twelfth and major tenth intervals in this group exhibits a strict adherence to ratios found in the natural overtone series.

Lloyd (1943) believed that other factors, in addition to pure intervals, contributed to the consonance perception of intervals:

> As a measuring instrument the ear has its natural limits of accuracy, just like the various means for measuring a penny. The accuracy of the ear depends on circumstances, such as the time allowed for making the measurement or the nature of the interval to be measured. (as cited in Averitt, 1973)

His belief that the time allowed for measuring an interval helped determine the accuracy of the ear was supported by Boomsliter and Creel (1961).

Lloyd's statement pointed to an additional issue of debate evident in the literature. Some writers believed that prolonged exposure to equal temperament had negatively impacted perception for pure intervals. On this subject, Yasser (1932) offered:

> The human ear is such, however, --at least in its present state of development--

> that it regards as "false" every intonation that is different from the one to which it is accustomed. It is a well-known fact that even the just intonation of the diatonic scale sounds partly "false" to an ear long accustomed to the tempered intonation and that special conditions are necessary for the ear to regain its natural ability to recognize the purity of just intonation and the acoustic inaccuracy of tempered intonation. (pp. 166-168)

As stated in the Introduction to the current study, the juxtaposition of just intonation and equal temperament in daily rehearsals and performance creates the potential for beats. Stoddard (1993) wrote:

> With the exception of the octave, not one interval [in equal temperament] is in tune. Hang on a minute, I hear you say, when I play chords on my keyboard they sound fine to me. That's because your ears have become so accustomed to these intervals that you don't notice the errors. (p. 1)

This phenomenon was discussed early by Rameau (1722) and in the twentieth century by Norden (1936) who complained that our ears had been "dulled by temperament" (p. 219)

Barbour (1953) disputed theories regarding perception of pure intervals. He wrote, "Scientific studies of intonation preferences show that the human ear has no predilection for just intervals, not even the pure major third" (p. 197). In contradiction to Barbour, Cazden (1972) wrote:

> The belief that the response of consonance is entirely due to an arbitrary choice or judgment of the "ear" cannot be sustained, since it turns out that those intervals whose special quality of agreement is noted by the "ear" are precisely

> those which can be expressed in terms of low integer values" (p. 205).

It was evident from a review of the literature that proponents of pure intervals for harmony intonation outnumber opponents.

Boomsliter and Creel

In 1963, a study was designed and conducted to collect data on the melodic intonation preferences of trained musicians. In that year, the *Journal of Music Theory* reported the results of a study in which Paul C. Boomsliter and Warren Creel presented theories regarding an intonation system they called extended reference. In that study, the term extended reference was used in contrast to direct reference:

> Modern hypotheses about musical scales drawn from the partial series have supported the same tendency to look for direct reference to the tonic. Consequently, investigators have tended to interpret variety in musical pitch as error in performance, or personal emotionalism, or some other type of unstable variation from direct reference, the assumed normal supplied by the formula. (p. 14)

Boomsliter and Creel developed the theory of extended reference through a study that utilized equipment designed "to identify the notes chosen by 'the ear of the musician' in playing melodies" and through the observation "that musicians, even in standard melodies, consistently use many notes that are 'unbelievably off pitch' if measured by the yardstick of the conventional scale" (p. 4). The investigation discovered that melodic tuning preferences chosen by study subjects did not conform to a conventional system of intonation, requiring the development of extended reference theory as a means of explaining the data.

In the study, trained musicians were placed at a specially constructed keyboard called a Search Organ. Construction of this keyboard allowed the musicians to select a specific tuning that they determined sounded best for the individual tones of a specified melody. The researchers then compiled data regarding the tuning selected for each note of the melody. Boomsliter and Creel concluded that the problem was not with the musician, but rather the manner in which tuning is measured.

A major component of extended reference theory explains that the human auditory system seeks out the simplest ratio prescribing an interval. The process of this study discovered that the simplest ratio is not necessarily related to the tonic. This would explain why some tones were produced that were off pitch when measured by a conventional intonation system:

> For example, in certain melodies subjects produce a sharp sixth, which we call Lay, rejecting the La of just intonation. Lay is 27/16 to Do. In the key of C it is A=445.5. Lay can be understood as produced by auditory organization in simple ratios. It stands a pure fifth, 3/2, above Re. If Re has become a temporary tonal center in the organization of the melody, and the ear is tuning in reference to Re, then the ear will call for the simple manageable 3/2 relationship to Re, producing Lay. (p. 10)

This relationship lead to the second major component of extended reference theory. This component is explained by the authors when they wrote, "The experimental results on melody patterns suggest that simple ratios also operate in melodic combination, with the modification that the neural system is capable of using simple ratios in chains, or linkages, as well as in direct relationship" (p. 10). To illustrate this concept, the authors offer more detail on the tone Lay:

> The complex ratio to Do, 27/16, is a mathematical convenience in a tuning formula, but musically irrelevant, because the note is not acting musically with Do. It acts as 3/2 over Re, and should be thought of as 3/2 over Re. La, A=440, is 5/3 to Do, but 40/27 to Re. (p. 10)

The last major component of extended reference theory is explained by the authors, "A melody typically uses direct reference at the start to establish the tonic, then goes into extended reference and stays there until the extended organization is resolved at the end, which normally is on the tonic" (p. 14).

Overtone Series

Discovery of the simple ratios of the overtone series is generally attributed to Rene Descartes in the middle of the 17th century, although his work detailing this discovery was not published until after his death. Also in the 17th century, Marin Mersenne connected the ratios of the overtone series with a major triad and a dominant seventh harmony. Joseph Sauveur published some of the first experimental evidence of the overtone series in the early 18th century.

Discovery of the overtone series occurred at a critical time in the history of tuning and temperament. Weyler and Gannon (1997) wrote, "And here, at the dawn of the 18th century, our historical irony is in full flower as precise equal temperament and the laws of harmonics were being simultaneously discovered and understood at exactly the same time" (p. 68). This unique time in music history created an issue for consonance debate that continues in present day conversations on tuning and temperament.

Difference Tones

The discovery of difference tones is generally attributed to eighteenth-century Italian composer and music theorist Guiseppe Tartini, with subsequent investigation by Sorge and Helmholtz. Leopold Mozart (1948/1988) wrote of using difference tones for proper intonation in violin performance:

> For if two notes, as I will indicate below, be so to speak drawn well and right out of the violin, one will be able at the same moment to hear a lower voice quite clearly, but as a muffled and droning sound. If on the contrary the notes be played out of tune, and one or the other be stopped even in the slightest degree too high or too low, then will be lower voice be false. (p. 164)

Mozart's statement establishes two conditions for the generation of difference tones. First, the primary tones must be played with sufficient volume. Secondly, the tones must comprise a pure interval. A third condition for the generation of difference tones is established by Helmholtz when he wrote, "They are most easily heard when the two generating tones are less than an octave apart, because in that case the differential is deeper than either of the two generating tones" (p. 153).

Leuba (1962) investigated the mathematical alignment of harmony comprised of pure intervals and the impact of resultant [difference] tones. He wrote, "It is the contention of the writer that unless resultant tones coincide exactly with the others present or implied in the harmonic structure of the music, the resultants will produce 'beats' with the other tones being played, and hence discord" (p. 4). Leuba's investigation primarily addressed difference tones generated by these types of harmonies and not the total matrix of primary tones, partials, and difference tones. His discussion presented information as a means of calculating difference tone frequencies generated by the triad. Figure 1 details the generation of difference tones by a major triad built on a C having a frequency of 264 cycles per second.

	Primary Tone	2	3	4	5	6	7	8
		Partials						
Note Name	C	C	G	C	E	G	Bb	C
Frequency (f_1)	264	528	792	1056	1320	1584	1848	2112
Difference Tones	*C*	*C*	*G*	*C*	*E*	*G*	*Bb*	*C*
$f_2 - f_1$	*66*	*132*	*198*	*264*	*330*	*396*	*462*	*528*
	Primary Tone							
Note Name	E	E	B	E	G#	B	D	E
Frequency (f_2)	330	660	990	1320	1650	1980	2310	2640
Difference Tones	*C*	*C*	*G*	*C*	*E*	*G*	*Bb*	*C*
$f_3 - f_2$	*66*	*132*	*198*	*264*	*330*	*396*	*462*	*528*
	Primary Tone							
Note Name	G	G	D	G	B	D	F	G
Frequency (f_3)	396	792	118 8	1584	1980	2376	2772	3168
Difference Tones	*C*	*C*	*G*	*C*	*E*	*G*	*Bb*	*C*
$f_3 - f_1$	*132*	*264*	*396*	*528*	*660*	*792*	*924*	*1056*

Figure 1. Fundamentals, Overtones, and Simple Difference Tones for a C Major Triad

Coincidence Theory

Whitcomb (1999) traced the origin of coincidence theory back to the writings of Galileo, Mersenne, and Descartes. Whitcomb (2005) attributed the first use of the term coincidence theory to H. F. Cohen in 1984. He wrote, "The degree of consonance of a group of sounds is determined by, and is proportional to, the rate at which the wave patterns of those sounds coincide" (p. 70).

The coincidence theory of consonance is complicated considering the number and variety of instruments in a wind ensemble, the complex tones produced by those instruments, and the number and variety of tones produced by these types of ensembles during rehearsals and performances. The possibility of beating exists between all of the primary frequencies, partials, and difference tones. This potential for beating delineated a need to utilize complex tones as auditory stimuli in an investigation of auditory perception of harmony intonation consonance.

Figure 1 further details the entire matrix of frequencies associated with a C major triad. A frequency of 264 cps was selected for the root of the triad due to the fact that a just diatonic scale built from this C allows an A of 440 cps. A pure major third above this C was calculated (264 x 5/4 = 330 cps) and a pure fifth was also calculated (264 x 3/2 = 396 cps). These primary tones are detailed in the first box of each row. Each primary tone is a fundamental and the frequency of seven overtones was calculated above the fundamental. The note name associated with that frequency is given. The frequency of simple difference tones between each primary tone was calculated using the formula: $f_1 - f_2$ = difference tone. This is not intended to prove that the difference tone and overtone frequencies are audible, rather to simply illustrate additional areas of mathematical alignment in the overall matrix.

The areas of frequency coincidence within this matrix are evident by examining similar note names. One interesting aspect of this frequency matrix is revealed by examining areas where frequency coincidence appears to be a problem. For example, partial six of the overtone series on C would logically seem to create a conflict with the fifth component of the series on E. Specifically, this is a G of 1584 cps against a G# of 1650 cps. While this conflict seems obvious, the fact that the frequency difference between the two is 66 cps, which is a C two octaves below the C at 264 cps, tends to reinforce the mathematical framework of the entire matrix.

To identify the frequencies that cause beats in intervals other than the unison, a theory of Hermann von Helmholtz (1877/1954) is utilized. Helmholtz believed that beats were created by the non-coincidental frequencies of upper partials. Plomp (1967) offered a simple illustration of this phenomenon utilizing a mis-tuned fifth with frequency ratio 301:200 cps. The second harmonic of the higher tone is 602 cps. The third harmonic of the lower tone is 600 cps. The beating frequency between these partials is 2 cps (p. 462).

Previous Studies

Johnson (1962) used "vocally trained persons who had ensemble singing experience" in his study (p. 6). In contrast to Boomsliter and Creel, Johnson did not attempt to map the intonation system study subjects utilized in performance situations. He selected a common chord progression (I-V-I) as the harmony example for his study. A traditional chord structure was used, meaning that the root was doubled and present in the bass voice, and each chord had a third and a fifth. Johnson varied the position of the chords in order to determine if any differences of preference would be caused by this variation. He used four different chord positions, two open and two closed, and utilized two different keys, the tonic and dominant.

Johnson used Pythagorean intonation, just intonation, and equal temperament in the study. He tested the preference of study subjects by playing the chord progressions in pairs. These pairings were explained as follows:

> The test items in Experiment I consisted of two playings [sic] of a chord progression, once in equal temperament and once in just intonation. Experiments II and III were similarly constructed, using Pythagorean tuning and just intonation, and equal temperament and Pythagorean tuning, respectively. (p. 17)

Johnson was also concerned about the concentration requirements placed on study subjects. He wrote:

> In order to keep to a minimum the problems of memory time span, the progressions had to be as short as possible. The chords of each progression had to be of sufficient duration that the listener could hear and absorb the timbre of their tuning, but the progression had to be brief enough that the timbre of the first progression of each pair was not forgotten while the second progression of the pair was being played. A three-chord progression with each chord having a duration of one second was found to be the optimum time span. (pp. 17-18)

Johnson separated the two tuning versions of the chord progression by a one second delay. The test items were separated by a three second silence during which the subject was asked to indicate a preference for one of the progressions. The subject entered a 1or 2 in the appropriate place on the answer sheet. Each test item was played a total of twenty times, ten times in one order of tuning systems and ten times in the reverse order. This combination of tuning versions and repetitions of the test items were spread out over three experiments. This resulted in 480 test items that were then "mixed in random order and recorded on four tapes, each twenty minutes in length" (p. 22).

The data in Johnson's study indicated that the subjects preferred equal temperament over just intonation and Pythagorean intonation over just intonation by decisive margins. In only one instance, on one chord progression, in comparison of just intonation to Pythagorean intonation, did study subjects express a preference for just intonation.

Bisel (1987) investigated a perceptual preference among Pythagorean intonation, just intonation, one-quarter meantone intonation, and equal temperament. The tonal music examples in Bisel's study were in the form of chorale harmonizations and unharmonized melodic material. In contrast to more contemporary compositions in which tonal relationships may be more nebulous, Bisel writes, "In types of music which do not have a single most stable pitch, slight deviations in pitch may not be as noticeable as they are in tonal music" (p. 7).

Bisel went to great lengths to document that intonation preference does not indicate a most suitable system in every situation. He wrote:

> The more obvious point is that there is no consensus on the superiority of any single system of tuning or temperament. In addition to the fact that different theorists disagree on which system is best, various theorists advocate two separate systems, or have held different viewpoints at various times in their lives. (p. 55)

Along this same line, Loosen (1995) investigated the effect of subjective musical preference on the perception of intonation. The investigation examined subjects with performance experience on the violin, piano, and a group of individuals with no performance experience. Loosen utilized a scale pattern tuned to Pythagorean intonation, just intonation, and equal temperament. He found that violin performers preferred Pythagorean intonation and attributed this finding to the fact that violin strings are tuned in perfect fifth intervals. He also found that piano players preferred equal temperament and attributed this to the fact that modern keyboard instruments are commonly tuned to this system. The group of individuals with no performance experience did not express a preference for any intonation system which clearly indicated that performance experience is an important determining factor in intonation preference.

The introduction to the current study does not offer a clear explanation for the presence of beats in intervals other than unisons that comprise the harmony produced by wind instrument ensembles. That explanation can be found in the work of Herman von Helmholtz (1877/1954) whose theory of consonance is based on frequency coincidence between partials of the primary interval tones and difference tones:

> Collecting the results of our investigations upon beats, we find that when two or more simple tones are sounded at the same time, they cannot go on sounding without mutual disturbance, unless they form with each other certain perfectly definite intervals. Such an undisturbed flow of simultaneous tones is called a *consonance*. When these intervals do not exist, beats arise, that is, the whole compound tones, or individual partial and combination tones contained in them

> or resulting from them, alternately reinforce and infeeble each other. (p. 204)

It is important to note that Helmholtz referred to simple tones, which are tones in which only one frequency is present. Hall and Kent (1957) referred to these types of sounds as pure tones, sinusoids, or sinusoidal waves (p. 4). The problem is that wind instruments generate complex tones that are often rich in partials above the fundamental frequency.

Plomp (1967) hinted at a solution to this dilemma:

> This phenomenon is incompatible with Ohm's acoustical law, which says, as formulated by von Helmholtz (1863), that the human ear is able to analyze a complex of tones into its sinusoidal components. Such an analysis implies that the two tones are perceived individually but fails to explain why beats are heard. (p. 462)

This concept was supported by Terhardt (1974) when he suggested the consideration of a complex tone as an array of sinus tones that represent the fundamental and subsequent partials (p. 1062). This solution facilitates objective examination for frequency coincidence among harmony components including primary tones, partials, and difference tones.

Helmholtz's theory establishes three concepts that are important to the current study. First, a logical explanation is presented as to why some harmonies contain beats and sound distorted when produced by a wind instrument ensemble. Second, as stated earlier in this introduction, any harmony containing minimal or zero beats should be perceived as more consonant than a harmony with beats. Last, Helmholtz's theory points to a need to utilize complex tones as stimuli in an investigation of harmony intonation consonance produced by a wind instrument ensemble.

Aural perception of harmony consonance has been a subject of debate for centuries. One long-standing theory maintains that harmony constructed of intervals prescribed by simple whole number ratios is aurally perceived as more consonant than harmony constructed of intervals prescribed by large or irrational number ratios. Cazden (1972) identified this concept as the "natural law theory of consonance" and wrote that "the expression of this natural law consists in the determination of musical consonance by relations which may be briefly stated in the form of simple number ratios" (p. 98). Terhardt (1973) argued that frequency distance, rather than simple ratio, is the determinant factor in consonant quality of an interval (p. 1061).

Seashore (1938/1967) wrote that "Consonance depends fundamentally on the degree of coincidence of sound waves" (p. 126). Helmholtz's theory of consonance was based on frequency coincidence of partials of the primary tones. Harmonies comprised of pure intervals exhibit a high degree of coincidence among components and should produce a low, if any, beat frequency.

Just Intonation

The system of just intonation utilizes pure intervals derived from the natural overtone series. The perception of intervals tuned to this system was discussed by Seashore (1938/1967), Hindemith (1945), Leuba (1962), Wilkinson (1988), and Monzo (1999). These discussions offered support to the concept that intervals tuned to simple ratios exhibit a high degree of coincidence among partials and other harmonic components and create minimal discordant beats. This degree of coincidence can be objectively examined through a graphic that details the frequencies of primary tones and partials. Figure 2 details frequency coincidence among primary tones and seven overtones for a C major triad tuned to intervals of just intonation. A frequency of 264 cps was selected for the root C since this would allow an A of 440 cps if a diatonic scale were based on this note.

1	2	3	4	5	6	7	8
264	528	792	1056	1320	1584	1848	2112
C	C	G	C	E	G	Bb	C
330	660	990	1320	1650	1980	2310	2640
E	E	B	E	G#	B	D	E
396	792	1188	1584	1980	2376	2772	3168
G	G	D	G	B	D	F	G

Figure 2. Fundamental frequencies and overtones for a C major triad tuned in just intonation

Equal Temperament

In contrast, the numerical ratios of equal temperament are not simple. Gannon, Weyler, and Coulombe (1997) described equal temperament as a compromise tuning system created to facilitate a simple piano keyboard and modulation between all keys. The numerical ratios of this system are determined by calculating the twelfth root of the number two, which represents the octave. The formula used to calculate the intervals of equal temperament may be expressed as $\sqrt[12]{2}^{\,n}$. This divides the octave into twelve equal intervals.

The literature does not dispute the idea that equal temperament opened up additional harmony possibilities. Equal temperament was primarily a theoretical concept prior to the invention of devices that allow precise measurement of the complicated ratios of that system. Until that time, tuning to equal temperament was accomplished by the human ear using beats as the determining factor. This is an interesting issue in that electronic tuners of the twentieth and twenty-first centuries subvert innate sensitivity for pure intervals in order to tune to equal temperament.

The juxtaposition of equal temperament and just intonation in music rehearsals and performance practice is a cause for concern among conductors of wind instrument ensembles. A portion of Helmholtz's earlier statement is recalled, "Justly-intoned chords, possess a full and saturated harmoniousness; they flow on, with a full stream, calm and smooth, without tremor or beat. Equally-tempered or Pythagorean chords sound beside them rough, dull, trembling, restless" (p. 319). In the same manner as Terhardt and Denckla, Helmholtz's reference to a rough quality describes a broader perception of consonance or dissonance and not just the distortion caused by frequencies that do not vibrate synchronously. His theory can be used to explain why harmony tuned to just intonation sounds different, if not perceptually better, to the human auditory system than harmony tuned to equal temperament.

Two reasons are evident as to why Helmholtz advocated the use of justly intoned chords, which is interpreted as descriptive of chords constructed using pure intervals. The first has already been discussed, that being frequency coincidence among upper partials of the primary tones. Helmholtz also believed that harmony comprised of pure intervals is more consonant because the frequencies of difference tones exhibit enhanced alignment and contribute to the overall coincidence.

By contrast, intervals tuned to equal temperament exhibit a high degree of non-coincidence among partials and additional harmonic components and have the potential to create many discordant beats. This degree of non-coincidence can be objectively examined through a graphic that details the frequencies of primary tones, partials, and additional harmonic components. Figure 3 details frequencies of primary tones and seven overtones for a C major triad tuned to intervals of equal temperament. The C with 264 cps was utilized for these calculations.

1	2	3	4	5	6	7	8
264 C	528 C	792 G	1056 C	1320 E	1584 G	1848 Bb	2112 C
332.62 E	665.24 E	997.86 B	1330.48 E	1663.10 G#	1995.72 B	2328.34 D	2660.96 E
385.55 G	771.10 G	1156.65 D	1542.20 G	1927.75 B	2313.3 D	2698.85 F	3084.40 G

Figure 3. Fundamental frequencies and overtones for a C major triad tuned in equal temperament

It can be illustrated that the intervals of equal temperament contain beats. For example, the G at 792 cps and the G at 771.10 cps are separated by 20.9 cps. There are also Ds at 2328.34 cps and 2313.30 cps, a difference of 15.04 cps. These differences are within the beating region that Helmholtz said would be objectionable. These differences also support the widely accepted tuning rule-of-thumb that requires fifths be raised in pitch by two cents and major thirds lowered by almost fourteen cents in order to achieve the most consonant harmony sound. (Fabrizo, 1994, p. 23)

Pythagorean Intonation

The Pythagorean scale is developed by projecting a series of pure fifths and then reducing the tones the number of octaves necessary to construct a one octave scale. This results in pure fifths, major thirds and seconds that are large, and minor seconds and thirds that are small. Johnson (1963) discovered specific musical examples in which Pythagorean intonation was preferred, particularly over just intonation. Figure 4 details frequencies of primary tones and seven overtones for a C major triad tuned to intervals of Pythagorean intonation.

1	2	3	4	5	6	7	8
264 C	528 C	792 G	1056 C	1320 E	1584 G	1848 Bb	2112 C
334.12 E	668.24 E	1002.36 B	1336.48 E	1670.6 G#	2004.72 B	2338.84 D	2672.96 E
396 G	792 G	1188 D	1584 G	1980 B	2376 D	2772 F	3168 G

Figure 4. Fundamental frequencies and overtones for a C major triad tuned in Pythagorean intonation

Context

Lloyd (1943) recognized that consonance judgment depends on the conditions, or context, in which intonation is perceived by the ear:

> As a measuring instrument the ear has its natural limits of accuracy, just like the various means for measuring a penny. The accuracy of the ear depends on circumstances, such as the time allowed for making the measurement or the

nature of the interval to be measured. (p. 366)

Context should include the harmonies, orchestrations, and compositional devices used in creating music. Jourdain (1997) wrote, "Every chord swims in an undulating sea of harmonic context. There is no considering the effects of a chord, or of a change of chord, apart from what has preceded it" (p. 104).

In this same area, Gann (1997) wrote, "Because it determines what sounds good, tuning has a pervasive influence on compositional tendencies. Every piece of pitched music is the expression of a tuning" (¶ 19). This statement delineates the importance of musical context in an investigation of harmonic intonation consonance. An accurate representation of musical context should approximate the conditions under which conductors encounter intonation problems in daily rehearsals and performances.

CHAPTER III

RESEARCH METHODOLOGY, PROCEDURES, AND TREATMENT OF DATA

The current study proposed that an investigation of the acceptability of intonation can be accomplished through a simple perceptual choice that addresses perception for harmonic intonation. Roberts and Mathews (1984) proposed a simple method by which a judgment of acceptability could be solicited from listeners when they wrote, "The operational test for intonation sensitivity is some form of judgment test in which listeners say, for example, which chord of a pair they prefer" (p. 952). Data collected in this manner was analyzed by chi square procedures.

The current study was divided into six phases. While the collection of data occurred in the fifth phase of the study, each preceding phase had the goal of facilitating the collection process among study subjects. The excerpts for this study were selected from the *MLR Instrumental Score Reading Series* published by G.I.A.Publications. Appendix A of this study details correspondence from G.I.A. Publications granting permission to use excerpts from this anthology in the study. In instances where the copyright was owned by someone other that G.I.A. Publications, permission was sought from that entity as well.

Three excerpts were selected based on information gathered from pre-study trials. The study was limited to three excerpts to fit within a 30 minute timeframe established in the scope and delimitations guidelines for the investigation. The first excerpt is for a brass quintet consisting of two trumpets, horn in F, and two trombones. The second is a duet for flute and bassoon. The third excerpt is a brass quintet for two trumpets, horn in F, trombone, and tuba.

The excerpts were converted to MUS and MIDI files using Finale 2008 software. The computer used in this process was a Compaq Presario C502US Notebook PC manufactured by Hewlett-Packard. The processor is an Intel® Celeron® M chip that operated at 1.86 gigahertz. The computer had 2038 mb of RAM and had Windows Vista Basic software installed. The installed sound device was a Conexant High Definition Audio card.

Justonic Pitch Palette 2.0 and Roland Virtual Sound Canvas 3.2 software were installed. Pitch Palette allowed the playback of MIDI files in various intonation systems including equal temperament, Pythagorean intonation, and just intonation. Pitch Palette added micro-tuning capability to the sound card installed in the computer. Virtual Sound Canvas enhanced the computer sound card and made it easier to prepare the audio compact disc for data collection

The final version of the compact disc utilized for data collection was created on a Superscope PSD340 compact disc recorder. The Compaq computer was connected directly to the auxiliary analog input on the recorder using an RCA stereo patch cord. This input has a signal-to-noise ratio of 85 decibels, total harmonic distortion of 0.01%, and an input sensitivity of 500mV/23K.

There were three tuning versions of each excerpt, these being equal temperament,

Pythagorean intonation, and just intonation. The tuning versions provided six pairings of the excerpts. The interval ratios used in the tunings are detailed in Figure 5. A comparison of the size of semitones in cents in contained in Figure 6. The pairs of excerpts were randomly mixed before being recorded onto the audio compact disc.

Just Intonation

Do		Re		Mi	Fa		Sol		La		Ti
	16/15		6/5			7/5		8/5		7/4	
1/1		9/8		5/4	4/3		3/2		5/3		15/8

Pythagorean Intonation

Do		Re		Mi	Fa		Sol		La		Ti
	256/243		32/27			729/512		128/81		16/9	
1/1		9/8		81/64	4/3		3/2		27/16		243/128

Equal Temperament (due to the irrational quantities created by multiples of $\sqrt[12]{2}$ these
numbers have been converted to their decimal equivalents in the following table.)

Do		Re		Mi	Fa		Sol		La		Ti
1	1.06		1.19			1.41		1.59		1.78	
1		1.22		1.26	1.33		1.50		1.68		1.89

Figure 5. Tables detailing ratios used for intonation systems in this investigation

Distance in semitones	Equal Temperament	Just Intonation	Pythagorean Intonation
1 Semitone	100	111.8	90.3
2 Whole tone	200	203.91	203.91
3 Minor third	300	315.64	294.14
4 Major third	400	386.31	407.82
5 Fourth	500	498.05	498.05
6 Tritone	600	582.51	611.73
7 Fifth	700	701.96	701.96
8 Minor sixth	800	813.69	792.18
9 Major sixth	900	884.36	905.87
10 Minor seventh	1000	968.83	996.09
11 Major seventh	1100	1088.27	1109.78
12 Octave	1200	1200	1200

Figure 6. Comparison of semitones in cents

Several settings were made to the Pitch Palette software prior to playback and recording of the excerpts. The MIDI output in the setup menu was set to *Roland VSC*. The output settings were set to *ALL Synthesizers Pitch Bend*. The key of each excerpt was set on the controls of the MicroTuner. In regards to the Tcherepnin quintet, the key was set to C since this was the first tonal center discerned in the music. The MicroTuner was set to *Auto root* which allowed the software to select a tuning root for each harmony detected. The *Hangar* was toggled to the *on* position which allowed the notes being played to be re-tuned instantaneously when a tuning message was received.

Pitch Palette software communicates with the computer through System Exclusive messages (SysEx). The tuning resolution of the sound card with the Roland Virtual Sound Canvas software installed was estimated at 1/4000 of a semitone. The author considered this resolution sufficient for purposes of the current study.

First Data Collection

The study group for data collection was comprised of wind and percussion instrument students at the University of Southern Mississippi. A copy of the Human Subjects Review Form is included as Appendix E to this study. More specifically, study subjects were members of the Wind Ensemble at the University of Southern Mississippi.

The Wind Ensemble is comprised of highly skilled undergraduate and graduate wind and percussion instrument performers.

The author traveled to the School of Music at the University of Southern Mississippi on March 26, 2008. The compact disc was played for the study subjects using the Compaq computer and a Phillips portable sound system hooked up to the computer. Study subjects were given verbal instructions and a copy of the data collection document. They were asked to listen to the compact disc and respond on the data collection document. A copy of the document is included as Appendix G to the study. Study subjects completed the document and returned it in the author. This process was completed within the thirty minute time limit established in the scope and delimitations for this investigation.

Nominal data collected in this study conformed to chi square (x^2) analysis. The decision to use chi square was based on the simple choice format used for data collection which yielded nominal data and made it necessary to determine whether or not the preference choices occurred by chance. Chi square is also ideal for this study since no population assumptions were required. The formula for calculation of chi square used in this study is defined by: $X^2 = \sum (f_0 - f_e)^2 / \mathrm{f}_e$.

The author was not satisfied, however, with the quality of the instrumental sounds recorded onto the compact disc. The primary reason the author selected Finale 2008 notation software for use in the study was that Garritan Personal Orchestra (GPO) sounds came as part of the package. However, it was discovered that the Justonic Pitch Palette software is not compatible with GPO. The timbre of the instrumental sounds were

therefore standard MIDI quality and not the enhanced sounds of GPO. This was considered by the author as a detriment to the authentic context sought in the current study.

A possible solution to this dilemma was detailed in an article by Roger Wibberley posted on *Music Theory Online* in February 2004. The article details a procedure for programming alternate tunings using the *Pitchwheel* function of Finale software. The author of the current study considered this a solution to improving the instrument sounds used for the excerpts in the study. The procedures for programming pitchwheel changes in Finale 2008 are slightly different than those detailed in the referenced article. However, the author was able to utilize the information to determine a method of programming alternate tunings of the excerpts utilized in the current study.

The procedure is time intensive, but allows precise pitch control of each note of the excerpt. The pitchwheel function is accessed through the *Expression* tool (*mf*) of Finale 2008. The procedure is to activate the expression tool by clicking on it and then clicking on the note that is to be adjusted.

Finale 2008 has two options available with the expression tool. One option creates an expression for the entire measure and is signified by an outlined arrow when the cursor is placed in a measure or near a note. The other option creates an expression for a single note and is signified by a solid black arrow, with a small note attached in the lower right hand corner, when the cursor is placed on the note that will be adjusted. The adjustment is initiated by placing the cursor on a note and double clicking on the mouse or keypad. The *Expression Selection* menu then appears with all of the expressions that

are available. In the lower left hand corner of this window, the user should ensure that the *Note Expression* box is selected.

The user clicks on *Create* which brings up the *Text Expression Designer* window. The window that appears gives the user an option of creating a label for the pitch wheel adjustment being created. The author of the current study used the numerical value of the pitch wheel adjustment as the label. For example, if the pitch wheel adjustment required a value of minus 320, the label was entered as -320.

The next step is to create the actual pitchwheel setting. The user clicks on the *Playback* tab in the window and then drops down the menu under *Type* and clicks on *Pitchwheel.* A numerical value is entered into the *Set to value* box, that value being the quantity required for pitch bend of the specified note. That quantity ranges from -8192 to +8192 and includes 0 which is the default value for equal temperament.

Each semitone can be divided into 8192 parts either above or below the default pitch in Equal Temperament. This means that each cent of tuning difference has a value of 81.92 on the pitch wheel. The pitch wheel setting is determined by multiplying 81.92 by the difference in cents between the desired pitch and the default pitch in equal temperament. For example, the difference between an equal temperament major third and a just intonation major third is 13.69 cents, with the just intonation third smaller by that amount. To obtain the pitchwheel value for lowering this third, 13.80 is multiplied by 81.92, which results in the quantity 1121.48. Finale does not allow decimals in the pitchwheel settings so the value entered would be rounded to -1121.

The pitchbend function of the MIDI keyboard needed to be set to a numerical value of 1. This setting allows each semitone to be divided into the 8192 parts that Finale pitchwheel settings allow. The procedures for setting keyboard pitchbend functions are contained in the user manual for that keyboard.

The final calculations to be considered are actual pitch wheel values corresponding to the intonation system desired for excerpt playback. As stated earlier, the pitch wheel values are determined by multiplying the interval difference in cents by 81.92. The value in cents of any interval can be obtained by using the formula: Cents = log (i) x (1200/log (2)). In this formula, "i" represents the interval ratio, which is readily pluged into the formula after conversion to a decimal equivalent. Once the pitchwheel value has been obtained, it must be determined whether the pitch wheel adjustment needs to be above or below the equal temperament value. The user enters a negative quantity to lower the pitch or a positive quantity to raise the pitch.

Figures 7 through 12 detail pitch wheel settings that were used in the second data collection effort. Due to the fact that each semitone can be divided into 8192 parts above or below the equal temperament default, a variety of pitch variants can be devised. The settings presented in these tables were calculated using the most common interval ratios of the intonation systems.

Some variations in the pitchwheel settings were allowed on this excerpt for listening purposes. In Figure 10, the pitchwheel setting for F was set to the equal temperament default to avoid a negative reaction by study subjects to the sharpness of that pitch if the calculated setting of +561 was utilized. In Figure 11, the pitch wheel settings were used to follow the harmonic rhythm of the excerpt. Also, the Trumpet in Bb 1 part was adjusted for the entire excerpt. The remaining parts were adjusted to the pitch wheel settings on beat one in measures 1, 3, 9, 11 and all notes in the last measure.

Excerpts in equal temperament required no pitch wheel settings. For this second set of recordings, the Compaq Presario notebook computer was connected to a Yamaha YPG-225 portable grand piano through a USB port. The pitchbend setting on the YPG-225 was set to 1. The MIDI settings in Finale were adjusted to accept the new setup. The MIDI setup menu in Finale 2008 was used to set the YPG-225 as the MIDI in and MIDI out device. The Superscope PSD340 compact disc recorder was connected to the YPG-225 through an RCA stereo patch cord. Several test recordings were made in order to set volume levels and ensure everything was working correctly.

Note Name	Pitchwheel Value
Bb	0
B	-400
C	160
C#/Db	-240
D	320
D#/Eb	-80
E	480
F	80
F#/Gb	655
G	240
G#/Ab	-160
A	400

Figure 7. Pitch Bend Settings for Excerpt One using Pythagorean Intonation

Note Name	Pitchwheel Value
Bb	0
B	481
C	160
C#/Db	641
D	-561
D#/Eb	-80
E	123
F	80
F#/Gb	561
G	-641
G#/Ab	721
A	-481

Figure 8. Pitch Bend Settings for Excerpt One using Just Intonation

Note Name	Pitchwheel Value
A	0
A#/Bb	-400
B	160
C	-240
C#/Db	320
D	-80
D#/Eb	480
E	80
F	655
F#/Gb	240
G	-160
G#/Ab	400

Figure 9. Pitch Bend Settings for Excerpt Two using Pythagorean Intonation

Note Name	Pitchwheel Value
A	0
A#/Bb	481
B	160
C	641
C#/Db	-561
D	-80
D#/Eb	123
E	80
F	561
F#/Gb	-641
G	721
G#/Ab	-481

Figure 10. Pitch Bend Settings for Excerpt Two using Just Intonation

Note Name	Pitchwheel Value
C	0
C#/Db	-400
D	160
D#/Eb	-240
E	320
F	-80
F#/Gb	480
G	80
G#/Ab	655
A	240
A#/Bb	-160
B	400

Figure 11. Pitch Bend Settings for Excerpt Three using Pythagorean Intonation

Note Name	Pitchwheel Value
C	0
C#/Db	481
D	160
D#/Eb	641
E	-561
F	-80
F#/Gb	123
G	80
G#/Ab	561
A	-641
A#/Bb	721
B	-481

Figure 12. Pitch Bend Settings for Excerpt Three using Just Intonation

It was at this point that an additional problem with program incompatibility was uncovered during recording of the compact disc. This problem affected the sound quality of the recorded excerpts. The author desired to use the

Garritan Personal Orchestra (GPO) sounds in the recordings. This was the plan during the first data collection effort as well. In order to use the GPO sounds, the user has to select *Play Finale through VST* from the MIDI/Audio menu in Finale 2008. The user then clicks on *VST Setup* in that same menu which will bring up the *Native Instruments VST Setup* window. Under *VST Instrument*, the user drops down the menu and selects *KontactPlayer2*. The edit menu is then clicked and the *KontactPlayer* window appeared that allowed assignment of GPO sounds to each channel in the Finale file.

GPO sounds were selected for each channel in each excerpt. For example, in the Ascendit Deus excerpt, *Trumpet Plr1* was assigned to channel 1, *Trumpet Plr2* to channel 2, *French horn Plr1* to channel 3, *Trombone Plr1* to channel 4, and *Trombone Plr2* to channel 5. However, when the file containing the pitch wheel changes was played using this setup, the sounds were distorted. Several trials were attempted and each file containing pitch wheel changes produced distorted sounds.

When the MIDI/Audio settings were changed to *Play Finale through MIDI* the pitch wheel settings are realized and the excerpts were recognizable. However, the instrumental timbres in the playback were not the desired GPO sounds but were instead generic midi sounds. These were the sounds that the author found troublesome in the first data collection effort.

A thorough examination of the Finale user manual and online help websites did not provide a solution to the problem. The author contacted Finale technical services by telephone and talked to a technical support representative. Following an explanation of the problem, and several minutes on hold, the representative advised the author to contact Native Instruments for technical support with the GPO sounds. The author contacted Native Instruments though their technical support website. An email describing the problem was sent using the contact form at that website. The response was prompt, but referred all technical questions regarding GPO sounds in Finale 2008 to the Finale technical support services.

With the aforementioned issues unresolved, the author made a decision to proceed with a second data collection effort. Part of this decision was based on an offer by the dissertation committee chairman to allow a second data collection using participants in a summer conducting workshop as study subjects. A second compact disc was prepared that used the MIDI instrument sounds rather than the desired GPO sounds. This was actually considered a strength of the second data collection effort since the sounds that the second study group would hear would be similar to the sounds used for the first study

group. The differences would be the pitchwheel adjustments that had been added to the Finale files and the MIDI sounds of the YPG-225.

Second Data Collection

The author traveled to Hattiesburg on June 16, 2008 for a second data collection effort. The study subjects for this session were participants in a summer conducting workshop at the University of Southern Mississippi. The subjects included music educators and graduate students working on a masters or doctoral degree at the University of Southern Mississippi.

The playback system used in the second data collection effort was a Sony MHC-GX99 Hi-Fi Component System. Following the delivery of instructions, study subjects responded to each excerpt pair in the same manner as the first data collection effort. The entire process was completed within the 30 minute time limit delineated in the scope and limitations established for this investigation. Analysis of the data collected in the second collection effort is detailed in Chapter Four of this study.

CHAPTER IV

ANALYSIS OF DATA

The purpose of this study was to investigate perceived preference for harmony consonance in musical excerpts tuned to equal temperament, just intonation, or Pythagorean intonation. The hypothesis stated that enhanced mathematical coincidence of harmony components would positively influence the expressed preference of study subjects. The null hypothesis stated that there would be no difference in expressed preference among the intonation systems utilized in the study.

Based on the results of pre-study trials, three excerpts were chosen that presented diverse patterns of preference responses and that allowed the collection of data to be accomplished within a 30 minute timeline. The first excerpt was a tonal brass quintet in the key of Bb major. The second excerpt was an imitative duet for flute and bassoon in A minor. The third excerpt was a brass quintet in a very polytonal, homophonic and chromatic style. The arrangement of excerpts on the compact disc is detailed in Figure 6.

The original number of study subjects in the first data collection effort was fifty-one (N=51). Three data collection documents were excluded from the statistical analysis due to the fact that those study subjects did not respond to all excerpt pairs. This provided the study with forty-eight data collection documents for analysis (N=48). Subjects were asked to respond to eighteen pairs of excerpts, choosing the version in each pair that they perceived as the most consonant for harmony. There were 864 (48x18) total responses in this study.

There were 288 (48x6) individual responses to each excerpt. Of the responses to

Excerpt One, 180 (62.5%) expressed a preference for equal temperament, 101 (35.07%)

Description	CD Track	Version A	Version B
	1	equal temperament	Pythagorean intonation
	2	just intonation	Pythagorean intonation
Excerpt #1	3	Pythagorean intonation	equal temperament
	4	just intonation	equal temperament
	5	Pythagorean intonation	just intonation
	6	equal temperament	just intonation
	7	Pythagorean intonation	just intonation
	8	equal temperament	just intonation
Excerpt #2	9	equal temperament	Pythagorean intonation
	10	just intonation	Pythagorean intonation
	11	just intonation	equal temperament
	12	Pythagorean intonation	equal temperament
	13	just intonation	equal temperament
	14	Pythagorean intonation	equal temperament
Excerpt #3	15	Pythagorean intonation	just intonation
	16	just intonation	Pythagorean intonation
	17	equal temperament	Pythagorean intonation
	18	equal temperament	just intonation

Figure 13. Arrangement of excerpts on compact disc

for Pythagorean intonation, and 7 (2.43%) for just intonation. Of the responses to Excerpt Two, 181 (62.85%) expressed a preference for equal temperament, 9 (3.12%) for Pythagorean intonation, and 98 (34.03%) for just intonation. Of the responses to Excerpt 3, 146 (50.7%) expressed a preference for equal temperament, 52 (18.05%) for Pythagorean intonation, and 90 (31.25%) for just intonation.

Description	CD Track	Version A		Version B	
	1	equal temperament	47	Pythagorean	1
	2	just intonation	2	Pythagorean	46
Excerpt #1	3	Pythagorean	10	equal temperament	38
	4	just intonation	1	equal temperament	47
	5	Pythagorean	44	just intonation	4
	6	equal temperament	48	just intonation	0
	7	Pythagorean	5	just intonation	43
	8	equal temperament	45	just intonation	3
Excerpt #2	9	equal temperament	47	Pythagorean	1
	10	just intonation	47	Pythagorean	1
	11	just intonation	5	equal temperament	43
	12	Pythagorean	2	equal temperament	46
	13	just intonation	16	equal temperament	32
	14	Pythagorean	8	equal temperament	40
Excerpt #3	15	Pythagorean	23	just intonation	25
	16	just intonation	40	Pythagorean	8
	17	equal temperament	35	Pythagorean	13
	18	equal temperament	39	just intonation	9

Figure 14. Number of responses for each intonation system (N=48)

Figures 15, 16, and 17 illustrate the excerpts utilized in the study. Permission to use these excerpts was granted by G.I.A. Publications and C. F. Peters Corporation. Documentation for this permission is contained in the appendixes to the current study.

Tables 1 through 18 detail chi square analyses performed on the responses to these excerpts. The chi square values for these calculations were obtained from the appendix in *Basic Statistical Analysis* (p. 452). Separate analyses were performed for each pair of excerpts on the compact disc. There were eighteen tracks on the compact disc.

Excerpt from Ascendit Deus

Gallus
Trans . by Wiskirchen

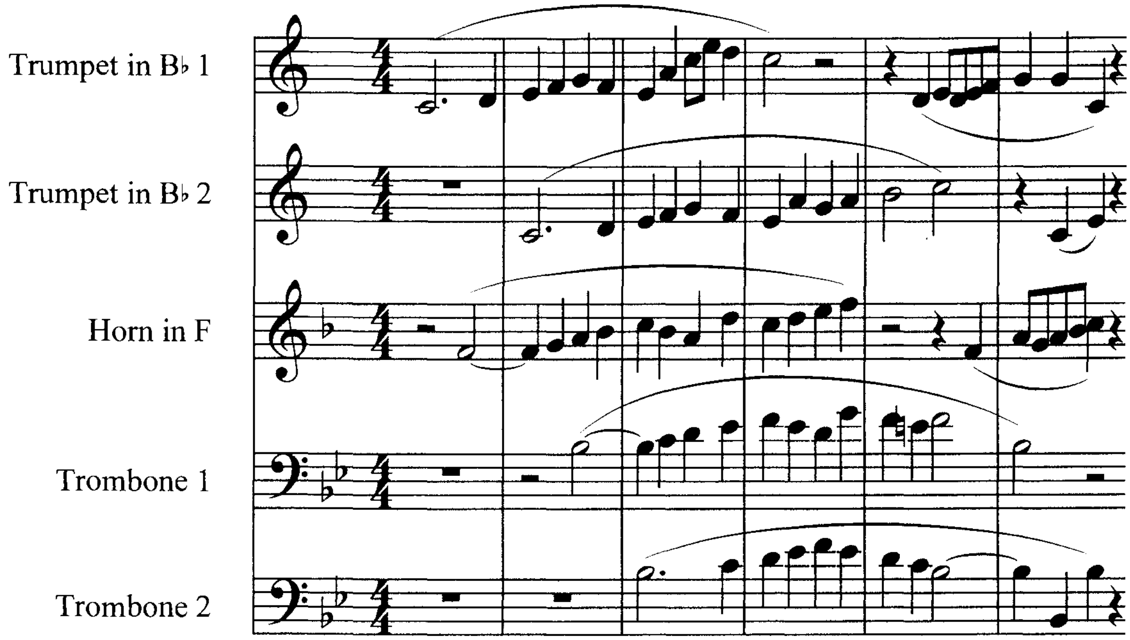

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Figure 15. Excerpt One, from Ascendit Deus

Table 1

Chi square analysis of responses to compact disc track *One*

	Equal Temperament	Pythagorean Intonation
f_o	47	1
f_e	24	24
$f_o - f_e$	23	-23
$(f_o - f_e)^2$	529	529
$(f_o - f_e)^2 / f_e$	22.04	22.04

Null hypothesis	$H_0 : f_o = f_e$	
Calculated chi square value	$\sum (f_0 - f_e)^2 / f_e$	44.08
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Reject H_0: significant at $P < .01$

Table 2

Chi square analysis of responses to compact disc track *Two*

	Just Intonation	Pythagorean Intonation
f_o	2	46
f_e	24	24
$f_o - f_e$	-22	22
$(f_o - f_e)^2$	484	484
$(f_o - f_e)^2 / f_e$	20.17	20.17

Null hypothesis	$H_0 : f_o = f_e$	
Calculated chi square value	$\sum (f_0 - f_e)^2 / f_0$	40.34
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Reject H_o: significant at $P < .01$

Table 3

Chi square analysis of responses to compact disc track *Three*

	Pythagorean Intonation	Equal Temperament
f_o	10	38
f_e	24	24
$f_o - f_e$	-14	14
$(f_o - f_e)^2$	196	196
$(f_o - f_e)^2 / f_e$	8.17	8.17

Null hypothesis	$H_0 : f_o = f_e$	
Calculated chi square value		16.34
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Reject H_0: significant at $P < .01$

Table 4

Chi square analysis of responses to compact disc track *Four*

	Just Intonation	Equal Temperament
f_o	1	47
f_e	24	24
$f_o - f_e$	-23	23
$(f_o - f_e)^2$	529	529
$(f_o - f_e)^2 / f_e$	22.04	22.04

Null hypothesis	$H_o : f_o = f_e$	
Calculated chi square value	$\Sigma (f_0 - f_e)^2 / f_0$	44.08
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Reject H_o: significant at $P < .01$

Table 5

Chi square analysis of responses to compact disc track *Five*

	Pythagorean Intonation	Just Intonation
f_o	44	4
f_e	24	24
$f_o - f_e$	20	-20
$(f_o - f_e)^2$	400	400
$(f_o - f_e)^2 / f_e$	16.67	16.67

Null hypothesis	$H_0 : f_0 = f_e$	
Calculated chi square value	$\sum (f_0 - f_e)^2 / f_0$	33.34
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Reject H_0: significant at $P < .01$

Table 6

Chi square analysis of responses to compact disc track *Six*

	Equal Temperament	Just Intonation
f_o	48	0
f_e	24	24
$f_o - f_e$	24	-24
$(f_o - f_e)^2$	576	576
$(f_o - f_e)^2 / f_e$	24	24

Null hypothesis	$H_o : f_o = f_e$	
Calculated chi square value	$\sum (f_0 - f_e)^2 / f_0$	48
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Reject H_o: significant at $P < .01$

Excerpt from Bouree

Johann Kreiger, 1651-1735
Edited by Fritz Rikko

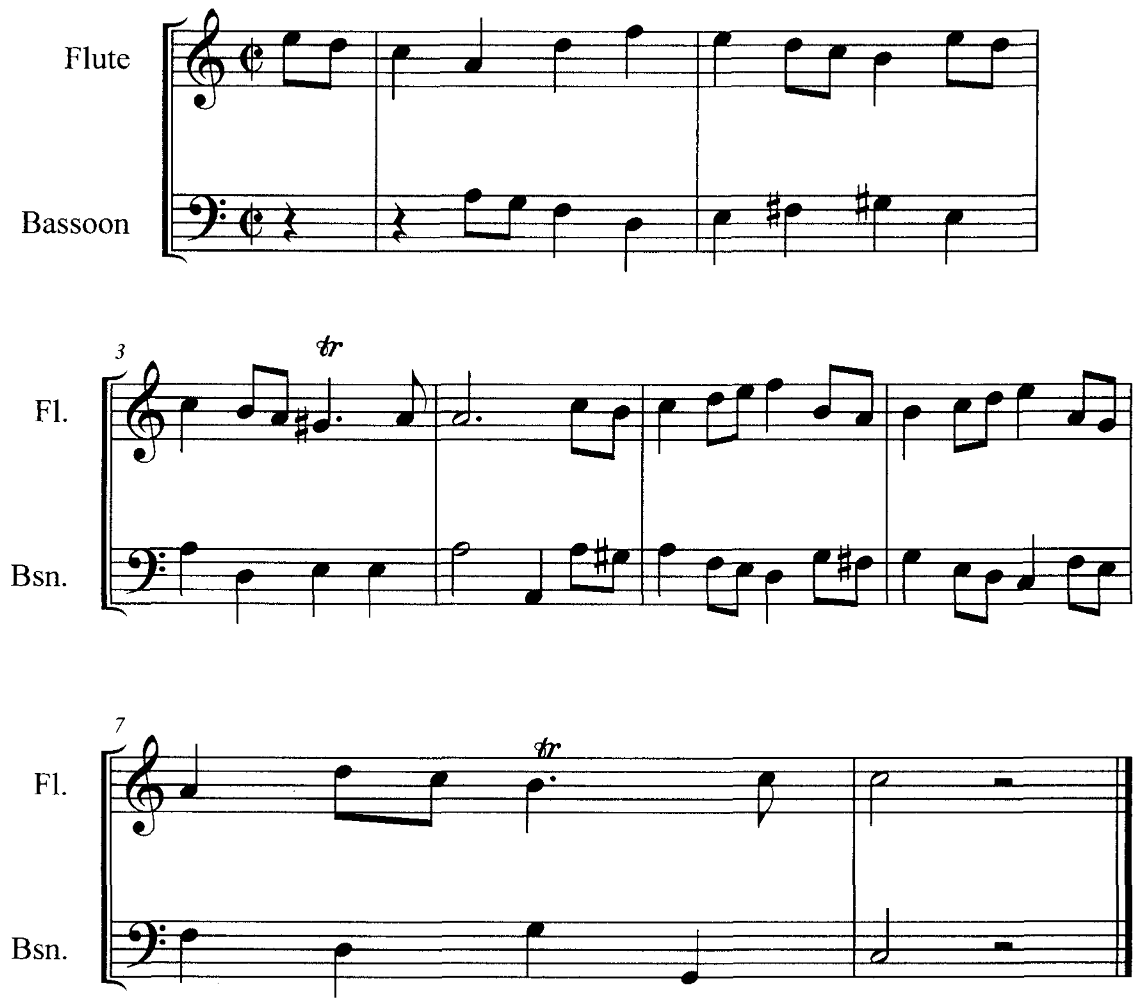

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Figure 16. Excerpt Two, from Bouree

Table 7

Chi square analysis of responses to compact disc track *Seven*

	Pythagorean Intonation	Just Intonation
f_o	5	43
f_e	24	24
$f_o - f_e$	-19	19
$(f_o - f_e)^2$	361	361
$(f_o - f_e)^2 / f_e$	15.04	15.04

Null hypothesis	$H_o : f_o = f_e$	
Calculated chi square value	$\sum (f_0 - f_e)^2 / f_0$	30.08
Degrees of freedom	$df = 1$	
Chi square value from table	$X^2_{.01(1)}$	6.64

Conclusion based on this chi square analysis;

Reject H_o: significant at $P < .01$

Table 8

Chi square analysis of responses to compact disc track *Eight*

	Equal Temperament	Just Intonation
f_o	45	3
f_e	24	24
$f_o - f_e$	21	-21
$(f_o - f_e)^2$	441	441
$(f_o - f_e)^2 / f_e$	18.38	18.38

Null hypothesis	$H_o : f_o = f_e$	
Calculated chi square value		36.76
Degrees of freedom	$df = 1$	
Chi square value		6.64

Conclusion based on this chi square analysis:

Reject H_o: significant at $P < .01$

Table 9

Chi square analysis of responses to compact disc track *Nine*

	Equal Temperament	Pythagorean Intonation
f_o	47	1
f_e	24	24
$f_o - f_e$	23	-23
$(f_o - f_e)^2$	529	529
$(f_o - f_e)^2 / f_e$	22.04	22.04
Null hypothesis	$H_0 : f_0 = f_e$	
Calculated chi square value	$\Sigma (f_0 - f_e)^2 / f_0$	44.08
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Reject H_o: significant at $P < .01$

Table 10

Chi square analysis of responses to compact disc track *Ten*

	Just Intonation	Pythagorean Intonation
f_o	47	1
f_e	24	24
$f_o - f_e$	23	-23
$(f_o - f_e)^2$	529	529
$(f_o - f_e)^2 / f_e$	22.04	22.04

Null hypothesis	$H_0 : f_o = f_e$	
Calculated chi square value	$\Sigma (f_0 - f_e)^2 / f_0$	44.08
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Reject H_o: significant at $P < .01$

Table 11

Chi square analysis of responses to compact disc track *Eleven*

	Just Intonation	Equal Temperament
f_o	5	43
f_e	24	24
$f_o - f_e$	-19	19
$(f_o - f_e)^2$	361	361
$(f_o - f_e)^2 / f_e$	15.04	15.04

Null hypothesis	$H_o : f_o = f_e$	
Calculated chi square value	$\sum (f_0 - f_e)^2 / f_0$	30.08
Degrees of freedom	$df = 1$	
Chi square	$X^2_{.01(1)}$	6.64

Conclusion based on this chi square analysis:

Reject H_o: significant at $P < .01$

Table 12

Chi square analysis of responses to compact disc track *Twelve*

	Pythagorean Intonation	Equal Temperament
f_o	2	46
f_e	24	24
$f_o - f_e$	-22	22
$(f_o - f_e)^2$	484	484
$(f_o - f_e)^2 / f_e$	20.17	20.17

Null hypothesis	$H_0 : f_o = f_e$	
Calculated chi square value	$\sum (f_0 - f_e)^2 / f_0$	40.34
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Reject H_o: significant at $P < .01$

Excerpt from Brass Quintet

A. Tcherepnin, Op. 105

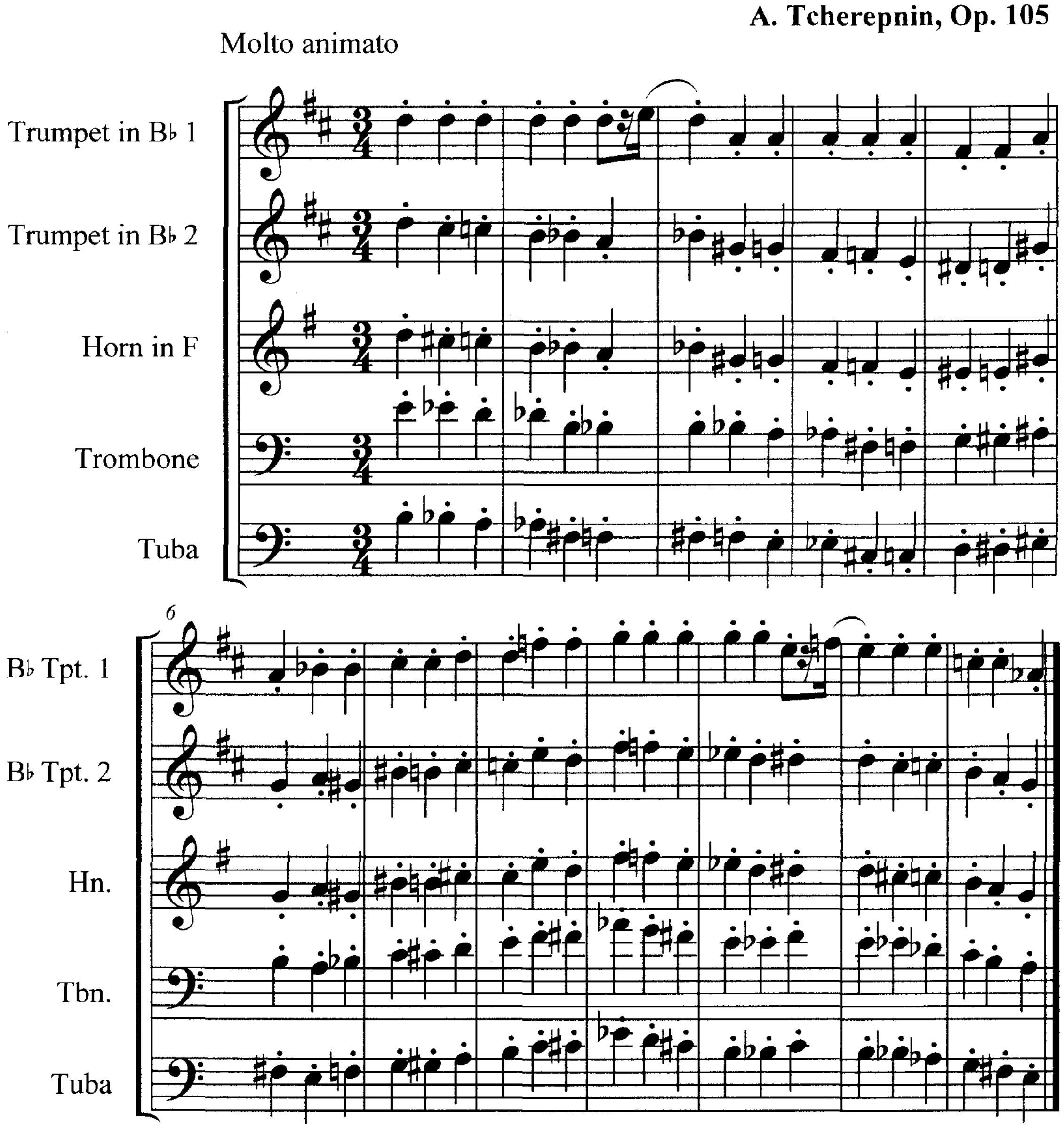

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Figure 17. Excerpt Three, from Brass Quintet

Table 13

Chi square analysis of responses to compact disc track *Thirteen*

	Just Intonation	Equal Temperament
f_o	16	32
f_e	24	24
$f_o - f_e$	-8	8
$(f_o - f_e)^2$	64	64
$(f_o - f_e)^2 / f_e$	2.67	2.67

Null hypothesis	$H_0 : f_0 = f_e$	
Calculated chi square value	$\Sigma (f_0 - f_e)^2 / f_0$	5.34
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Accept H_o

Table 14

Chi square analysis of responses to compact disc track *Fourteen*

	Pythagorean Intonation	Equal Temperament
f_o	8	40
f_e	24	24
$f_o - f_e$	-16	16
$(f_o - f_e)^2$	256	256
$(f_o - f_e)^2 / f_e$	10.67	10.67

Null hypothesis	$H_0 : f_o = f_e$	
Calculated chi square value	$\Sigma (f_0 - f_e)^2 / f_0$	21.34
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Reject H_o: significant at $P < .01$

Table 15

Chi square analysis of responses to compact disc track *Fifteen*

	Pythagorean Intonation	Just Intonation
f_o	23	25
f_e	24	24
$f_o - f_e$	-1	1
$(f_o - f_e)^2$	1	1
$(f_o - f_e)^2 / f_e$	.04	.04
Null hypothesis	$H_0 : f_0 = f_e$	
Calculated chi square value	$\sum (f_0 - f_e)^2 / f_0$	.08
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Accept H_o

Table 16

Chi square analysis of responses to compact disc track *Sixteen*

	Just Intonation	Pythagorean Intonation
f_o	40	8
f_e	24	24
$f_o - f_e$	16	-16
$(f_o - f_e)^2$	256	256
$(f_o - f_e)^2 / f_e$	10.67	10.67

Null hypothesis	$H_o : f_o = f_e$	
Calculated chi square value	$\sum (f_0 - f_e)^2 / f_0$	21.34
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Reject H_o: significant at $P < .01$

Table 17

Chi square analysis of responses to compact disc track *Seventeen*

	Equal Temperament	Pythagorean Intonation
f_o	35	13
f_e	24	24
$f_o - f_e$	11	-11
$(f_o - f_e)^2$	121	121
$(f_o - f_e)^2 / f_e$	5.04	5.04

Null hypothesis	$H_0 : f_o = f_e$	
Calculated chi square value	$\Sigma (f_0 - f_e)^2 / f_0$	10.08
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Reject H_0: significant at $P < .01$

Table 18

Chi square analysis of responses to compact disc track *Eighteen*

	Equal Temperament	Just Intonation
f_o	39	9
f_e	24	24
$f_o - f_e$	15	-15
$(f_o - f_e)^2$	225	225
$(f_o - f_e)^2 / f_e$	9.38	9.38

Null hypothesis	$H_0 : f_o = f_e$	
Calculated chi square value	$\sum (f_0 - f_e)^2 / f_0$	18.76
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Reject H_0: significant at $P < .01$

In summation of the first data collection effort, in sixteen of the chi square analyses the null hypothesis was rejected. In the remaining two analyses, the null hypothesis was accepted. Study subjects exhibited an overall preference for equal temperament, followed by just intonation, and then Pythagorean intonation. The total body of chi square analyses leads to rejection of the null hypothesis.

In the two excerpts pairings where the null hypothesis was accepted, the excerpt utilized was the Tcherepnin brass quintet. The excerpt is polytonal, homophonic, and chromatic. In the responses to compact disc track 13, 32 subjects preferred equal temperament and 16 preferred just intonation. This was the strongest preference expressed for any intonation system when paired against equal temperament. It is interesting to note that in the responses to compact disc track 18, 39 subjects preferred Equal Temperament which in that case was the first intonation system heard. The null hypothesis was rejected for compact disc track 18.

In the responses to compact disc track 15, the preference responses exhibited the most even split of the study. Twenty-three study subjects preferred Pythagorean intonation and twenty-five preferred just intonation. However, in compact disc track 16, using the same excerpt, just intonation was presented first and was preferred by 40 study subjects while Pythagorean intonation was preferred by eight.

The order of presentation had an apparent effect on the preference responses in compact disc tracks seven through eighteen. For example, in track seven Pythagorean intonation was presented first and received five preference responses. In track ten, Pythagorean intonation was presented second and received one preference response.

The number of study subjects in the second data collection effort was 26 (N=26).

Similar to the first data collection effort, subjects were asked to respond to 18 pairs of excerpts, choosing the version in each pair that they perceived as the most consonant for harmony. There were 468 (26x18) total responses in this part of the study.

Description	CD Track	Version A	Version B
	1	equal temperament	Pythagorean intonation
	2	just intonation	Pythagorean intonation
Excerpt #1	3	Pythagorean intonation	equal temperament
	4	just intonation	equal temperament
	5	Pythagorean intonation	just intonation
	6	equal temperament	just intonation
	7	Pythagorean intonation	just intonation
	8	equal temperament	just intonation
Excerpt #2	9	equal temperament	Pythagorean intonation
	10	just intonation	Pythagorean intonation
	11	just intonation	equal temperament
	12	Pythagorean intonation	equal temperament
	13	just intonation	equal temperament
	14	Pythagorean intonation	equal temperament
Excerpt #3	15	Pythagorean intonation	just intonation
	16	just intonation	Pythagorean intonation
	17	equal temperament	Pythagorean intonation
	18	equal temperament	just intonation

Figure 18. Arrangement of excerpts on compact disc

There were 156 (26x6) individual responses to each excerpt. Of the responses to Excerpt One, 88 (56.41%) expressed a preference for equal temperament, 67 (42.95%)

for Pythagorean intonation, and 1 (0.64%) for just intonation. Of the responses to Excerpt Two, 104 (66.67%) expressed a preference for equal temperament, 14 (8.97%) for Pythagorean intonation, and 38 (24.36%) for just intonation. Of the responses to Excerpt Three, 86 (55.13%) expressed a preference for equal temperament, 28 (17.95%) for Pythagorean intonation, and 42 (26.92%) for just intonation.

Description	CD Track	Version A		Version B	
	1	equal temperament	19	Pythagorean	7
	2	just intonation	0	Pythagorean	26
Excerpt #1	3	Pythagorean	8	equal temperament	18
	4	just intonation	1	equal temperament	25
	5	Pythagorean	26	just intonation	0
	6	equal temperament	26	just intonation	0
	7	Pythagorean	5	just intonation	21
	8	equal temperament	26	just intonation	0
Excerpt #2	9	equal temperament	26	Pythagorean	0
	10	just intonation	17	Pythagorean	9
	11	just intonation	0	equal temperament	26
	12	Pythagorean	0	equal temperament	26
	13	just intonation	6	equal temperament	20
	14	Pythagorean	4	equal temperament	22
Excerpt #3	15	Pythagorean	7	just intonation	19
	16	just intonation	16	Pythagorean	10
	17	equal temperament	19	Pythagorean	7
	18	equal temperament	25	just intonation	1

Figure 19. Number of responses for each intonation system (N=26)

The following pages detail chi square analyses performed on responses to the excerpts. The chi square values for these calculations were obtained from the appendix in *Basic Statistical Analysis.* (p 452) Separate analyses were performed for each pair of excerpts on a compact disc track. There were eighteen tracks on the compact disc.

Table 19

Chi square analysis of responses to compact disc track *One*

	Equal Temperament	Pythagorean Intonation
f_o	19	7
f_e	13	13
$f_o - f_e$	6	-6
$(f_o - f_e)^2$	36	36
$(f_o - f_e)^2 / f_e$	2.77	2.77

Null hypothesis	$H_0 : f_0 = f_e$	
Calculated chi square value		5.54
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Accept H_o

Table 20

Chi square analysis of responses to compact disc track *Two*

	Just Intonation	Pythagorean Intonation
f_o	0	26
f_e	13	13
$f_o - f_e$	-13	13
$(f_o - f_e)^2$	169	169
$(f_o - f_e)^2 / f_e$	13	13

Null hypothesis	$H_0 : f_o = f_e$	
Calculated chi square value		26
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Reject H_o: significant at $P < .01$

Table 21

Chi square analysis of responses to compact disc track *Three*

	Pythagorean Intonation	Equal Temperament
f_o	8	18
f_e	13	13
$f_o - f_e$	-5	5
$(f_o - f_e)^2$	25	25
$(f_o - f_e)^2 / f_e$	1.92	1.92

Null hypothesis	$H_0 : f_0 = f_e$	
Calculated chi square value		3.85
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Accept H_o:

Table 22

Chi square analysis of responses to compact disc track *Four*

	Just Intonation	Equal Temperament
f_o	1	25
f_e	13	13
$f_o - f_e$	-12	12
$(f_o - f_e)^2$	144	144
$(f_o - f_e)^2 / f_e$	11.08	11.08

Null hypothesis	$H_0 : f_0 = f_e$	
Calculated chi square value		22.16
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Reject H_0: significant at $P < .01$

Table 23

Chi square analysis of responses to compact disc track *Five*

	Pythagorean Intonation	Just Intonation
f_o	26	0
f_e	13	13
$f_o - f_e$	13	-13
$(f_o - f_e)^2$	169	169
$(f_o - f_e)^2 / f_e$	13	13

Null hypothesis	$H_o : f_o = f_e$	
Calculated chi square value		26
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Reject H_o: significant at $P < .01$

Table 24

Chi square analysis of responses to compact disc track *Six*

	Equal Temperament	Just Intonation
f_o	26	0
f_e	13	13
$f_o - f_e$	13	-13
$(f_o - f_e)^2$	169	169
$(f_o - f_e)^2 / f_e$	13	13

Null hypothesis	$H_0 : f_o = f_e$	
Calculated chi square value		26
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Reject H_o: significant at $P < .01$

Table 25

Chi square analysis of responses to compact disc track *Seven*

	Pythagorean Intonation	Just Intonation
f_o	5	21
f_e	13	13
$f_o - f_e$	-8	8
$(f_o - f_e)^2$	64	64
$(f_o - f_e)^2 / f_e$	4.92	4.92

Null hypothesis	$H_0 : f_o = f_e$	
Calculated chi square value		9.85
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Reject H_o: significant at $P < .01$

Table 26

Chi square analysis of responses to compact disc track *Eight*

	Equal Temperament	Just Intonation
f_o	26	0
f_e	13	13
$f_o - f_e$	13	-13
$(f_o - f_e)^2$	169	169
$(f_o - f_e)^2 / f_e$	13	13

Null hypothesis	$H_o : f_o = f_e$	
Calculated chi square value		26
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Reject H_o: significant at $P < .01$

Table 27

Chi square analysis of responses to compact disc track *Nine*

	Equal Temperament	Pythagorean Intonation
f_o	26	0
f_e	13	13
$f_o - f_e$	13	-13
$(f_o - f_e)^2$	169	169
$(f_o - f_e)^2 / f_e$	13	13

Null hypothesis	$H_o : f_o = f_e$	
Calculated chi square value		26
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Reject H_o: significant at $P < .01$

Table 28

Chi square analysis of responses to compact disc track *Ten*

	Just Intonation	Pythagorean Intonation
f_o	17	9
f_e	13	13
$f_o - f_e$	4	-4
$(f_o - f_e)^2$	16	16
$(f_o - f_e)^2 / f_e$	1.23	1.23

Null hypothesis	$H_0 : f_0 = f_e$	
Calculated chi square value		2.46
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Accept H_0

Table 29

Chi square analysis of responses to compact disc track *Eleven*

	Just Intonation	Equal Temperament
f_o	0	26
f_e	13	13
$f_o - f_e$	-13	13
$(f_o - f_e)^2$	169	169
$(f_o - f_e)^2 / f_e$	13	13

Null hypothesis	$H_o : f_o = f_e$	
Calculated chi square value		26
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Reject H_o: significant at $P < .01$

Table 30

Chi square analysis of responses to compact disc track *Twelve*

	Pythagorean Intonation	Equal Temperament
f_o	0	26
f_e	13	13
$f_o - f_e$	-13	13
$(f_o - f_e)^2$	169	169
$(f_o - f_e)^2 / f_e$	13	13

Null hypothesis	$H_0 : f_o = f_e$	
Calculated chi square value		26
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Reject H_0: significant at $P < .01$

Table 31

Chi square analysis of responses to compact disc track *Thirteen*

	Just Intonation	Equal Temperament
f_o	6	20
f_e	13	13
$f_o - f_e$	-7	7
$(f_o - f_e)^2$	49	49
$(f_o - f_e)^2 / f_e$	3.77	3.77

Null hypothesis	$H_0 : f_o = f_e$	
Calculated chi square value		7.54
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Reject H_0: significant at $P < .01$

Table 32

Chi square analysis of responses to compact disc track *Fourteen*

	Pythagorean Intonation	Equal Temperament
f_o	4	22
f_e	13	13
$f_o - f_e$	-9	9
$(f_o - f_e)^2$	81	81
$(f_o - f_e)^2 / f_e$	6.23	6.23

Null hypothesis	$H_o : f_o = f_e$	
Calculated chi square value		12.46
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Reject H_o: significant at $P < .01$

Table 33

Chi square analysis of responses to compact disc track *Fifteen*

	Pythagorean intonation	Equal Temperament
f_o	7	19
f_e	13	13
$f_o - f_e$	-6	6
$(f_o - f_e)^2$	36	36
$(f_o - f_e)^2 / f_e$	2.77	2.77

Null hypothesis	$H_o : f_o = f_e$	
Calculated chi square value		5.54
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Accept H_o

Table 34

Chi square analysis of responses to compact disc track *Sixteen*

	Just Intonation	Pythagorean Intonation
f_o	16	10
f_e	13	13
$f_o - f_e$	3	-3
$(f_o - f_e)^2$	9	9
$(f_o - f_e)^2 / f_e$	.69	.69

Null hypothesis	$H_o : f_o = f_e$	
Calculated chi square value		1.38
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Accept H_o

Table 35

Chi square analysis of responses to compact disc track *Seventeen*

	Equal Temperament	Pythagorean Intonation
f_o	19	7
f_e	13	13
$f_o - f_e$	6	-6
$(f_o - f_e)^2$	36	36
$(f_o - f_e)^2 / f_e$	2.77	2.77

Null hypothesis	$H_o : f_o = f_e$	
Calculated chi square value		5.54
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Accept H_o

Table 36

Chi square analysis of responses to compact disc track *Eighteen*

	Equal Temperament	Just Intonation
f_o	25	1
f_e	13	13
$f_o - f_e$	12	-12
$(f_o - f_e)^2$	144	144
$(f_o - f_e)^2 / f_e$	11.08	11.08

Null hypothesis	$H_0 : f_0 = f_e$	
Calculated chi square value		22.16
Degrees of freedom	$df = 1$	
Chi square value from table		6.64

Conclusion based on this chi square analysis:

Reject H_o: significant at $P < .01$

In summation of the second data collection effort, the results were similar to the first set of data. Equal temperament was preferred over just intonation and Pythagorean intonation. However, this set of data accepted the null hypothesis six times whereas the first set of data accepted the null only two times. In the first set of data, the null hypothesis was accepted twice on Excerpt 3, which was the Tcherepnin quintet. In the second set of data, the null hypothesis was accepted three times on Excerpt 3, with one of those being compact disc track 17 which paired equal temperament with Pythagorean intonation. This is an interesting development considering the fact that the null hypothesis was accepted on compact disc track one also, which also paired equal temperament and Pythagorean intonation.

The conclusion that can be drawn from both sets of data is that polytonal harmony is more difficult to perceive for consonance. Both sets of data indicate a preference for equal temperament in the Excerpt 1, followed by Pythagorean intonation, and then just intonation. Both sets of data indicate a preference for equal temperament in Excerpt 2, followed by just intonation, the Pythagorean intonation.

The overall preference for equal temperament is viewed by the author as a comfort zone for study subjects. It can also be concluded that Pythagorean intonation fares better than just intonation in the tonal excerpt due to the fact that the Pythagorean intonation major third is much closer to an equal temperament third than a just third. Additional research in these areas is recommended, particularly when an investigation can be designed that utilizes instrumental sounds that are more authentic than those used in the current study.

CHAPTER V

SUMMARY AND CONCLUSIONS

A stated purpose of this study was to investigate perceived preference for harmony consonance in musical excerpts tuned to equal temperament, just intonation, or Pythagorean intonation. The study accomplished this purpose by asking study participants to simply choose one from a pair of excerpts that they perceived as the most consonant. Coincidence theory stated that the more consonant sound would be one in which the mathematical components are in enhanced alignment. The null hypothesis stated that there would be no difference in expressed preference among intonation systems utilized in the study.

Two sets of data were collected in the course of the study. In the first data set, the null hypothesis was rejected in 16 of 18 chi square analyses. In the remaining two analyses, the null hypothesis was accepted. In the second data set, the null hypothesis was rejected 12 times and accepted 6 times. The chi square analyses for both sets of data indicate that the choice of one intonation system over another did not occur by chance. Summarily, the null hypothesis for the study was rejected.

Both sets of data follow the same pattern of expressed preference. The results do not support coincidence theory. The results of this study also do not agree with Bisel's conclusion that no intonation system is preferred over another. Equal temperament was preferred when paired with any other intonation system. In Excerpt 1, a simple tonal quintet, Pythagorean intonation was preferred over just intonation when the two systems were paired. In Excerpt 2, an imitative duet, just intonation was preferred over Pythagorean intonation when the two systems were paired. The results of the study do not support coincidence theory due to the fact that the mathematics of equal temperament is more complex than any other intonation system.

One of the most interesting discoveries of the study occurred using Excerpt 3, which was a Tcherepnin brass quintet. That excerpt is polytonal, homophonic, and chromatic. In data set one, the null hypothesis was accepted two times. In data set two, the null hypothesis was accepted three times on that excerpt. Although the study subjects expressed preference for equal temperament over the other intonation systems, the numbers were not as one-sided with the Tcherepnin quintet as with the other two excerpts. The conclusion drawn from these findings indicated that consonance perception in polytonal harmony is more difficult than in simpler, tonal music.

In data set one of the current study, the responses to compact disc track thirteen detailed 32 subjects who preferred equal temperament and 16 preferred just intonation. This was the strongest preference expressed for any intonation system when paired against equal temperament in either of the sets of data. In data set two, the intonation system that fared best against equal temperament was Pythagorean intonation. At no point in the second data collection effort did just intonation fare as well as in the first data set. A possible explanation for the overall closer connection between equal temperament and Pythagorean intonation lies in the distance between the major thirds in those systems. The Pythagorean third is higher than the equal temperament third, which in turn is over 13 cents sharper than a just intonation third. This would possibly explain why Pythagorean intonation was preferred over just intonation in Excerpt 1 which was more tonal than the other excerpts. Additional study in this area is recommended.

In the responses to compact disc track 15 of data set one, the preference responses exhibited one of the most even split of the study. Twenty-three study subjects preferred Pythagorean intonation and twenty-five preferred just intonation. However, in compact disc track 16, utilizing the same excerpt, just intonation was presented first and was preferred by 40 study subjects while Pythagorean intonation was preferred by 8. This led the author to conclude that the order of presentation has an influence on preference response. This is an area recommended for further study.

In the first data collection effort, the author was not satisfied with the quality of sounds on the playback compact disc, although the collection of data was accomplished. It was discovered during the course of the study that the Justonic Pitch Palette software and Garritan Personal Orchestra (GPO) sounds, that are packaged with Finale 2008 notation software, are not compatible. Additional research in this area is recommended with programs that are compatible and more closely approximate the complex tones created by wind instruments.

The second troublesome part of the sounds utilized in the first data collection effort centered on the fact that the scales utilized for the excerpts were all adjusted to an A of 440 cycles-per-second. This had the unfortunate result of creating tonic notes that were sometimes noticeably different pitches. Depending on the excerpt pairing, study subjects on occasion were asked to listen to the same excerpt starting on different tunings of the tonic pitch. This also created inconsistencies in some of the unisons and octaves during excerpt playback. For some of the study subjects, this was a distraction.

The unsatisfactory sounds of the playback disc prompted the author to explore options for improving the quality of the excerpt playback. A member of the dissertation committee suggested an article by Wibberley (2004) that detailed the process of adjusting pitchwheel values in Finale software to accommodate alternate tunings. This was considered a possible method of utilizing the GPO sounds. The methods and procedures used to produce new versions of the excerpts are detailed in Chapter III of the current study.

However, this new approach to recording the excerpts did not meet with the desired result. The author was not able to use the pitchwheel settings in Finale 2008 with the GPO sounds. The sounds that were used in a second data collection effort were the same generic MIDI sounds that were problematic in the first data collection. However, use of the pitchwheel function did allow control of pitch adjustments, particularly of the tonic, octaves, and unisons which were questionable in the first recordings.

An additional goal of the study was to provide information that could be used by conductors and performers to create and maintain consonant intonation in wind instrument ensembles. This goal was to be facilitated by using authentic wind instrument sounds in an authentic musical context. The MIDI sounds that were utilized in the study were not sounds that replicate the complex tones of wind instruments to the degree desired by the author. While the collection of data was accomplished, it became apparent during the course of the study that the technology to micro-tune the most authentic instrumental sounds does not exist.

The results of this study show an overall preference for equal temperament. This was the most surprising discovery of the investigation. The author expected just intonation to fare better in the results. The fact that the results of this study do not support coincidence theory points to the need for further investigation in this area.

The utilization of equal temperament in rehearsal and performance is readily accomplished due to the fact that woodwind instruments and melodic percussion instruments play primarily in equal temperament. It is also easy to compare the intonation of all instruments to an equal temperament standard since most electronic tuning devices are programmed with equal temperament as the default intonation system.

However, the fact that just intonation and Pythagorean intonation were preferred in some areas of the study indicates that those intonation systems also have a place in rehearsal and performance. One determining factor would appear to be context, in particular the style of composition. Another determining factor could be conditioning of the human auditory system over a period of time to a particular intonation system. The centuries spent listening to equal temperament as the preferred intonation system could have conditioned the human auditory system to accept that as the most consonant sound.

The author does not refute the fact that just intonation is an ideal system to use for tonal harmonies that sound sufficiently long for a consonance judgment to be accomplished by the listener. The author believes that highly trained instrumentalists and singers adjust harmony to just intonation ratios instinctively in order to eliminate beats and create the most consonant sound. Additional research could uncover musical contexts in which Pythagorean intonation would illicit the preferred consonance response. The current study encourages research in this area.

In the author's opinion, the primary ingredient that was lacking in the current study was the flexibility to create the most accurate intonation using complex tones that approximate the sounds of wind instruments. The combination of hardware and software did not possess the flexibility necessary to produce recorded versions of the excerpts in the context desired. It is an interesting parallel that flexibility is one of the most important skills that wind instrument performers can possess. Through a combination of embouchure, air support, slide length, alternate fingerings, and listening skills, a performer can adjust any pitch to conform to any intonation system. The ultimate objective is to eliminate the beats that add roughness to the sound.

A stated goal of the project was to examine harmonies that are perceived as consonant to the human ear and provide information as to why those sounds are preferred over others. The first part of this goal was accomplished. Study subjects picked an excerpt from each pair that they perceived as most consonant. The sounds that study subjects picked, however, do not support the basic tenets of coincidence theory. In fact, study subjects overwhelmingly picked the intonation system that is derived from complicated mathematics. The null hypothesis for the study was rejected, but not based on any evidence derived from coincidence theory.

Finally, results of the current study constitute one small step in a direction full of possibilities for further research. The investigation created more questions than it answered. This is deemed a beneficial by-product, both in terms of additional areas of study directly related to the study topic in additional areas of inquiry that more resourceful investigators might be able to design. Additional research in the following areas is recommended:

1. Comparison of perceptual responses to intonation systems utilizing instrumental sounds that more closely approximate the timbres of actual wind instruments.

2. The use of Finale pitchwheel settings to explore alternate tunings if GPO sounds can be used with pitchwheel settings in the investigation.

3. Use of Justonic Pitch Palette software to explore alternate tunings if GPO sounds can be used in the investigation.

4. A study that explores the order of presentation of intonation systems as a way of investigating if that order influences perceptual preference for one system over another.

5. A study that explores the amount of time it takes to make a consonance judgment. This interval of time is identified as the attack transient in the literature.

6. A modern version of Boomsliter and Creel's extended reference study could be designed to investigate the tuning of individual lines of a larger work and then recombine the lines to determine the correlation between melodic and harmony intonation.

7. A study could be designed that separates instrumentalists in groups of string, woodwinds, brass, and keyboard performers. It would be interesting to determine if the type of intonation the performer is most accustomed to influences their tuning preference.

APPENDIX A

August 6, 2007

Music Learning Research Division of G.I.A. Publications
7404 S. Mason Ave.
Chicago, IL 60638

Dear Sirs,

My name is Derle R. Long and I am Director of Bands at the University of Louisiana at Monroe. I am also a candidate for the degree Doctor of Philosophy in Music Education from the University of Southern Mississippi. Dr. Thomas V. Fraschillo is my dissertation committee chairman. I am A.B.D. in my pursuit of this degree.

My dissertation title is "An investigation of tuning preferences of a selected group of instrumentalists with reference to Just Intonation, Equal Temperament, and Pythagorean Intonation." My study had its genesis in a study by Hugh Bailey Johnson, Jr., at the Indiana University in 1963 in which he investigated the tuning preferences of a select group of singers.

The study will present short musical excerpts to selected listeners. The excerpts will be notated using Finale music notation software. The excerpts will be reproduced using Cakewalk Dimension Pro virtual synthesizer, Justonic Pitch Palette, and a yet-to-be-determined sound system. The Justonic software allows the excerpt to be tuned and played back in an infinite variety of intonation systems. The three systems selected for this study are Just Intonation, Equal Temperament, and Pythagorean Intonation, in the same manner as Hugh Johnson.

This is where G.I.A. Publications comes into the picture. For several years I have used the MLR Instrumental Score Reading Program in my conducting classes here at ULM. One of the things I like best is the sequential nature of the excerpts, from Level One to Level Three. I believe this sequential approach would work well in my study.

I am seeking permission to utilize no more than eight musical examples from the MLR program as the excerpts for my study. I propose to use two excerpts from Levels One and Two and three excerpts from Level Three. I realize that G.I.A. does not own all of the copyrights to the music in the MLR. However, since the excerpts are contained in an anthology that is copyrighted by G.I.A., I am seeking your permission first. Should that be granted and an excerpt be selected in which the copyright is owned by someone other than G.I.A., I will seek permission to use that excerpt from the copyright owner.

I will, of course, provide full disclosure and gratitude to G.I.A. Publications in text of the study should you find it possible to grant this permission. I will also make the data and conclusions available to you as soon as the study is accepted by my committee and the U.S.M. Graduate School. Should you need more information on this request, please do not hesitate to call me at 318.342.1594 or communicate by email at long@ulm.edu.

Sincerely,

Derle R. Long
Director of Bands

APPENDIX B

Derle R. Long

From: Chris Giebel [chrisg@giamusic.com]
Sent: Thursday, August 09, 2007 8:38 AM
To: long@ulm.edu
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APPENDIX C

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September 28, 2007

Derle R. Long
ULM Band Department
700 University Ave.
Monroe, LA 71209-0250

Dear Mr. Long,

Thank you for your e-mail correspondence in which you have requested permission to include excerpts from Alexander Tcherepnin's Opus 105 in your dissertation.

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APPENDIX D

THE UNIVERSITY OF SOUTHERN MISSISSIPPI

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HUMAN SUBJECTS PROTECTION REVIEW COMMITTEE
NOTICE OF COMMITTEE ACTION

The project has been reviewed by The University of Southern Mississippi Human Subjects Protection Review Committee in accordance with Federal Drug Administration regulations (21 CFR 26, 111), Department of Health and Human Services (45 CFR Part 46), and university guidelines to ensure adherence to the following criteria:

- The risks to subjects are minimized.
- The risks to subjects are reasonable in relation to the anticipated benefits.
- The selection of subjects is equitable.
- Informed consent is adequate and appropriately documented.
- Where appropriate, the research plan makes adequate provisions for monitoring the data collected to ensure the safety of the subjects.
- Where appropriate, there are adequate provisions to protect the privacy of subjects and to maintain the confidentiality of all data.
- Appropriate additional safeguards have been included to protect vulnerable subjects.
- Any unanticipated, serious, or continuing problems encountered regarding risks to subjects must be reported immediately, but not later than 10 days following the event. This should be reported to the IRB Office via the "Adverse Effect Report Form".
- If approved, the maximum period of approval is limited to twelve months. Projects that exceed this period must submit an application for renewal or continuation.

PROTOCOL NUMBER: **28032502**
PROJECT TITLE: **Seeking a Perceptual Preference for Harmony Consonance Using Equal Temperament, Just Intonation, and Pythagorean Intonation is Selected Excerpts for Wind Instruments**
PROPOSED PROJECT DATES: **03/26/08 to 06/24/08**
PROJECT TYPE: **Dissertation or Thesis**
PRINCIPAL INVESTIGATORS: **Derle R. Long**
COLLEGE/DIVISION: **Gruaduate Studies**
DEPARTMENT: **Music**
FUNDING AGENCY: **N/A**
HSPRC COMMITTEE ACTION: **Expedited Review Approval**
PERIOD OF APPROVAL: **03/31/08 to 03/30/09**

Lawrence A. Hosman — 4-2-08
Lawrence A. Hosman, Ph.D. — Date
HSPRC Chair

APPENDIX E

HUMAN SUBJECTS REVIEW FORM
UNIVERSITY OF SOUTHERN MISSISSIPPI
(SUBMIT THIS FORM IN DUPLICATE)

Protocol # 28032502
(office use only)

Name Darle R. Long Phone 318-345-7887

E-Mail Address long@ulm.edu

Mailing Address 18 Magnolia Dr., Monroe, LA 71203
(address to receive information regarding this application)

College/Division Graduate Studies Dept Music

Department Box # 5081 Phone 601-266-4990

Proposed Project Dates: From March 26, 2008 To June 24, 2008
(specific month, day and year of the beginning and ending dates of full project, not just data collection)

Title Seeking a perceptual preference for harmony consonance using Equal Temperament, Just Intonation, and Pythagorean Intonation is selected excerpts for wind instruments.

Funding Agencies or Research Sponsors ________

Grant Number (when applicable) ________

____ New Project

x Dissertation or Thesis

____ Renewal or Continuation: Protocol # ____

____ Change in Previously Approved Project: Protocol # ____

Darle R. Long 3/10/08
Principal Investigator Date

Advisor 3/18/08
Date

x Charles Elliott 3/18/08
Department Chair Date

RECOMMENDATION OF HSPRC MEMBER

____ Category I, Exempt under Subpart A, Section 46.101 ( )( ), 45CFR46.

X Category II, Expedited Review, Subpart A, Section 46.110 and Subparagraph (B).

____ Category III, Full Committee Review.

3-26-08
HSPRC College/Division Member DATE

4-2-08
HSPRC Chair DATE

APPENDIX F

Consent Sample (Short Form)

THE UNIVERSITY OF SOUTHERN MISSISSIPPI

AUTHORIZATION TO PARTICIPATE IN RESEARCH PROJECT
(Short Form - to be used with oral presentation)

Particpant's Name ______________________________

Consent is hereby given to participate in the research project entitled
Seeking a perceptual preference. All procedures and/or investigations to be followed and their purpose, including any experimental procedures, were explained by Derle R. Long. Information was given about all benefits, risks, inconveniences, or discomforts that might be expected.

The opportunity to ask questions regarding the research and procedures was given. Participation in the project is completely voluntary, and participants may withdraw at any time without penalty, prejudice, or loss of benefits. All personal information is strictly confidential, and no names will be disclosed. Any new information that develops during the project will be provided if that information may affect the willingness to continue participation in the project.

Questions concerning the research, at any time during or after the project, should be directed to researcher(s) name(s) at telephone number(s). This project and this consent form have been reviewed by the Human Subjects Protection Review Committee, which ensures that research projects involving human subjects follow federal regulations. Any questions or concerns about rights as a research participant should be directed to the Chair of the Institutional Review Board, The University of Southern Mississippi, 118 College Drive #5147, Hattiesburg, MS 39406-0001, (601) 266-6820.

Use the following only if applicable: The University of Southern Mississippi has no mechanism to provide compensation for participants who may incur injuries as a result of participation in research projects. However, efforts will be made to make available the facilities and professional skills at the University. Information regarding treatment or the absence of treatment has been given. In the event of injury in this project, contact treatment provider's name(s) at telephone number(s).

A copy of this form will be given to the participant.

Signature of participant ______________________ Date ______________

D. R. L.
Signature of person explaining the study

3/26/08
Date

APPENDIX G

Data Collection Document

Number ____________________

This project has been reviewed by the Human Subjects Review Committee, which ensures that research projects involving human subjects follow federal regulations. Any questions or concerns about rights as a research subject should be directed to the chair of the Institutional Review Board, The University of Southern Mississippi, 118 College Drive, #5147, Hattiesburg, MS 39406-0001, (601)266-6820.

Instructions – On each CD track you will hear pairs of each excerpt, identified on this document as "A" or "B". Choose the version that you think sounds best for harmony consonance. Place an "X" in the box for the version you choose. You may make comments about the versions in the space provided.

Description	CD Track	A	B	Comments
Excerpt # 1	1			
	2			
	3			
	4			
	5			
	6			
Excerpt # 2	7			
	8			
	9			
	10			
	11			
	12			
Excerpt # 3	13			
	14			
	15			
	16			
	17			
	18			
Excerpt # 4	19			
	20			
	21			
	22			
	23			
	24			

APPENDIX H

Scale and frequencies utilized for Bouree

	Just Intonation	Equal Temperament	Pythagorean Intonation
Do	220	220	220
	234.67	233.08	231.77
Re	247.5	246.94	247.5
	264	261.63	260.74
Mi	275	277.18	278.44
Fa	293.33	293.66	293.33
	308	311.13	313.24
So	330	329.63	330
	352	344.23	347.65
La	366.67	369.99	371.25
	385	392	391.11
Ti	412.5	415.30	417.66
Do	440	440	440

APPENDIX I

Scale and frequencies utilized for Ascendit Deus

	Just Intonation	Equal Temperament	Pythagorean Intonation
Do	234.67	233.08	231.77
	250.31	246.94	244.17
Re	264	261.63	260.74
	281.60	277.18	274.69
Mi	293.33	293.66	293.33
Fa	312.89	311.13	309.03
	328.53	329.63	330
So	352	344.23	347.65
	375.47	369.99	366.25
La	391.11	392	391.11
	410.67	415.30	412.03
Ti	440	440	440
Do	469.34	466.16	463.54

APPENDIX J

Scale and frequencies utilized for Brass Quintet

	Just Intonation	Equal Temperament	Pythagorean Intonation
Do	264	261.63	260.74
	281.6	277.18	274.69
Re	297	293.66	293.33
	316.80	311.13	309.03
Mi	330	329.63	330
Fa	352	349.23	347.65
	369.60	369.99	371.25
So	396	392	391.11
	422.4	415.30	412.03
La	440	440	440
	462	466.16	463.54
Ti	495	493.88	495.00
Do	528	523.26	521.48

APPENDIX K

Finale 2008 Pitchwheel Settings for Ascendit Deus

Note Name	Pythagorean Intonation	Just Intonation
Bb	0	0
B	-400	481
C	160	160
C#/Db	-240	641
D	320	-561
D#/Eb	-80	-80
E	480	123
F	80	80
F#/Gb	655	561
G	240	-641
G#/Ab	-160	721
A	400	-481

APPENDIX L

Finale 2008 Pitchwheel Settings for Bouree

Note Name	Pythagorean Intonation	Just Intonation
A	0	0
A#/Bb	-400	481
B	160	160
C	-240	641
C#/Db	320	-561
D	-80	-80
D#/Eb	480	123
E	80	80
F	655	561
F#/Gb	240	-641
G	-160	721
G#/Ab	400	-481

APPENDIX M

Finale 2008 Pitchwheel Settings for Tcherepnin Quintet

Note Name	Pythagorean Intonation	Just Intonation
C	0	0
C#/Db	-400	481
D	160	160
D#/Eb	-240	641
E	320	-561
F	-80	-80
F#/Gb	480	123
G	80	80
G#/Ab	655	561
A	240	-641
A#/Bb	-160	721
B	400	-481

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