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Spatial Visualization of High School Geometry Students

Jennifer Crissey

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SPATIAL VISUALIZATION OF HIGH SCHOOL GEOMETRY STUDENTS

by

Jennifer Crissey

A Dissertation
Submitted to the Graduate School,
the College of Arts and Sciences
and the School of Science and Mathematics Education
at The University of Southern Mississippi
in Partial Fulfillment of the Requirements
for the Degree of Doctor of Philosophy

Committee:

Dr. Anna Wan, Committee Chair
Dr. Kendrick Buford
Dr. Julie Cwikla
Dr. James Lambers
Dr. Rebecca Robichaux-Davis

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ABSTRACT

Spatial ability has been defined as “the innate ability to visualize that a person has before any formal training has occurred, i.e., a person is born with ability” (Sorby, 1999, p. 21) and is comprised of spatial orientation and spatial skills. Each of these facets are used in everyday life. High school students can enhance their spatial visualization skills through experiences and instruction. The purpose of this study was to test an intervention with the aim of increasing high school geometry students’ spatial visualization skills. The participants in this study were high school geometry students who were randomly placed in either a technology or manipulative group. Participants were given the Revised Purdue Spatial Visualization Test: Rotations (PSVT:R) at the beginning of the study to collect baseline data of students’ spatial visualization skills. A few weeks later, the PSVT:R was given as a pretest followed by the implementation of the intervention. The intervention consisted of spatial activities namely Quick draw, Quick images, Quick blocks, and an instructional unit that consisted of five major class activities designed to increase spatial visualization skills. The manipulative groups completed the activities using concrete, tangible models during instruction. The technology groups completed the activities using Desmos and GeoGebra. After the intervention, participants were given the PSVT:R as a posttest. A few weeks after the intervention was completed, the PSVT:R was administered one last time and a Repeated Measures ANOVA was conducted to determine if there were significant differences between the technology and manipulative groups. The results indicated the manipulative group scored higher than the technology group on all four implementations of the PSVT:R.

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DEDICATION

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CHAPTER I – INTRODUCTION

Background

“We live in a three-dimensional world; hence, improving spatial ability is part of our obligation as mathematics teachers” (Patkin & Dayan 2013; p. 179). People are impacted by their spatial ability and spatial skills as they learn to navigate our three-dimensional world. According to Sorby (1999), “Spatial ability is defined as the innate ability to visualize that a person has before any formal training has occurred... However spatial skills are learned or acquired through training” (p. 21). Early in the study of spatial ability with Piaget, the focus was on spatial skills because spatial ability is the skill set with which an individual is born. More recently, Stieff & Uttal (2015) stated, “There is now considerable evidence that spatial ability is malleable- that a variety of experiences, ranging from life experiences to specific, intensive training, can improve spatial ability” (p. 609). And therefore, spatial ability should be able to be improved through mathematics teaching. Because spatial skills could be enhanced through instruction and experiences, research has been conducted to show how to increase spatial ability in students.

Spatial skills can be divided into three components (1) spatial relations, (2) spatial visualization, and (3) spatial orientation (Pittalis & Christou, 2010). Spatial visualization involves our ability to navigate the world. “It affects our ability to navigate from place to place, identify an object moving towards us, estimate quantities, understand drawings and charts and compose various items” (Patkin & Dayan, 2013, p. 179-180). Spatial visualization has been described as the ability to complete “difficult spatial tasks that require a sequence of transformations of a spatial representation and more complex

stimuli” (Pittalis & Christou, 2010 p. 195). This could be done by imagining what a piece of paper would look like after folding and unfolding a certain number of times. Spatial visualization is also described as moving an object mentally (Sorby, 1999). Spatial visualization tasks have included two categories of tasks, mental rotation and mental transformation. According to Sorby (1999), “The difference between these two categories is that with mental rotation, the entire object is transformed by turning in space, whereas with mental transformation, only part of the object is transformed in some way” (p. 22). Both aspects equally contribute to one’s spatial visualization skills that are used to help one function in our three-dimensional world.

Spatial visualization skills are used in everyday life. For instance, spatial visualization skills are used to plan alternate routes home when an accident has blocked the regular route home, giving directions to someone, or locating keys. Many adults lack these skills and demonstrate this by driving around the parking lot multiple times to find a place to park because they “can’t” parallel park. Because many adults lack the basic spatial visualization skills, instruction should focus on methods that will help enhance students’ spatial visualization skills.

Gender also plays a role in spatial visualization. Research has shown that males outperform females on spatial visualization tasks. According to Fennema & Sherman (1977), “Sometimes this difference is attributed to underlying ability and other times it is attributed to a social climate that does not encourage girls to study mathematics” (p. 51-52). When comparing the spatial visualization of males and females, Fennema & Sherman (1977) stated, “The findings suggest that the existing opinion that females have less aptitude for mathematics needs to be modified” (p. 65). Upon comparing males and

females with similar mathematic backgrounds the differences between the two groups were small (Fennema & Sherman, 1977). The research of Ben-Chaim et al. (1988) contradicted Fennema & Sherman (1977) by stating, “In general, boys seem to outperform girls with respect to spatial visualization ability” (p. 64). Quaiser-Pohl & Lehmann (2002) showed differences in spatial abilities favoring males. Tzuriel & Egozi (2010) conducted a study that used a “Spatial Sense” intervention in which used Quick Draw and flash cards to improve spatial skills. The results stated, “Comparison of the pre-to postintervention improvement of boys and girls showed that girls’ improvement in the experimental group was greater than that of boys and that the girls closed the initial gap” (Tzuriel & Egozi, 2010; p. 1426). More recently, the research conducted by Harris et al. (2021) used 84 fourth grade students to analyze the relationship between spatial reasoning and mathematics. These results contrasted with existing literature finding a favor towards females in spatial visualization. Because of these mixed gender results about the spatial visualization skills in boys and girls, further research was needed.

In addition to gender, there are other spatial experiences that can impact students’ spatial visualization skills. Those other spatial experiences include playing sports, musical instruments, or video games and playing with blocks as a child. Students can improve their spatial visualization skills through participation in sports, playing video games, playing musical instruments, and playing with construction-based toys (Gold et al. 2018). According to Pietsch & Jansen (2012), “A long-time activity in these disciplines, such as playing an instrument or sport for many years, has an enhancing effect on a specific cognitive task” (p. 162). Cherney (2008) claimed, “The results suggest that even a very brief practice (4 h) in computer game play does improve

performance on mental rotation measures” (p. 783). Lastly, Brosnan’s work required 50 nine year old students to use Lego blocks to build three-dimensional models. Brosnan (1998) stated, “The finding that those who finished the model also scored higher on spatial ability than those who did not complete the model suggests support for a relationship between children’s play with Lego blocks and their spatial ability” (p. 25). These findings show that the classroom is not the only place for improvement of spatial visualization skills. Experiences students have outside of school can contribute to increased spatial visualization skills before they enter the classroom.

Spatial visualization skills can be gained through life experiences and in the classroom. There are aspects of the United States K-12 curriculum that allow students the opportunity to learn and enhance their spatial skills. Uttal (2000) discussed the use of maps to enhance spatial visualization. According to Uttal (2000), “Maps allow us to look at, and study, sets of spatial relations without actually navigating through the space” (p. 249). Another way to develop spatial visualization is through science. Jackson et al. (2015) examined students’ spatial visualization through an earth/space unit and found that instruction in this area increased spatial visualization skills of the students in the study. Julia & Antoli (2018) examined spatial ability through a Science Technology Engineering and Math (STEM) course and found that 50 sixth grade students’ spatial ability improved because of the STEM course.

The mathematics classroom has been used frequently to enhance spatial visualization skills. Ben-Chaim et al. (1988) involved fifth and sixth grade students in a spatial visualization unit that was geometry-centered. The students used small cubes to build things and then drew representations from various perspectives. Similarly,

Clements et al. (1997) implemented a mathematics unit with third graders to enhance spatial thinking. Cheng & Mix (2014) stated, "...our study is the first to show a direct effect of spatial training per se on math performance in early elementary-aged children. We found that even a single session of spatial training led to significant improvement on certain problems" (p. 7). High school geometry is a course that has the potential to develop logic and spatial visualization skills depending on the type and quality of instruction. According to Clements & Battista (1992), "Clearly, geometry and spatial reasoning are strongly interrelated, and most mathematics educators seem to include spatial reasoning as part of the geometry curriculum" (p.1). These previous studies supported the selection of geometry as the course used in this study.

Within geometry instruction, technology is a useful tool to enhance instruction. When discussing the use of computers in the classroom, Chang et al. (2016) stated, "Students can thus gain experience visualizing and manipulative geometric figures and develop the ability to reason from those geometric figures" (p. 918). According to Danielson & Meyer (2016), "Computers can provide students with an understanding of the implications of their thinking" (p. 260). This allows students to receive instant feedback enabling them to adjust their work as they solve problems.

Some of the tools available for student use in the geometry classroom include Dynamic Geometry Software (DGS) such as Cabri 3D, Geometer's Sketchpad, and GeoGebra. According to Baki et al. (2011), "The DGS Cabri 3D was designed for exploring 3D geometry. This software is believed to revolutionize computer-assisted visualization and reasoning in 3D geometry, just as earlier DGS revolutionized the study of plane geometry" (p. 294). Cabri 3D can be used to help students manipulate three-

dimensional figures to facilitate the development of spatial visualization skills. Another useful tool for geometry instruction is Geometer's Sketchpad (GSP). According to McClintock (2002), "The way the GSP environment differs from the traditional environment is that the GSP environment can provide students with dynamic two-dimensional representations of three-dimensional objects, powerful transformation tools, and some other useful features such as animation and the use of buttons" (p. 753). In reference to GSP, Idris (2007) stated, "Geometer's Sketchpad provides a flexibly structured mathematics laboratory that supports the investigation and exploration of concepts at a representational level, linking the concrete to the abstract" (p. 170). GeoGebra combines some of the features of Cabri 3D and Geometer's Sketchpad. According to Saha et al. (2010), "GeoGebra provides students in various visualization ability levels to learn geometric concepts and to explore relationships easily" (p. 692). Each of these tools provides students a way to solve problems in a visual way with instant feedback.

Research has shown each of these tools can be used to enhance spatial visualization as well as mathematics achievement. Baki et al. (2011) used Cabri 3D to increase spatial visualization skills in pre-service mathematics teachers. McClintock (2002) explored the use of GSP on middle and high school students to effectively increase spatial visualization skills. Idris (2007) used GSP to examine the van Hiele levels of geometric thought of secondary students. Hannafin et al. (2008) studied how GSP impacts the geometry achievement and spatial ability of sixth grade students. Tieng & Eu (2014) assessed van Hiele levels of geometric thought of third grade students using GSP. Saha et al. (2010) studied mathematics achievement in secondary students using

GeoGebra. Each of these studies showed success except for Hannafin et al. (2008) which showed no significant difference between the GeoGebra group and control group.

Statement of the Problem

Research has documented the advantages of using various interventions to increase students' spatial visualization skills across age groups. Cheng & Mix (2014) and Lowrie et al. (2019) used spatial visualization interventions to increase students' spatial visualization skills, which in turn improved their mathematics performance. Patkin & Dayan (2013) used an intervention unit to improve spatial visualization skills of high school students. Chang et al. (2016) used an intervention unit to improve spatial visualization and increase geometry performance for twelfth grade students. The students in this study had a geometry course the previous year. Sorby et al. (2013) used an intervention to increase spatial visualization skills of freshman engineering students. Each of these studies documented success in increasing students' spatial visualization skills, but the literature did not include an intervention unit to increase spatial visualization of tenth grade geometry students, specifically.

Gender has been documented by research to impact spatial visualization skills. According to research there has been a gap in spatial visualization and mathematics achievement between males and females. Each of the following studies found that males outperformed females in spatial visualization skills (Ben-Chaim et al. 1988); Quaiser-Pohl, 2002; Julia & Antoli, 2018; & Ogunkola & Knight, 2019). Other research by Harris et al. (2021) showed mixed results such as gender differences in spatial visualization favoring females over males, but in terms of spatial orientation the research favored males over females. According to Fennema & Sherman (1977), "The data do not support

either the expectations that males are invariably superior in mathematics achievement or spatial visualization...” (p. 69). Tzurriel & Egozi (2010) also found gender differences in favor of females. Logan & Lowrie (2017) found no significant difference between males and females regarding spatial visualization. These differing results suggested more research needed to be conducted on the topic to investigate both type of intervention and gender differences.

Research has also shown benefits of using technology in the classroom to enhance students’ spatial visualization. Baki et al. (2011) used Cabri 3D to enhance spatial visualization skills of pre-service mathematics teachers. McClintock (2002) used Geometer’s Sketchpad to increase three-dimensional spatial visualization skills of students beginning in seventh grade and following the same students each year through tenth grade. Idris (2007) and Tieng & Eu (2014) used Geometer’s Sketchpad to increase students’ van Hiele levels of geometric thought with Idris (2007) using high school students and Tieng & Eu (2014) using third grade students. Hannafin et al. (2008) used Geometer’s Sketchpad to increase the spatial visualization skills of sixth grade students. Saha (2010) used GeoGebra to increase geometry learning of high school students. Each of these studies successfully used technology for enhancing aspects of geometric thinking, geometry learning or spatial visualization. Each of these studies successfully used technology with diverse ages of students ranging from third grade to pre-service teachers. However, there is a lack of research that explores these tools’ efficacy to enhance spatial visualization of high school students. Thus, more research should be conducted.

The present study sought to use an intervention to increase the spatial visualization skills of high school geometry students. The present study also sought to address the gap in spatial visualization between genders in high school geometry students. Lastly, this study sought to compare the use of technology tools during instruction to instruction with concrete, tangible models to enhance spatial visualization skills of high school geometry students.

Purpose of the Study

The first purpose of this study was to investigate and compare teaching methods aimed at increasing spatial visualization skills of high school geometry students using multiple interventions. This study also examined whether there was a gap in spatial visualization skills based on gender. Finally, the purpose of this study was to compare the use of technology enhanced instruction to instruction with concrete, tangible models to facilitate spatial visualization skills of high school students.

Research Questions

The research questions that guided this study helped the researcher determine if there was a gender gap in the spatial visualization ability of high school geometry students and compare technology interventions with concrete, tangible models for improving spatial visualization skills of high school geometry students. The research questions that were addressed by this study included:

1. Which learner characteristics predict spatial visualization as measured by the Revised Purdue Spatial Visualization: Rotations Test?
2. To what extent does geometry instruction using technological tools increase the spatial visualization skills of high school geometry students?

3. To what extent do concrete, tangible models used during geometry instruction without technological tools increase the spatial visualization skills of high school geometry students?

4. Which method of instruction has a more significant impact on students' spatial visualization skills, instruction using technological tools or instruction that does not use those tools?

Hypotheses

The hypothesis for the first research question was that students who have more spatial based experiences or are males will have higher spatial visualization skills than those who do not. Those other spatial experiences include involvement in sports, playing musical instruments, playing video games, playing with building blocks as a child, course preference (Algebra or Geometry), and the time of day the students have geometry. Quaiser-Pohl & Lehmann (2002), Julia & Antoli (2018), and Ogunkola & Knight (2019) found males outperform females in spatial visualization. Harris et al. (2021) found mixed results of male and female performance in spatial visualization. Fennema & Sherman (1977) and Tzuril & Egozi (2010) found females outperform males in spatial visualization. Logan & Lowrie (2017) found no significant difference in gender in spatial visualization. Thus, the hypothesis for this study was that students with previous spatial experiences or who are male will have higher levels of spatial visualization than students who do not have the previous spatial experiences or who are female.

The hypothesis for the second research question focused on the extent to which geometry instruction using technological tools increased the spatial visualization skills of high school geometry students. According to McClintock (2002), "The results of the

research suggest that GSP and the associated activities were effective in helping the students develop three-dimensional visualization and achieve conceptual understanding of geometry content” (p. 743). Thus, the hypothesis for the second research question was that the spatial visualization scores of the students using technology will be higher at the end of the intervention than they were at the beginning.

The hypothesis for the third research question focused on the extent to which concrete, tangible models used during geometry instruction without technology increase spatial visualization of high school geometry students. According to Baki et al. (2011), “Results of the study showed that PSVT scores of the students with physical manipulatives in the Manipulatives Group were significantly higher at the end of the semester than at the beginning” (p. 304). Thus, the hypothesis for the third research question was that spatial visualization scores for the students using concrete, tangible models without technology would be higher than they were before the instructional unit.

The hypothesis for the fourth research question focused on which method of instruction would have a more significant impact on the spatial visualization scores of high school geometry students, the instruction with technology or the instruction with concrete, tangible models. According to Baki et al. (2011), “We obtained the unexpected result that there was a significant difference between the Computer Group and Manipulatives Group in the views section of PSVT” (p. 305). Thus, the hypothesis for the fourth research question was that instruction using technology would be more effective for enhancing the spatial visualization skills of high school geometry students than instruction with concrete, tangible models.

Significance

The significance of this study was that it sought to combine multiple aspects of previous literature to determine if claims in recent research apply to current tenth grade high school geometry students. It also aimed to investigate different interventions each with the goal of increasing spatial visualization of skills in high school geometry students. Lastly this study explored any gender differences in the spatial visualization of high school geometry students.

Assumptions and Delimitations

The participants in this study were high school geometry students enrolled in tenth grade geometry with the researcher. One assumption of this study was that the participants understood the language used within the Revised Purdue Spatial Visualization Test: Rotations (RPSVT:R). Participants who did not understand all the terminology used on the assessment would have a hard time answering questions correctly. Another assumption was that all students would put forth their best effort on every assessment and activity in which they participated. At times, effort levels matched moods resulting in greater effort or decreased effort into their work based on factors that occurred outside of the classroom.

One delimitation of this study was that only the researcher's students were used for this study. This limited generalizability to other high school geometry students in the researcher's school district because the sample only included participants from one suburban high school. The students in this school had greater access to experiences that allowed them to build their spatial visualization skills compared to students in rural schools, such as access to sports teams, music classes, and other extracurricular activities

that may not have been offered in rural areas. The classes used in this study were a representative sample of the students in this high school.

Another delimitation of this study was that all students came from diverse backgrounds affecting their levels of spatial visualization when they entered the class. Students who participated in band, athletics, or video games may have had higher levels of spatial visualization before the intervention than those who did not.

Definitions of Terms

The terms used in this research study that may be unfamiliar to those outside the realm of mathematics education are included in this section. Many terms have multiple definitions so the definitions relevant to this study are described below.

1. *Dynamic Geometry Software (DGS)*- “tools that one can use to create and support student-centered learning environments” (Hannafin et al., 2008, p. 148).
2. *GeoGebra*- “a free open-source dynamic software for mathematics teaching and learning that offers geometry and algebra features in a fully connected software environment” (Saha et al., 2010, p. 688).
3. *Geometer’s Sketchpad (GSP)*- “a dynamic geometry construction and exploration tool” (McClintock, 2002, p. 739)
4. *Quick block*- task “creating 3D cubic images, incorporating different colors, shapes, and orientations...to challenge our students’ sense of visual orientation, we designed images from different points of view... students have two opportunities to view the image then build what they saw” (Matney et al., 2020, p. 11-12).

5. *Quick draw*- “an engaging mathematical activity that helps students develop their mental imagery” (Wheatley, 2007, p. 1).
6. *Quick image*- “classroom routine that...promotes subitizing” (Matney et al., 2020, p. 11-12).
7. *Spatial ability*- “the innate ability to visualize that a person has before any formal training has occurred, i.e., a person is born with ability” (Sorby, 1999, p. 21).
8. *Spatial orientation*- “knowing where you are and how to get around in the world; that is understanding and operating on relationships between different positions in space, especially with respect to your own position” (Clements, 1998, p. 11).
9. *Spatial reasoning*- “consists of the set of cognitive processes by which mental representations for spatial objects, relationships, and transformations are constructed and manipulated” (Clements & Battista, 1992, p. 1).
10. *Spatial skills*- “skills learned or acquired through training” (Sorby, 1999, p. 21).
11. *Spatial structuring*- “mental operation of constructing an organization or form for an object or set of objects” (Battista, et al., 1999, p. 503)
12. *Spatial visualization*- “mentally moving an object” (Sorby, 1999, p. 22).
13. *Visual thinking*- “as in the first van Hiele level of geometric thinking, is thinking that is tied down to limited, surface-level, visual ideas” (Clements, 1998, p. 18).

CHAPTER II – LITERATURE REVIEW

This chapter begins with research that has shown interventions enhancing spatial visualization in a variety of students. The ages of the students ranged from elementary children up to pre-service teachers in college. Next, the chapter describes research that examined gender differences in spatial visualization. Lastly, the chapter reports research that focused on dynamic geometry software to improve spatial visualization skills of students. The dynamic geometry software programs included Cabri 3D, Geometer's Sketchpad, and GeoGebra.

Spatial Visualization

Patkin & Dayan (2013) conducted a case study to determine if an intervention unit would improve spatial ability in high school students. The purpose of the study was to examine the impact of an intervention unit on spatial ability in high school students. The research objectives included: “examine to what extent it is possible to improve 12th graders’ spatial ability by means of an intervention unit” and “examine to what extent the use of an intervention unit can improve gender-related performance and reduce gender difference in spatial ability.” (p. 183). The participants included 46 twelfth grade students divided into the experimental and control groups. The experimental group had 22 students, 7 girls and 15 boys. The control group had 24 students, 11 girls and 13 boys. The researchers began by giving a pretest to both groups, then the experimental group spent three weeks participating in the intervention unit based on a methodology by Sorby and Barrmans (2000) and a book for spatial visualization development combined with art. The control group participated in the regular curriculum during the time of the intervention. At the end of the intervention, a posttest was given to both groups. A two-

way ANOVA was used to analyze pre and posttest scores. The results indicated that the intervention unit improved spatial visualization abilities in the students (Patkin & Dayan, 2013).

Sorby et al. (2013) examined the role spatial training plays in spatial skills and calculus performance for engineering students. The purpose of the study was to explore the benefits of an intervention unit on freshman engineering students who failed the pretest on their spatial ability. “The study sought to determine whether the spatial intervention course would be effective in improving the spatial skills of freshmen engineering students” (Sorby, 2013 p. 22). The participants of the study included 675 freshmen students from Michigan Tech, 542 males and 133 females. All students were given the Purdue Spatial Visualization Test: Rotations (PSVT:R) as a pretest. Of the students who took the pretest, those who scored 18 or lower (60%) were required to enroll in a course designed to help them improve their 3D spatial skills. The students were then separated into intervention and comparison groups. Students who scored 18 or lower were placed in the intervention group and the students who scored 19 or higher were assigned to the comparison group. The course for spatial skills development included a workbook and multimedia software. Each session started with a mini lecture on the topic and was followed by modules in the software for students to work individually or in small groups. After completing the modules, students completed workbook pages. A Regression Discontinuity was used to analyze the data. The results indicated that the intervention was effective in raising posttest scores compared to those who had no intervention (Sorby et al., 2013).

Cheng & Mix (2014) investigated the impact spatial training has on mathematics ability in young children. The purpose of their study was to determine if spatial training had an impact on the mathematics ability of young children. The research question addressed by this study was, “Can improved spatial ability improve mathematical ability?” (Cheng & Mix, 2014 p. 4) The participants in the study included 58 children ages 6-8 years old from a predominantly Caucasian middle-class school in Michigan. Researchers randomly assigned participants to the treatment group or spatial training group and the others were assigned to the control group. There were 31 students in the treatment group and 27 students in the control group. This included 17 boys and 14 girls in the treatment group and 17 boys and 10 girls in the control group. The researchers gave three pretests, two of which were spatial tests (Mental rotation test and Spatial relations subtest) and the third was a math test. Then the students completed a 40-minute training session that was followed by three posttests. The students in the treatment group received a session of mental rotation practice that involved students looking at two parts of a flat shape and identifying which one of four pictures represented the shape. The students in the control group did crossword puzzles. A multivariate analysis of covariance was conducted to compare posttest results to pretest results. The results indicated that there was a direct effect of spatial training on math performance in young children (Cheng & Mix, 2014).

Chang et al. (2016) explored interactive learning for spatial geometry using mobile devices. The purpose of their study was to facilitate the learning of geometry using a hands-on spatial geometry learning system. The research question for the study asked how effectively the Hands-On Learning by Doing geometry system (HOLD)

enhanced the ability of high school students to think geometrically. The participants in the study included 58 high school students who were randomly assigned to experimental or control groups. The students were from Taipei City. The experimental group had 31 students of which 19 were boys and 12 were girls. The control group consisted of 27 students of which 13 were boys and 14 were girls. The average age of the students was 18 years old and in the 12th grade. Participants had spatial geometry classes the previous year. The researchers used a pretest and posttest to collect data before and after the intervention. The treatment group participated in the hands-on learning activities while the control group participated in lecture-based instruction. Both groups had the same teacher to help control the variable of teacher impact. The experiment was conducted three times for 85 minutes each. An analysis of covariance was used to analyze the data for effects of various learning outcomes. The results were divided into four sections: perceptual apprehension, sequential apprehension, operative apprehension, and overall spatial geometry scores. For perceptual apprehension the results indicated no significant difference for the experimental or control group. Sequential apprehension was developed by students in the experimental group. Operative apprehension scores were the lowest of all categories in this study. There was a significant difference between the experimental and control groups for overall geometry scores which showed that HOLD facilitated spatial geometry teaching (Chang et al., 2016).

Lowrie, et al. (2019) observed that spatial visualization can predict mathematics performance in elementary and middle school aged children. The purpose of their study was to determine if a spatial visualization intervention would increase students' spatial reasoning and mathematics performance. The researchers "hypothesized that spatial

training would improve performance on math problems” (Lowrie et al., 2019, p. 733).

The participants of the study included 327 fifth and sixth grade students from ten primary schools in rural Australia. This included 93 boys and 84 girls from nine intact classes for the intervention group and 79 boys and 71 girls from a separate eight classes for the control group. Over 85% of students spoke English as their first language. The schools selected for this study were pulled from broad sociodemographic areas. The researchers used a Spatial Reasoning Instrument (SRI) to measure spatial reasoning and a mathematics test developed from items on Australia’s National Assessment Program (NAPLAN) Numeracy Test. The teachers from the schools participating in the study chose to be a part of the intervention or control group. The study took place during the fall semester of the school year during regular school hours. Students took the SRI and the mathematics test one week before the three-week intervention began as a pretest and again one week after the intervention as a posttest. The intervention included lessons implemented in 60-minute class periods twice a week. Students learned about reflections, symmetry, paper folding and cutting, nets of solids, hidden blocks, and cross sections of 3D objects. Digital tools that were used in the implementation of these lessons included Symmetria: Path to Perfection, Illuminations: Cube Nets, and GeoGebra. The control groups’ activities were taken from the teachers’ regular curriculum program. Scores on the pretest and posttests were compared using hierarchical linear models. The three-week program showed significant gains for the intervention group’s math performance compared to the control group. The program focused on spatial visualization making the intervention groups’ gain scores improve across two out of three constructs compared to the control groups’ gain scores (spatial visualization and spatial orientation gain scores).

The students in the intervention group showed significant performance increases on the math test compared to the control group as well. This study supports shorter intervention programs and a focus on spatial visualization to improve mathematics performance as well as other spatial constructs (Lowrie et al., 2019).

The research in this section showed the impact of various intervention units on spatial visualization skills of a variety of students. In each study, the intervention unit was successful in improving students' spatial visualization skills. The types of students across each of these studies included young children, elementary and middle grades, high school students, and engineering students in college. The variation of students across the studies showed that spatial visualization skills can be improved through intervention at any grade level. Some interventions included technology and others used physical manipulatives, but both types were successful in improving spatial visualization skills of students.

Gender Differences in Spatial Visualization

Males outperform females

Fennema & Sherman (1977) researched differences among males and females in math achievement and spatial visualization. The purpose of their study was to get new insight into gender related differences in math achievement when the variable of previous mathematics study was controlled. The participants in the study included 1,233 students in grades 9-12 in Wisconsin. Of these students, 589 were female and 644 were male. Students who were in grade level mathematics courses were used in this study to control for mathematics background. Students were given the Quick Word Test, Test of Academic Progress, Space Relations Test of the Differential Aptitude Test, measures of Math Related Courses and Space Related Courses, the Fennema-Sherman Mathematics

Attitude Scales, and a scale measuring Mathematics Activities. An analysis of variance was used to analyze the data. The results showed that students with similar mathematics backgrounds showed no significant differences between male and female groups in mathematics achievement. Males scored higher on spatial visualization. Females reported fewer positive perceptions of attitude towards mathematics. The study also found that socio-cultural factors played a major role in sex-related differences in mathematics achievement and spatial visualization. (Fennema & Sherman, 1977).

Ben-Chaim et al. (1988) studied gender differences and the effect of instruction on spatial visualization. The purpose of the study was to address concerns in the literature of administering an instructional unit to students to improve spatial visualization. The research questions addressed by this study were: “Do sex and grade level differences exist, before instruction, in spatial visualization ability? Will instruction in spatial visualization tasks affect the spatial visualization performance of students in grades 5, 6, 7, and 8? Will these effects be different for boys than for girls? Will these effects differ by grade level? Do these effects of instruction on spatial visualization skills persist over time for each grade level and sex?” (Ben-Chaim, et al., 1988, p. 53-54). The participants of the study included approximately 1,000 students and 21 teachers over three locations in a midwestern city. The first location was an inner-city school that had 65% white, 23% black, and 10% Latino students at the time. The second location was a rural middle-class school that was predominantly white students. The third location was a suburban middle school that served upper middle-class university and state government professionals and was also predominantly white. All students were classified as being in middle grades. A Spatial Visualization Test (SVT) from the Middle Grades Mathematics Project (MGMP)

was given as a pretest. A Spatial Visualization Unit was taught during the regular math class over a three-week period. Then the posttest was given after instruction was over. The results indicated differences in spatial visualization across grade levels as expected for age and maturity. Before the instructional unit there were differences in spatial visualization that showed boys outperforming girls. After the instructional unit all students showed significant gains in spatial visualization with boys outperforming girls (Ben-Chaim et al. 1988).

Quaiser-Pohl & Lehmann (2002) focused on the spatial abilities of girls through undergraduate students from a range of majors. The purpose of their study was to highlight differences in spatial abilities among a variety of academic subgroups as well as the role experience and attitude play in spatial ability for males and females. The research questions addressed were: “Do the members of these academic subgroups remarkably differ in their spatial abilities? Do these academic groups differ in experiences and attitudinal variables that could be related to the differences in spatial ability? How are experiential as well as attitudinal variables and spatial abilities related to each other and does this relationship differ between males and females?” (Quaiser-Pohl & Lehmann, 2002, p. 248). The participants of the study consisted of 183 (112 females and 71 males) freshmen from Otto-von-Guericke University in Magdeburg, Germany. All students were enrolled in various degree programs including 53 majoring in arts, humanities, and social sciences (38 female and 15 male), 19 majoring in sports (11 female and 8 male), 69 majoring in psychology (51 female and 18 male) and 42 majoring in computational visualistics (12 female and 30 male). The students were given the Mental Rotation Test to assess mental rotation ability, the Spatial Activities Questionnaire to indicate the

frequency in which they participated in leisure activities that required spatial ability, the Attitudes towards mathematics and physics questionnaire, the Spatial Abilities in Everyday Life questionnaire and the Computer experience questionnaire to ask students if they gained their computer skills formally in school or informally at home. To analyze the results of the Mental Rotation test, an analysis of variance was used to determine significant effects for gender that favored males over females. When analyzing data from the academic subgroups, there were no differences in spatial ability. For actual life experiences and attitude there was some importance in that the computer experience enhanced performance on technical activities (Quaiser-Pohl & Lehmann, 2002).

Julia & Antoli (2018) examined spatial ability and mechanical reasoning in STEM courses. The purpose of their study was to evaluate if participation in a STEM course improved the spatial ability of students. The research questions addressed were: “Do the students that attend the STEM course improve their spatial ability? Do the participants in the STEM course enhance their mechanical reasoning? Do these results depend on the gender of the participants? Do these results depend on the grade level?” (Julia & Antoli, 2018, p. 960). The participants included 26 students (15 boys and 11 girls) in sixth grade and 24 students (11 boys and 13 girls) in seventh grade. These students were from a small city in Spain. The students participated in a STEM course that was taught one hour per week for 26 sessions for seventh grade and 28 sessions for sixth grade. Students were given a pretest to determine spatial ability and another pretest to determine mechanical reasoning. During the course, students participated in hands-on activities to solve problems. The results indicated that the spatial ability of the students

improved for those enrolled in the STEM course and that male participants outperformed female participants (Julia & Antoli, 2018).

Ogunkola & Knight (2019) investigated the spatial ability of high school students in Barbados. The purpose of the study was to determine the effect of a technical drawing course, video games, gender, and type of school on the spatial ability of secondary students. The participants in the study included students ages 13-15 from nine public schools in Barbados. All students were from a range of lower to upper class families. There was a total of 420 students participating in this study (269 boys and 151 girls). Each of the students selected had completed two years of a technical and vocational program but had not been exposed to technical drawing. The Spatial test battery (STB) was used to collect information regarding gender and whether or not students played video games. The tests given were modified versions of the Mental Rotation Test to measure spatial visualization ability and the Object Perspective Test to measure spatial orientation. After the pretest was given to both groups, the technical drawing course was implemented for 10 weeks with the treatment group. This included students reproducing sketches or drawings of two- and three-dimensional figures each week. The students in the control group participated in their regular classes. Then the STB was administered again as a posttest to both the treatment and control groups. A four-way analysis of covariance and multiple classification analysis were used to analyze the data. The results indicated that the spatial ability of the students enrolled in the technical drawing class. The results also showed boys outperforming girls (Ogunkola & Knight, 2019).

Females showed greater improvement than males

Tzuriel & Egozi (2010) examined gender differences in the spatial ability of first grade students. The purpose of their study was to determine whether there were gender differences in the spatial ability of first grade students. The research questions addressed were: “Do young children in first grade show gender differences on mental rotation tasks at an appropriate level of difficulty for their age? To what degree does training of representation and transformation of perceptual stimuli, by viewing them from different perspectives reduce initial gender differences? Is the magnitude of the hypothesized gender differences influenced or moderated by global versus local strategies of representing and transforming visuospatial information? What are the interactive effects of gender, training, and task difficulty (degree of rotation and test level) on performance of mental rotation tasks?” (Tzuriel & Egozi, 2010, p. 1418). The participants in the study included 116 first grade students (58 boys and 58 girls) from Israeli public schools. Students were randomly assigned to the experimental and control groups. The experimental group had 60 students (30 boys and 30 girls) and the control group had 56 students (28 boys and 28 girls). Three different assessments were given to measure students’ spatial ability. The Spatial Relations (SR) subtest from the Primary Mental Abilities- Children’s Version was used to measure the ability to rotate objects in two-dimensional space. The Windows Test (WT) based on the Mental Rotation test has three levels of difficulty (WT1, WT2, WT3) which allows for students’ responses to be analyzed separately based on the level of the test. The Global-Local Process Strategies (GLPS) was implemented to assess visual-spatial processing styles. The intervention program was designed to promote spatial sense in students through the mathematics

curriculum. This program was based on Wheatley's (1996) "Quick Draw" activities. The program consisted of eight 45-minute sessions given once a week in small groups. Students were given the assessments as a pretest then students participated in the intervention program for three months. The students in the experimental group participated in the Spatial program and the control group participated in a substitute program that focused on pictographic and fine motor skills without teaching, guidance, or group discussion. Posttests were given after the intervention. The data from the study included the scores from the three different pre- and posttests. The results indicated that the treatment group showed a significant improvement in spatial ability compared to the control group. The gender comparison showed that girls' improvement in the experimental group was greater than the improvement of boys. In the control group, there were minor improvements, but the boys showed a slight improvement over the girls (Tzurriel & Egozi, 2010)

Jackson et al. (2015) investigated spatial development in sixth grade students with respect to gender and race. The purpose of their study was to examine two groups of sixth grade students' scientific knowledge and geometric spatial visualization (GSV). The research questions addressed were: "To what extent do sixth-grade middle school students' GSV change after participating in an integrated Earth/Space unit? What gender differences exist in GSV between students as they reason and consider lunar phases within and between control and experimental groups? What differences exist in GSV among sixth-grade middle-level students from different racial backgrounds as they reason and consider lunar phases between control and experimental groups?" (Jackson et al., 2015, p. 331). The participants of the study included sixth grade students from two

middle schools in the south-central region of the United States. The students from the first school included 890 students in which 83% were White, 6% African American, 4% Hispanic, and 3% Other. The participants selected included 141 students from the first school in the experimental group and 65 in the control group. The students in the second school included 1,131 students of which 74% were White, 10% African American, 8% Hispanic, 5% Asian, and 3% Other. The participants from this school included 221 students in the experimental group and 28 students in the control group. The data collected were the results of three assessments: the Lunar Phases Concept Inventory, the Geometric Spatial Assessment, and the Purdue Spatial Visualization- Rotation Test. Students were given the assessments at the beginning of the study. Students in the control group participated in their regular Earth/Space curriculum while the experimental group participated in a NASA based curriculum called Realistic Exploration in Astronomical Learning that integrated math and science. Both groups studied the Solar System as part of their units. The instruction lasted four weeks in one school and nine weeks in the other. A repeated measures analysis of variance (RM-ANOVA) was used to analyze the data. The results indicated that students in the experimental group showed significant gains from pretest to posttest on GSV compared to the control group. The females in the control group showed significant gains from pretest to posttest (Jackson et al., 2015).

Females scored higher than males

Harris et al. (2021) examined the role of gender in mathematics performance and spatial reasoning. The purpose of the study was to examine the contributions of mental rotation, spatial visualization, and spatial orientation on mathematics performance. The research questions addressed by the study were: “What are the unique contributions of

different spatial constructs, namely MR, SV, and SO, to mathematics performance for students in fifth- and eighth-grade cohorts? Specifically, does SO play a role in the spatial-math relationship, independently of MR and SV? Does gender influence the relationship between spatial reasoning and mathematics at two middle-school time points?” (Harris, et al., 2021, p. 415). The participants of the study included 84 fifth-grade students from four classes in one school in Canberra, Australia. The students ages ranged from 10-12 years old. Of the 84 students in the study, 43 were female and 41 were male. Students were given the Math T as an assessment of mathematics content and a Spatial Reasoning Instrument (SRI) to measure spatial reasoning. All scores were dummy coded for gender. The results indicated no significant gender differences on the MathT and SRI measures using t-tests. When each aspect of spatial ability was examined, males outperformed females in mental rotation. There were no significant differences for spatial visualization. A second study was conducted with 903 eighth-grade students from nine schools in Canberra, Australia who ranged from 12-15 years of age. Students in the study also took the MathT for content knowledge. A variety of spatial tests were given which included The Card Rotation Test, Paper Folding Test, the Perspective-Taking Test, and the Verbal Classification Test. The results indicated no significant gender differences on the MathT. When examining the spatial results, there were significant gender differences for SV in favor of females and the differences in SO favored males. There were no significant differences for MR. Females scored significantly higher than males on the verbal reasoning test (Harris et al., 2021).

No significant difference for gender

Logan & Lowrie (2017) examined gender differences on spatial tasks of a national assessment. The purpose of the study was to examine summative assessment results of spatial numeracy tasks over time with a distinct focus on gender. The research questions addressed were: “Is there a relationship between performance on spatial tasks and performance on other mathematics tasks for males and females? Are there gender differences in performance on spatial tasks? What do the participant responses tell us about cohort behaviors and ways of processing?” (Logan & Lowrie, 2017, p. 207). The participants included a group of students from Australia who were studied over time. The participants who completed the third-grade assessment were 9,415 third graders (4,622 females and 4,793 males). Two years later the fifth-grade assessment tested 4,347 students (2,061 females and 2,286 males). Then, two years later the seventh-grade assessment tested 19,044 students (9,533 females and 9,511 males). The assessment given was the Australian National Assessment Program-Literacy and Numeracy (NAPLAN). The Knowledge Discovery Data (KDD) process was used to gather information from this set of data. The researchers identified questions that required spatial manipulations including mental rotation, spatial orientation, and spatial visualization. Independent samples t-tests were used to identify differences in scores between males and females. The results indicated no significant gender differences for spatial visualization items except for small differences at the seventh-grade level. There were some differences between males and females on spatial orientation items with males scoring higher (Logan & Lowrie, 2017).

The research in this section compared gender differences in spatial visualization over a variety of age groups and in a variety of courses. The studies reported had participants as young as first grade up to college freshmen. The study that observed first graders showed girls outperforming boys in spatial visualization. All the other studies from upper elementary school, middle school, high school, and college showed boys outperforming girls in spatial visualization. For each of these studies, spatial visualization was facilitated through a variety of subjects including mathematics, Earth/Space science, STEM, and technical drawing. Each of the subject areas used in these studies showed gains in students regarding spatial visualization, but boys outperformed girls. The research showed girls making improvements in spatial visualization after participating in the various treatments, but girls did not make as significant of gains as boys.

Table 1 *Summary of Literature on Gender Differences*

Males outperforming females in spatial visualization	Females showed greater improvement than males	Females scored significantly higher than males	No significant difference for gender
Fennema & Sherman (1977)	Tzuriel & Egozi (2010)	Harris et al. (2021)	Logan & Lowrie (2017)
Ben-Chaim et al. (1988)	Jackson et al. (2015)		
Quaiser-Pohl & Lehmann (2002)			
Julia & Antoli (2018)			
Ogunkola & Knight (2019)			

Other Spatial Experiences

There are many factors that can influence spatial visualization skills outside of gender and classroom instruction. Those factors are classified in this study as other spatial experiences. For this study other spatial experiences are defined as playing sports, playing musical instruments, playing video games, and playing with building blocks as a child. The purpose of this consideration of other spatial experiences is to “...explore the environment that has shaped the cognitive collection of students’ spatial reasoning skills... to help identify critical life experiences where spatial skills appear to be trained” (Gold et al. 2018, p. 669).

Playing Sports

Playing sports has been shown to help improve students’ spatial ability. Athletes often calculate or interpret the path of a ball or puck with the intention of swinging a bat or stick to make contact or calculate the path of putting a ball through a goal post or into a net on a regular basis. According to Pietsch & Jansen (2012), “It is evident that sports students who have been training since the age of 14 have a higher motor performance than those students who have not.” (p. 162). Their study examined the effects of long-term training in sports or music of students who are not professional athletes or musicians. The participants of the study included 120 students from one university. Of those students 60 were male and 60 were female. The students were in music (20 male, 20 female), sports (20 male, 20 female), and education science (20 male, 20 female). Each participant was given a questionnaire to tell how much time they spent practicing. Participants were given a cognitive processing speed test and mental rotation test to assess differences in cognitive process speeds and spatial visualization skills. Two

analyses of variance were used to analyze the data. According to Pietsch & Jansen (2012), “This study has shown that students of sports and music demonstrate a better performance on mental rotation tasks when compared to students of education science who did not have additional sports or musical training” (p. 161).

Ozel et al. (2002), conducted a study to observe the relationship between participation in sports and spatial skills of the participants. The participants of this study included 36 males who were divided into three subgroups. One subgroup included gymnasts, the second subgroup included athletes, and the third group consisted of nonathletes. Participants were given mental rotation items on the computer. Matlab computer programming was used to assess the participants. The results “confirmed the hypothesis that athletes, whatever their discipline, perform better than nonathletes” (Ozel et al., 2002, p. 1150). According to Ozel et al. (2002), “This relation can be explained by the extensive use of the processes of transformation imagery during sport activities to improve their efficiency” (p. 1150). According to Ozel et al. (2002), “This study provides some support to the idea that the regular practice of spatial activities, such as sports, could be in relation to the mental spatial capacities of participants” (p. 1151). These types of experiences help to improve mental rotation and spatial visualization skills in students.

Playing Musical Instruments

According to Pietsch & Jansen (2012), “Playing a musical instrument might have positive physical and cognitive effects” (p. 162). Playing instruments while reading sheet music requires spatial ability because the eyes are looking at the music while the hands and feet are operating the instrument separately. According to Gaser & Schlaug (2003), “Musicians repetitively practice this visual-spatial to motor transformation by reading

musical notation and translating it into motor plans accompanied by simultaneous auditory feedback that aids the matching of the visual patterns to the motor program” (p. 9243). This repeated exposure to these spatial skills helps increase spatial visualization in students who play musical instruments.

Playing Video Games

Playing video games gives students the opportunity to virtually manipulate objects and move them in a virtual space. According to Cherney (2008), “The present study confirms that computer game practice may differentially improve the scores of males and females in mental rotation” (p. 784). The skills acquired from playing computer games or video games can have lasting effects for spatial visualization skills. According to Terlecki et al. (2008), “The greater spatial experience experienced by the videogame training group led to transfer effects that were maintained over several months (p. 1009). More specifically, action games improve spatial visualization skills as students are participating in virtual activities. According to Feng et al. (2007), “Non-action games may be less likely to have a beneficial effect because they do not sufficiently exercise spatial attentional capacities” (p. 853).

Playing with Building Blocks as a Child

Playing with blocks gives children the opportunity to manipulate objects to build anything they can imagine. This helps them to develop spatial visualization skills as they build and interact with their objects from different sides. Gold et al. (2018) examined spatial skills in 345 geology students with consideration for previous experience with construction-based blocks or video games in childhood. According to Gold et al. (2018), “Our data suggest a significant positive association between frequent play with

construction-based toys and spatial skills” (p. 677). Brosnan (1998) conducted a study examining spatial ability in children using Lego blocks. Participants were asked to build a three-dimensional model of a bridge as part of the intervention. According to Brosnan (1998), “The finding that those who finished the model also scored higher on spatial ability than those who did not complete the model suggests support for a relationship between children’s play with LEGO blocks and their spatial ability” (p. 25).

Playing sports, musical instruments, video games, or playing with blocks each have ways of impacting spatial visualization skills through repeated exposure. These experiences combined showed the potential for increasing spatial ability and spatial visualization skills. Playing sports, musical instruments, and video games helps students build mental rotation skills as they interact with others or with balls or instruments. The video games provide opportunities to navigate virtual worlds which helps students see images and figures from a variety of viewpoints which helps build mental rotation skills as well. Playing with blocks gives children the hands-on experience with physical objects to manipulate and build this allows them to begin building their spatial skills which can be carried forward as they get older and then applied in classroom settings.

Dynamic Geometry Software

Cabri 3D

Baki et al. (2011) described the impacts of using Dynamic Geometry Software (DGS) and physical manipulatives on spatial visualization skills. The purpose of their study was to examine the effects of using DGS Cabri 3D and physical manipulatives on pre-service mathematics teachers’ spatial visualization skills. The research questions included: “Does DGS-based instruction help the students to improve their spatial

visualization? Does physical manipulative-based instruction help the students to improve their spatial visualization skills? Is there a significant difference among groups related to students' spatial visualization skills?" (p. 295). The participants included 96 pre-service mathematics teachers in their first year of college. All students were Elementary Education majors at the Karadeniz Technical University. All students had taken Solid Geometry during the 2007-08 school year. The pre-service teachers were randomly assigned to one of three groups. The groups included DGS-based instruction (Computer Group), physical manipulative-based instruction (Manipulative Group), and the traditional instruction-based group (Traditional Group). The Computer Group had 34 students, the Manipulative Group had 32 students, and the Traditional Group had 30 students. Data was collected using the Purdue Spatial Visualization Test (PSVT) as the pretest and posttest. During the intervention, students in the Computer Group were instructed in the computer lab where they studied solid geometry topics using Cabri 3D. The students in the Manipulatives Group studied solid geometry in the classroom with the use of physical manipulatives. Each group used a total of 21 worksheets to accompany their activities during instruction. The Traditional Group received teacher-centered instruction in which the teacher would explain topics and draw on the board. All groups covered the same topics with the same teacher over the course of ten weeks. The results indicated a significant difference in spatial visualization skills in the Computer and Manipulative Groups who both outperformed the Traditional Group. An ANOVA was used to analyze the scores between the groups. The findings showed that spatial visualization was significantly improved when using DGS and physical manipulatives (Baki et al., 2011).

Geometer's Sketchpad

McClintock et al. (2002) studied the development of three-dimensional visualization using Geometer's Sketchpad (GSP). The purpose of their study was to examine the effects of GSP on middle and high school students in three-dimensional visualization. The research questions addressed were: "What role can the GSP dynamic instructional environment play in the development of students' three-dimensional visualization? Is there evidence to indicate improvement in the students' three-dimensional spatial ability when GSP is used to teach three-dimensional geometry? What role can the GSP dynamic instructional environment play in the development of students' geometric thinking as defined by the van Hiele theory? Is there evidence to indicate improvement in students' geometric thinking when GSP is used to teach three-dimensional geometry?" (McClintock, et al., 2002, p. 740). The participants included 24 students over a four-year period. The students were part of the Partnership Academic Community (PAC) program which was a collaboration between Florida International University (FIU) and Miami-Dade County Public Schools. Through the partnership, a program was created for at-risk minority students in Miami. These students are African American and Hispanic from lower socio-economic families in middle and high schools. Students were engaged in problem solving in geometry using GSP constructions, investigations, and experiments. The students were in seventh grade at the beginning of the study and participated through tenth grade. Instructional sessions were provided to engage students in hands-on explorations using GSP, then students had to make conjectures based on what they learned in the exploration and be able to fully explain their conjectures in meaningful ways. The study was qualitative in nature, so detailed

notes were taken during each of the sessions. Some sessions were recorded and transcribed. A constant comparative approach was used to analyze the data. The results indicated that GSP was effective in helping students develop three-dimensional visualization skills and develop a conceptual understanding of geometry content.

Idris (2007) studied the use of Geometer's Sketchpad (GSP) on geometry achievement and van Hiele levels of geometric thought. The purpose of the was to examine van Hiele levels of geometric thinking using GSP. The research questions explored the effects of GSP on geometry achievement and the effects of GSP-based instruction on van Hiele levels of thinking. The participants included 65 Form Two students in a secondary school in Kuala Lumpur. Thirty-two students were placed in the treatment group and participated in ten weeks of instruction using GSP. The other 33 students were placed in the control group and were instructed using a traditional approach. The students in both groups were given a van Hiele Geometry Test (VHGT) to assess their levels of geometric thought as a pre and posttest. Then students were given a questionnaire to determine their response toward the use of GSP. The activities for the treatment group included the use of GSP in which students explore, investigate, discover, etc. to help improve their mathematical understanding. Descriptive statistics of frequencies were used to compare difference in students' van Hiele levels of thinking for both the treatment and control groups. The results indicated significant differences in geometry achievement for the treatment group compared to the control group. The use of GSP can help move students to a higher level of geometric thinking.

Hannafin et al. (2008) examined how spatial ability impacts geometry achievement using GSP. The purpose of their study was to assess the effects of spatial

ability and type of instructional program implemented on geometry achievement. The study examined three hypotheses: “Students in the Sketchpad treatment group will score higher on the posttest than will students working in the Tutorial program; high-spatial students will outperform low-spatial students on the posttest and particularly in the Sketchpad treatment; and students in the Sketchpad condition will like the instruction better than will students in the Tutorial group” (p. 150). The participants included 66 sixth-grade students of which 27 were boys and 39 were girls. These students were enrolled in four different classes in a middle-class suburb close to a northeastern university. The study took place during regular mathematics classes. The students were randomly assigned to a treatment group, referred to as the Sketchpad Group and a control group, referred to as the Tutorial Group in the study. Students were given the Raven’s Colored Progressive Matrices as a pretest to measure spatial problem-solving skills and a geometry pretest to assess content knowledge. The Sketchpad Group participated in an online geometry program and used a booklet of activities to guide them using Geometer’s Sketchpad. The Tutorial Group only participated in the online geometry program. Upon completion of the intervention, students were given a posttest and an attitude questionnaire. An analysis of covariance, ANCOVA was used to assess the posttest data of the instructional program used and spatial ability levels and used a multivariate analysis of variance, MANOVA across the attitude items to see how the program used impacted students’ attitudes. The results indicated that the students in the Sketchpad Group did not score higher on the posttest than students in the Tutorial Group. Students who were classified as high-spatial students did outperform low-spatial students on the

posttest. Students who participated in the Sketchpad Group did not prefer their instruction to the Tutorial group, possibly because it required more work (Hannafin et al., 2008).

Tieng & Eu (2014) used GSP to assess geometric thinking in young students. The purpose of their study was to explore the use of GSP on students' van Hiele level of geometric thought. The following research questions were addressed: "Is there any significant difference in the students' van Hiele level of geometric thinking between experimental group and control group before the intervention using GSP?, Is there any significant effect of using the GSP on students' van Hiele level of geometric thinking on the topic of angles?, Is there any significant effect of using the traditional approach on students' van Hiele level of geometric thinking on the topic of angles?, Is there any significant difference in the students' van Hiele level of geometric thinking between experimental group and control group after the instruction using GSP?, Is there any correlation between students' ICT literacy and their van Hiele level of geometric thinking?" (Tieng & Eu, 2014, p. 21). The participants of the study included 31 year three students (age 9) from intact classes in a primary school in Pahang, Malaysia. The students were randomly assigned to the treatment and control groups. The treatment group was instructed using Geometer's Sketchpad and the control group was instructed using traditional instruction. Three different sets of pretests and posttests were used to determine students' van Hiele levels of geometric thinking. Each pretest and posttest pair was designed to assess geometric thinking in different areas. One was assessing knowledge of right angles, the next was knowledge of acute angles, and the third was knowledge of obtuse angles. The pretests were given at the beginning of the study to get baseline levels of geometric thinking. Then students in the experimental group were

instructed using GSP for two activities in which they identified properties of angles while the students in the control group completed the same tasks without GSP. Then the posttest was given after the implementation of these activities. The data collected included students' van Hiele levels of geometric thought. These scores were analyzed using a t-test and an analysis of variance, ANOVA to address each of the research questions. The results indicated no significant differences between the two groups for pretests 1 and 2, but the results for pretest 3 showed a significant difference between the two groups with the experimental group outperforming the control group. This indicated that using the GSP helped students improve their van Hiele level of geometric thought (Tieng & Eu, 2014).

GeoGebra

Saha et al. (2010) studied the effects of GeoGebra on mathematics achievement. The purpose of their study was to investigate the effects of GeoGebra on learning Coordinate Geometry in secondary students. The research objectives for this study were: "To identify differences in the mean posttest scores between students utilizing GeoGebra and conventional instruction; to identify differences in the mean posttest scores of students utilizing GeoGebra and conventional instruction among high visual-spatial ability (HV) students; To identify differences in the mean posttest scores of students utilizing GeoGebra and conventional instruction among the low visual-spatial ability (LV) students" (Saha et al., 2010, p. 689). The participants included 53 students who were ages 16-17 and who resided in Kuala Lumpur. The students were assigned to two groups, the GeoGebra Group and the Conventional Group. The GeoGebra Group had 27 students and the Conventional Group had 26 students. Within each group students were

categorized by spatial ability, specifically high visual-spatial ability (HV) or low visual-spatial ability (LV). Students were given the Spatial Visualization Ability Test Instrument (SVATI) to determine if they classified as HV or LV. Students in the GeoGebra Group were instructed on Coordinate Geometry using GeoGebra software while the Conventional Group received instruction on the same topics through a traditional teacher-centered approach. The data were analyzed using a factorial design to determine changes in HV and LV within the GeoGebra Group and Conventional Group. The results indicated a significant difference between the scores of the two groups with the GeoGebra group outperforming the Conventional Group. This supported the notion that technology used as part of instruction is more effective than traditional instruction alone.

The research shared in this section showed three different types of Dynamic Geometry Software (DGS); Cabri 3D, GSP, and GeoGebra and their uses to improve spatial visualization, van Hiele levels of geometric thought, and mathematics achievement. These studies were mostly conducted with middle and high school students, but there were some that included elementary students and preservice teachers. Third grade students showed advancements in their van Hiele levels of geometric thought with GSP while 6th grade students did not show improvements with the use of GSP. The use of GSP did improve spatial visualization for middle and high school students and advanced the van Hiele levels of geometric thought of high school students. Cabri 3D was shown to improve spatial visualization in pre-service teachers. GeoGebra increased mathematics achievement in secondary students. Each form of DGS was shown to increase student achievement at all age ranges, except for sixth grade. This makes each tool beneficial for mathematics instruction with specific uses for enhancing spatial visualization.

CHAPTER III – METHODOLOGY

This chapter describes the theoretical framework and research design selected for this study. The purpose of this study was to increase the spatial visualization skills of high school geometry students using an intervention comparing technology use to instruction using concrete, tangible models. The research questions addressed by this study were: Which learner characteristics predict spatial visualization as measured by the Revised Purdue Spatial Visualization: Rotations Test? To what extent does geometry instruction using technological tools increase the spatial visualization skills of high school geometry students? To what extent do concrete, tangible models used during geometry instruction without technological tools increase the spatial visualization skills of high school geometry students? Which method of instruction has a more significant impact on students' spatial visualization skills, instruction using technological tools or instruction that does not use those tools? This chapter includes a description of the theoretical framework that guided the study, the research design, the purpose of the study, an explanation of the population, sample, and instrumentation used in the study. This chapter also includes the intervention, data collection, and data analysis, and methodological limitations of this study.

Theoretical Framework

The theoretical framework that influenced all decisions made for this study included a decomposition of spatial ability into its components. The discovery that spatial skills are malleable and can be developed in children lead to instructional decisions. Constructivist teaching principles led to the methods of instruction and activities selected for intervention. Technological Pedagogical and Content Knowledge (TPACK) and

Substitution, Augmentation, Modification, and Redefinition (SAMR) inspired technology integration into lessons.

Spatial Ability

Spatial ability guides people through their lives daily. Being spatially aware helps people navigate a three-dimensional world. Spatial ability is the innate ability to visualize or manipulate figures in the mind before any formal training occurs (Sorby 1999).

Spatial skills are the skills one can learn through activities or training. Spatial skills are classified as spatial visualization and spatial orientation. Spatial visualization is further classified as mental rotation and mental transformation groups (Sorby 1999). These skills help people navigate by car in unfamiliar places, plan alternate routes home when a regular route is blocked, give directions to others, or locate items at home.

Development of Spatial Skills

Spatial skills are the skills that can be learned and improved through instruction or other experiences. According to Stieff & Uttal (2015), “There is now considerable evidence that spatial ability is malleable- that a variety of experiences, ranging from life experiences to specific, intensive training, can improve spatial ability” (p. 609) Because spatial skills can be taught, it is important to know how they are developed. According to Bishop (1978), “Piaget and his colleagues theorize that individuals progress through a series of three stages in their understanding of spatial relationships, acquiring sequentially topological, projective, and Euclidean abilities” (p. 20). The first stage is topological relations which involves recognizing and classifying simple figures, putting objects in order, and recognizing their closeness to other things. This is typically developed around ages 3-5. The second stage is projective representation which is usually developed in

adolescence. This phase includes the ability to visualize what objects will look like from different perspectives. The third phase is Euclidean abilities which is gained in high school or beyond. This phase includes measuring and manipulating distance, length, area, volume, and angles (Bishop, 1978). There are several activities that can be useful to help increase spatial skills. According to Sorby (1999),

“Activities that have been found to develop spatial skills include: 1) playing with construction toys as a young child, 2) participating in classes such as shop, drafting, or mechanics as a middle school or secondary student, 3) playing 3-dimensional computer games, 4) participating in some type of sports, and 5) having well-developed mathematical skills” (p. 24).

Each of these activities can be conducted in the formal setting of a classroom or the informal setting of play at home. Thus, the variety of spatial experiences is unique to each child and contribute to such diverse spatial visualization skill levels among participants in this study.

Constructivism

Constructivism is a theory of learning that puts students at the center of their learning, as active participants in creating, constructing, and building new knowledge and new understandings. According to Van de Walle, Karp, & Bay-Williams (2014),

“At the heart of constructivism is the notion that learners are not blank slates but rather creators or constructors of their own learning. Integrated networks, or cognitive schemas, are both the product of constructing knowledge and the tools with which additional new knowledge can be constructed. As learning occurs the networks are rearranged, added to, or otherwise modified” (p. 26)

As students interact with and manipulate new ideas, they create new knowledge that builds off previous knowledge. Constructivism also requires students to reflect on their learning. Taking this time to reflect helps give them the opportunity to connect new ideas

to old ones or piece together new bits of information to help them make sense of what they are learning.

Constructivism incorporates a method of instruction that includes teaching through problem solving, using collaborative groups, employing technology, and using a variety of other techniques. According to Norton & D'Ambrosio (2008), "The implication in many cases is that these techniques facilitate students' constructions, which might otherwise be replaced by passive absorption" (p. 220). This suggests that students learn best by doing.

Technology Pedagogy and Content Knowledge (TPACK)

Technology, Pedagogy, and Content Knowledge (TPACK) is used to describe teachers' knowledge and understanding of how to integrate technology into instruction. There are three major components of TPACK according to Koehler & Mishra (2009). Those components include knowledge of content, knowledge of pedagogy, knowledge of technology, and the ways teachers provide instruction using the combination of these things. Content knowledge is the knowledge one has about the subject being taught such as Geometry, Algebra, etc. If the teacher does not know the content, they will certainly not be able to teach it. Pedagogical knowledge describes the knowledge the teacher has about the strategies and methods of instruction that will be best suited for their students. This requires an understanding of how students create meaning and gain new skills. Technology knowledge includes the ability to successfully implement technology in the classroom. This requires knowing which tools will help students learn current concepts and which tools may hinder learning.

The most important aspect of this TPACK model is where the three main aspects of a teacher's knowledge converge. According to Koehler & Mishra (2009), "Underlying truly meaningful and deeply skills teaching with technology, TPACK is different from knowledge of all three concepts individually (p. 66). For example, pedagogical content knowledge determines the way in which a teacher will interpret the content, find ways to represent it, and provide instruction that will meet the needs of all their students. The integration of all three aspects reflects a teacher's ability to be flexible, so they can constantly update the ways in which they provide instruction to meet the ever-changing needs of diverse groups of students.

Saha et al. (2010) used GeoGebra to effectively teach geometry. This study placed 53 high school students into two groups; the experimental group was taught using GeoGebra while the control group received traditional instruction. Students were given a Spatial Visualization Ability Test Instrument (SVATI) to categorize them into low and high visual-spatial ability groups. An independent samples t-test determined there was a significant difference between the students using GeoGebra and those who did not.

Forsythe (2007) used GSP to teach geometry concepts. She found that students were more successful when using GSP than when using traditional paper-pencil tasks. According to Forsythe (2007), when students worked in pairs using GSP, they learned to be systematic in what they were doing. The students had to use problem solving skills to get the computer to produce desired images. This led to meaningful discussions and explanations of geometric concepts.

Hannafin et al. (2008) observed the effects of activities in Geometer's Sketchpad on geometry achievement. The participants included 66 sixth-grade students from four

different math classes. Students were placed into two groups; the experimental group was the group using GSP, while the control group used an online tutorial for instruction. Students were given a pretest and posttest before and after treatment. The results showed that the students in the Sketchpad group did not outscore the students in the tutorial group as predicted.

Substitution, Augmentation, Modification, Redefinition (SAMR)

The Substitution, Augmentation, Modification, and Redefinition (SAMR) model is another way to describe teacher learning with educational technology. This model is shown as a ladder to indicate levels of teaching with technology from lowest levels of technology integration to the highest levels of technology integration. Substitution and Augmentation are at the bottom and are classified as methods of enhancement through technology. Within substitution, technology takes the place of another tool but doesn't change the function of the activity. For example, offering a digital copy of an assessment instead of a paper copy, students are still receiving the assessment so there is no change in the function of the task. At the augmentation level, technology takes the place of another tool, but does offer functional improvement. For example, in a math class students could be given calculators to do arithmetic calculations to eliminate the possibility of calculation errors allowing students to focus on higher order problem solving skills (Hamilton, Rosenber, & Akcaoglu, 2016).

Modification and Redefinition are both classified as methods of transformation with technology meaning the use of the technology requires a redesign of a task. An example of this would be substituting a picture or diagram with a computer simulation of a concept or skill. Redefinition uses technology to create tasks such as making videos or

creating presentations. For example, students can create a video or presentation to share thoughts and ideas.

Summary

Each method for incorporating technology discussed above is part of the framework that guided the decisions made in this study. Knowing that spatial skills can be developed and enhanced through instructional activities led to the decision to try to improve spatial visualization skills. Constructivist teaching methods guided the selection of activities and the type of instruction used in this study. TPAK and SAMR influenced the decision to use technology and the ways in which technology were used in the intervention. Critical Test Theory and Item Response Theory guided the decision to use the Revised Purdue Spatial Visualization Test: Rotations that was used to measure spatial visualization skills in participants of this study.

Research Design

The research philosophy of this study was positivism. According to Ryan (2018), “positivists value objectivity and proving or disproving hypotheses” (p. 1). This philosophy fit this study because the goal was to collect data and analyze it to accept or reject stated hypotheses. The research type for this study was deductive. According to Johnson-Laird (1999), by definition, deduction yields valid conclusions, which must be true given that their premises are true” (p. 110). The results of this study intended to provide the researcher with valid conclusions which lead to accepting or rejecting hypotheses. The study was a quasi-experimental quantitative study. According to Shadish et al. (2002), “Quasi-experiments share with all other experiments a similar purpose- to test descriptive causal hypotheses about manipulable causes...but, by definition, quasi-

experiments lack random assignment” (p. 13-14). The participants in this study were students in the researcher’s six intact geometry classes, thus preventing random assignment. This study was experimental because it involved two groups receiving different forms of instruction to determine which method was more effective. Because students were enrolled in intact classes and could not be randomly assigned to groups, each class was randomly assigned to be a technology group or a manipulatives group by rolling a single die. This was a longitudinal study. According to Rajulton (2001), “...The term ‘longitudinal data’ denotes repeated measurements of the same individuals over a time span long enough to encompass a detectable change in their developmental status” (p. 170). The participants in this study were assessed four times over a period of twelve weeks which made this study longitudinal.

Purpose of the Study

The first purpose of this study was to investigate and compare teaching methods aimed at increasing spatial visualization skills of high school geometry students using multiple interventions. This study also examined whether there was a gap in spatial visualization skills based on gender. Finally, the purpose of this study was to compare the use of technology enhanced instruction to instruction with concrete, tangible models to facilitate spatial visualization skills of high school students.

Research Questions

- **Research Question 1:** Which learner characteristics predict spatial visualization as measured by the Revised Purdue Spatial Visualization: Rotations Test?

- **Research Question 2:** To what extent does geometry instruction using technological tools increase the spatial visualization skills of high school geometry students?
- **Research Question 3:** To what extent do concrete, tangible models used during geometry instruction without technological tools increase the spatial visualization skills of high school geometry students?
- **Research Question 4:** Which method of instruction has a more significant impact on students' spatial visualization skills, instruction using technological tools or instruction that does not use those tools?

Hypotheses

- H_0 : The intervention will have no impact on students' spatial visualization levels.
- H_1 : Students with previous spatial experiences will have higher spatial visualization scores and those scores will increase during the intervention.
- H_2 : Technology use during instruction will improve spatial visualization scores.
- H_3 : Concrete, tangible models used during geometry instruction without technology will improve spatial visualization scores.
- H_4 : Students using technology will have greater improvements to spatial visualization scores than students using concrete, tangible models alone.

Population Description

The population from which the researcher obtained a sample was the geometry students at Germantown High School, located in Madison, MS. In grades 9-12 there were over 1,380 students enrolled from diverse backgrounds. Of these students, 52% were male and 48% were female. The racial makeup included 59% White, 37% African

American, 2% Asian, and 2% Hispanic. The participants were a representative sample of the population.

Sample

The sample chosen for this study included geometry students who were enrolled in the researcher's six geometry classes. All students were in grades 9-12 with most students in tenth grade because the geometry course was part of the regular track for mathematics classes.

Instrumentation

The Revised Purdue Spatial Visualization Test: Rotations (Yoon, 2011) was used to assess the participants' spatial visualization levels before the intervention to obtain baseline data. Then the same assessment was implemented as a pretest immediately prior to the intervention and again as a posttest immediately following the intervention. Lastly, the assessment was given for post posttest data four weeks after the intervention was over. This test consisted of 30 multiple choice questions designed to assess students' ability to visualize the rotation of three-dimensional objects. This test was designed to be completed in 30 minutes. According to Yoon (2011), "The PSVT:R has 2 practice items followed by 30 test items which consist of 13 symmetrical and 17 asymmetrical figures of 3D objects, which are drawn in a 2D isometric format. All the figures contain shapes of cubes or cylinders with varied truncated slots" (p. 33). The instructions and example question of this assessment were included in Appendix A based on the author's request. The psychometric properties of this test were based on Classical Test Theory (CTT) and Item Response Theory (IRT). According to Yoon (2011), "Data from this study yielded a Cronbach's $\alpha = .862$, indicating good internal consistency in using the Revised PSVT:R

with undergraduate students” (p. 85). Yoon (2011) also stated, “All fit indexes (RMSEA, CRI, and TLI) except the Chi-square and WRMR had an acceptable range of values” (p. 86).

Intervention

The researcher used Geometry class time with the learning goals to enhance participants’ spatial visualization skills. An increase in spatial visualization skills helped to contribute to increased performance and success in other subject areas and other aspects of life because spatial visualization skills are used for the rest of the students’ lives and are not skills limited to high school instruction. Once IRB approval was obtained, the researcher obtained parental consent and student assent from those who chose to participate in the study. Once consent and assent were obtained, the research study began. Each participant was assigned a code name to ensure their identity was not known or discoverable by anyone other than the researcher. Those code names were entered into SPSS and coded for gender (0-male and 1-female). Each of the researcher’s six geometry classes were randomly assigned to the technology or manipulative groups by rolling a single die.

During week 1 of the study, the Revised PSVT:R was given to all participants in the study to obtain baseline data of spatial visualization levels. During weeks 2-4 the participants engaged in regular classroom activities not part of the intervention. During week 5, the participants completed the Revised PSVT:R as a pretest then the intervention began. After the pretest was administered, the participants participated in small tasks during each class block designed to enhance their spatial visualization skills through week 8. These tasks included Quick Draw, Quick Image, and Quick Blocks. Each task

lasted approximately 5-10 minutes of class and was implemented at the beginning of class as a warmup activity. Quick Draw was taken Wheatley's (2007) work and required pencil, paper, and the showing of an image like the one shown in Figure 1. The image was shown for approximately three seconds and removed. Then the participants attempted to draw what they saw. After about one minute, the image was shown again for approximately five seconds then the participants tried to complete their drawings. After about one minute to finish drawing, the participants and researcher engaged in a meaningful conversation in which they discussed what they saw and strategies they used to complete their drawings. Quick Image was similar to Quick Draw following the same format for implementation, but the image shown was different. Participants were shown an image like the one in Figure 2 for a few seconds. Then the participants attempted to figure out how many dots were in the image. Some images had a single color, some had two colors as in figure 2, or more colors. After taking a few minutes to share what they saw, the image was shown again for a few seconds followed by meaningful discussion. Some questions that were asked during this activity included asking what the participants saw and how they knew how many dots were pictured. The different colored dots helped participants think about quantity and grouping. Quick Blocks (Matney et al., 2020) followed the same procedure as Quick Draw and Quick Image but required the use of linking cubes instead of paper and pencil. Participants were shown an image like the one in figure 3 for just a few seconds and then used the linking cubes to attempt to build the same image. After a few minutes, the participants were shown the image again and adjustments to their figure were made as needed. Quick Blocks were differentiated in terms of increasing levels of difficulty in which the figures were shown from different

viewpoints, or there were different colored blocks included such as the ones shown in Figure 4. According to Matney et al. (2020), “The design of each image incorporates the elements of shape, pattern, and color so that students have a variety of spatial pathways to envision the number of blocks and their orientation to one another” (p.12). This warmup activity over the first three weeks gave the participants some exposure to activities designed to increase spatial visualization skills.

During week 6, the instructional unit began and continued for three weeks. All participants continued to engage in Quick Draw, Quick Images, and Quick Blocks for the entire intervention. During week 6, instruction on Transformations began. For Activity 1, participants in the technology group engaged in the GeoGebra activity Transformation Practice (Appendix D) in which the participants used GeoGebra to move the objects on the graph digitally. The participants in the manipulatives group engaged in the Transformation Exploration Activity (Appendix E) in which they used graph paper and tracing paper to move objects on the graph. Both groups discovered consistent methods for translating, reflecting, and rotating figures on the graph while the technology group used technology, and the manipulative group used manipulatives. For Activity 2, the participants explored dilations. The participants in the technology group engaged in the GeoGebra activity “Exploring Dilations in the Coordinate Plane” (Appendix F) in which they manipulated the graph to adjust the center of dilation, used a slide tool to adjust the scale factor and they could click a box to show the image and remove it. The participants in the manipulatives group engaged in a dilations assignment in which they used graph paper to draw their own figure and created a scale factor to enlarge or reduce the image (Appendix F).

During week 7, the participants completed activities 3 and 4. For Activity 3, the participants used transformations to create tessellations. The participants in the technology group completed in the tessellations that use rotations task using GSP (Appendix H). The participants in the manipulatives group completed a tessellations activity using paper (Appendix I). For activity 4, the participants completed in the activity “Building a 3D Object” (Appendix J). The participants in the technology group drew their figures using GeoGebra or GSP while participants in the manipulatives group drew their figures on paper.

During week 8, participants completed in Activity 5. Activity 5 was “Proportional Reasoning with a Pyramid.” According to Mamolo, et al. (2011), “Described here is an activity designed to foster learners’ ability to use visualization, spatial reasoning, and geometric modeling to solve problems” (p. 545). Participants used clear plastic pyramids and filled them partially with water to observe the shape like the base (Figure 5). The more water added, the bigger the shape. This allowed students to see proportional changes to the surface polygon. Next, the participants anchored the pyramid using Play-Doh such that it was not perpendicular to the floor (Figure 6). This created different polygons for the base. The participants then completed the prompts provided with the activity to give them a chance to manipulate the activity and reflect on what they were doing. The participants in the technology group created a sketch of the pyramid in GSP and used the features of GSP to manipulate the figure to notice how the angles and ratios were preserved, but the lengths of the sides changed (Appendix K). The manipulatives group tried to sketch the pyramid and made changes on paper.

At the end of week 8, when the instructional unit was complete the participants completed the Revised PSVT:R as a posttest. During weeks 9-11, the participants will partake in regular classroom activities not part of the intervention for this study. During week 12, the participants completed the Revised PSVT:R as a post posttest to determine if their spatial visualization scores had improved as a result of the instructional unit.

Data Collection

Once the International Review Board (IRB) approved this study, the participants were given assent and consent forms and the researcher explained the expectations of the study. There was no risk involved in participating in this study. The participants were engaged in regular classroom activities that did not require them to do any additional work outside of class. The first assessment was given during Week 1 of the study and provided a baseline data of participants' spatial visualization skills. The pretest was given during Week 5 before the instructional unit was taught. Then, the posttest was administered during Week 8 upon completion of the instructional unit. A post posttest was administered during Week 12. Thus, four separate data points were obtained for this study.

Data Analysis

The scores on the Revised Purdue Spatial Visualization Test: Rotations were entered into Statistical Package for Social Sciences (SPSS) after each testing date. The statistical analysis used on this data set included: Correlation and Multiple Regression, Descriptive Statistics, a Dependent t-test, and Repeated Measures Analysis Repeated Measures Analysis of Variance (ANOVA). The repeated measures ANOVA is useful for within-subjects designs and was beneficial for this study because the same group of

students was assessed four times throughout the school year. A repeated measures ANOVA requires an assumption of sphericity. One way to assess this was using Mauchly's test. If Mauchly's test was significant ($p < .05$) then there were significant differences between variances, and this violated sphericity. If sphericity is not violated, then the research hypothesis was supported.

To address research question 1, "Which learner characteristics predict spatial visualization as measured by the Revised Purdue Spatial Visualization: Rotations Test?" A correlation and multiple regression Analysis was used. The results indicated if gender or previous spatial factors such as playing sports, playing musical instruments, playing video games, playing with building blocks, course preference, or the time of day they had geometry had an impact on participants' spatial visualization scores for Week 1, the initial assessment.

To address research question 2, "To what extent does geometry instruction using technological tools increase the spatial visualization skills of high school geometry students?" Descriptive statistics were used to compare the mean test scores of the technology group throughout the intervention. The results indicated if the participants in the technology group had increased spatial visualization scores from Week 1 through Week 12.

To address research question 3, "To what extent do concrete, tangible models used during geometry instruction without technological tools increase the spatial visualization skills of high school geometry students?" Descriptive statistics were used to analyze the mean test scores of the manipulatives group. The results indicated if the

participants in the manipulatives group had increased spatial visualization scores from Week 1 through Week 12.

To address research question 4, “Which method of instruction has a more significant impact on students’ spatial visualization skills, instruction using technological tools or instruction that does not use those tools?” A repeated measures ANOVA was used. The results of the test indicated which group, technology or manipulatives, had more significant increases in spatial visualization skills from Week 1 through Week 12.

Methodological Limitations

Some limitations of this methodology included that not all participants began the intervention with the same spatial visualization skills which resulted in a variety of levels of improvement throughout the intervention. Another limitation of this methodology was participant attendance. The researcher could not guarantee that all participants were present for each assessment date and for each day of the intervention. This was an external variable for this study that could not be controlled by the researcher.

Concluding Summary

The first purpose of this study was to investigate and compare teaching methods aimed at increasing spatial visualization skills of high school geometry students using multiple interventions. The research design for this study was positivist and deductive. This was quasi-experimental longitudinal study that used two treatment groups to determine the effectiveness of an intervention designed to increase spatial visualization skills of high school geometry students. The Revised PSVT:R was used as an assessment to gather baseline data. It was also used as a pretest and posttest before and after the intervention, and as a post posttest after the intervention was complete to obtain a total of

four data points for analysis. Correlation, multiple regression analysis, descriptive statistics, and a repeated measures ANOVA were used to analyze the data to answer each of the research questions.

CHAPTER IV – RESULTS

The purpose of this chapter was to report the findings from data analysis of a study of the spatial visualization skills of high school geometry students. This chapter begins with a description of the sample. Following this, the research questions are restated, and the tests used to analyze the results for each question are described. Each hypothesis was accepted or rejected based on the results of statistical analysis. Data tables are also included to support the results.

Description of Sample

The population for in this study included the researcher's 133 high school geometry students. This population were enrolled in one of six geometry classes. Of this population, 63 (47%) were male and 70 (53%) were female. There was one student in 9th grade (1%), 112 in tenth grade (84%), 19 in eleventh grade (14%), and one in twelfth grade (1%). The student ages ranged from 15-18. From that population, 65 participants returned consent forms and those became the sample used for this study. Table 2 shows the frequencies of gender for those participating in the study. Within the sample used for this study, 39% were male and 61% were female.

Table 2 *Descriptive Statistics for Gender*

<i>Gender</i>				Valid	Cumulative
		Frequency	Percent	Percent	Percent
Valid	Male	25	38.5	38.5	38.5
	Female	40	61.5	61.5	100.0
	Total	65	100.0	100.0	

The researcher’s six geometry classes were randomly assigned to technology or manipulative groups by assigning each class a number 1-6 and rolling a physical die.

Table 3 shows the frequencies for how many participants were in each group. The manipulatives group had 48% of the participants and the technology group had 52% of the participants.

Table 3 *Descriptive Statistics for Instructional Method*

<i>Instructional Method</i>				Valid	Cumulative
		Frequency	Percent	Percent	Percent
Valid	Manipulatives group	31	47.7	47.7	47.7
	Technology group	34	52.3	52.3	100.0
	Total	65	100.0	100.0	

Results for Research Question One

Research question one stated: “Which learner characteristics predict spatial visualization as measured by the Revised Purdue Spatial Visualization: Rotations Test?”

The null hypothesis stated: “The intervention will have no impact on students’ spatial visualization scores.” The alternate hypothesis stated: “Students with previous spatial experiences will have higher spatial visualization scores and those scores will increase

during the intervention.” The “learner characteristics” and “previous spatial experiences” as mentioned in the question and hypothesis are defined as playing sports, musical instruments, video games, playing with building blocks, gender, a course preference of algebra or geometry, and the time of day they were enrolled in geometry (AM or PM). Statistical Package for Social Sciences was used to perform correlation and multiple regression analyses to determine the relationship between the results of the first administration of the Revised Purdue Spatial Visualization Test: Rotations (PSVT:R) and the self-reported learner characteristics. A regression model can be used with categorical variables, particularly when the categorical variables are binary, as is the case in this study.

Regression analysis created a model that was evaluated using a repeated measures ANOVA. The multiple regression model with all seven predictors produced $R^2 = .278$, $F(7, 57) = 3.14$, $p = .007$ as shown in Table 4. This showed that 27.8% of the variance in the data was explained by this study and the other 72.2% was unexplained. The F-statistic was used to determine if any predictors were statistically significant. The F-statistic of $F(7, 57) = 3.14$ showed the data aligned with the model. This allowed the researcher to reject the null hypothesis and accept the alternate hypothesis. The other spatial experiences that showed a significant impact on this model were “Gender” $p = .012$ and “PlayInstrument” $p = .001$. The adjusted R Square shows 19% of the model was explained, indicating some variables did not impact the model at all or they had a negative impact on the model.

Table 4 *Regression Analysis*

Model Summary^b

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Durbin-Watson
1	.528 ^a	.278	.190	5.15432	2.117

a. Predictors: (Constant), GeometryTime, PlayVideoGames, CoursePreference, Sports, PlayInstrument, BuildingBlocks, Gender

b. Dependent Variable: PSVT 1

Of the seven variables included in the correlation analysis for question one, four had negative relationships and three had positive relationships. The following variables showed negative relationships when correlated with the first administration of the Revised PSVT:R: “Gender” (-4.132), “Sports”(-0.637), “Playinstrument” (-4.589) or “Buildingblocks” (-2.081). The following variables showed a positive relationship when correlated with the first administration of the Revised PSVT:R:

“Playvideogames”(0.948), “CoursePreference (0.857), and “GeometryTime” (0.837).

As shown in Table 5, “PlayInstument” and “Gender” had a significant negative regression weight, indicating students who played an instrument were expected to score lower on the Revised PSVT: R, after controlling for the other variables in the model. Each of the other variables did not contribute to the multiple regression model.

Table 5 *Multiple Regression Analysis Results*

Coefficients^a

Model	Unstandardized		Standardized		Sig.	Collinearity	
	Coefficients	Std. Error	Beta	t		Tolerance	VIF
1 (Constant)	18.299	1.659		11.03	.000		
Gender	-4.132	1.598	-.354	-2.586	.012	.676	1.478
Sports	-.637	1.331	-.056	-.479	.634	.928	1.077
PlayInstrument	-4.589	1.351	-.403	-3.396	.001	.897	1.115
PlayVideoGame	.948	1.682	.083	.564	.575	.589	1.697
BuildingBlocks	-2.081	2.073	-.132	-1.004	.320	.731	1.368
CoursePreference	.857	1.601	.068	.536	.594	.797	1.255
GeometryTime	.837	1.407	.070	.595	.554	.922	1.085

a. Dependent Variable: PSVT1

Results for Research Question Two

Research question two stated: “To what extent does geometry instruction using technological tools increase the spatial visualization skills of high school geometry students?” The null hypothesis stated: “The intervention will have no impact on students’ spatial visualization scores.” The alternate hypothesis stated: “Technology use during instruction will improve spatial visualization scores.” SPSS was used to calculate descriptive statistics to compare the means of the technology group for each of the four administrations of the Revised PSVT:R. Table 5 shows the descriptive statistics obtained.

Table 6 *Descriptive Statistics for Technology group*

<i>Descriptive Statistics</i>				
	Instructional Method	Mean	Std. Deviation	N
PSVT1	Technology	14.0588	5.52647	34
PSVT 2	Technology	15.0000	6.37229	34
PSVT 3	Technology	15.0588	6.21797	34
PSVT 4	Technology	15.3235	6.66388	34

Based on Table 6 the mean score for the Technology Group increased each time the Revised PSVT:R assessment was administered. This allowed the researcher to reject the null hypothesis and accept the alternate hypothesis. It appeared that the intervention for the participants using technology during geometry instruction improved their spatial visualization scores as measured by the Revised PSVT. A dependent samples t-test was performed on the Technology Group to compare the first assessment of the PSVT (M = 14.058, SD = 5.526) to the last assessment of the PSVT (M = 15.323, SD = 6.663) to determine if there was a significant change. The results of the t-test supported that there was a significant difference between mean scores, $t(33) = -1.815$, $p > .001$. The results of this t-test supported the hypothesis that there was a significant difference in Revised PSVT:R scores for the Technology Group. Without a control group, the researcher could not fully determine if the intervention using technology was the single determinant of increased spatial visualization scores, but it is evident that spatial visualization scores did not decrease with the use of technology. The standard deviation for these means was high compared to the means which indicated more variability in the data. The coefficient of

variation (CV) for each mean was: PSVT 1, CV= 2.5439, PSVT 2, CV=2.3539, PSVT 3, CV= 2.42181, PSVT 4, CV=2.29948. Each of these CV values were greater than 1 indicating they were high. This showed that the data values were spread out in relation to the mean.

Results for Research Question Three

Research question three stated: “To what extent do concrete, tangible models used during geometry instruction without technological tools increase the spatial visualization skills of high school geometry students?” The null hypothesis stated: “The intervention will have no impact on students’ spatial visualization scores”. The alternate hypothesis stated: “Concrete, tangible models used during geometry instruction without technology will improve spatial visualization scores.” SPSS was used to calculate descriptive statistics to compare the means of the Manipulatives Group for each of the four administrations of the Revised PSVT:R. Table 6 shows the descriptive statistics obtained.

Table 7 *Descriptive Statistics for Manipulatives group*

<i>Descriptive Statistics</i>				
	Instructional		Std.	
	Method	Mean	Deviation	N
<u>PSVT 1</u>	Manipulatives	14.2581	6.02754	31
<u>PSVT 2</u>	Manipulatives	16.1290	6.74656	31
<u>PSVT 3</u>	Manipulatives	16.4516	6.20666	31
<u>PSVT4</u>	Manipulatives	16.7419	5.89897	31

Based on Table 7, the mean score for the Manipulatives Group increased each time the Revised PSVT:R assessment was administered. This allowed the researcher to reject the null hypothesis and accept the alternate hypothesis. It appeared that the intervention for the participants using manipulatives during geometry instruction

improved their spatial visualization scores as measured by the Revised PSVT:R. A dependent samples t-test was performed on the Manipulatives Group to compare the first assessment of the PSVT (M = 14.258, SD = 6.027) to the last assessment of the PSVT (M = 16.741, SD = 5.898) to determine if there was a significant change. The results of the t-test supported that there was a significant difference between mean scores, $t(330) = -2.8976$, $p > .001$. The results of this t-test supported the hypothesis that there was a significant difference in Revised PSVT:R scores for the Manipulatives Group. This suggested that the use of manipulatives during geometry instruction did not cause spatial visualization scores to decrease. The standard deviation for these means was also high compared to the means which indicated more variability in the data. The coefficient of variation (CV) for each mean was: PSVT 1, CV= 2.36549, PSVT 2, CV=2.39069, PSVT 3, CV= 2.65063, PSVT 4, CV=2.838105. Each of these CV values were greater than 1 indicating they were high. This showed that the data values were spread out in relation to the mean.

Results for Research Question Four

Research question four stated: “Which method of instruction has a more significant impact on students’ spatial visualization skills, instruction using technological tools or instruction that does not use those tools?” The null hypothesis stated: “The intervention will have no impact on students’ spatial visualization scores.” The alternate hypothesis stated: “Students using technology will have greater improvements to spatial visualization scores than students using concrete, tangible models alone.” SPSS was used to conduct a repeated measures ANOVA to compare the effect of instructional methods on Revised PSVT:R scores over time. There was no statistically significant difference in

PSVT scores between the two groups ($F(3, 189) = .582, p=.627$). Table 8 shows the results of the repeated measures ANOVA. The results of this ANOVA allowed the researcher to accept the null hypothesis and reject the alternate hypothesis.

Table 8 *Repeated Measures ANOVA*

Tests of Within-Subjects Effects

Measure: Score

Source		Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
PSVTScore	Sphericity	135.760	3	45.253	4.96	.002	.073
	Assumed				3		
	Greenhouse- Geisser	135.760	2.78 7	48.717	4.96 3	.003	.073
	Huynh-Feldt	135.760	2.97 5	45.634	4.96 3	.003	.073
	Lower- bound	135.760	1.00 0	135.760	4.96 3	.029	.073
PSVTScore * InstructionalMe thod	Sphericity	15.929	3	5.310	.582	.627	.009
	Assumed						
	Greenhouse- Geisser	15.929	2.78 7	5.716	.582 3	.615	.009
	Huynh-Feldt	15.929	2.97 5	5.354	.582 3	.626	.009
	Lower- bound	15.929	1.00 0	15.929	.582 3	.448	.009

Summary

For this study, the data were analyzed using a variety of statistical methods to answer four research questions. Research question 1 stated, “Which learner characteristics predict spatial visualization as measured by the Revised Purdue Spatial Visualization: Rotations Test?” Correlation and multiple regression analyses were used to

analyze the data and the alternate hypothesis was accepted. This indicated that other spatial experiences contributed to increased spatial visualization scores of high school geometry students. The learner characteristics that predicted spatial visualization scores were gender and playing a musical instrument.

Research question 2 stated, “To what extent does geometry instruction using technological tools increase the spatial visualization skills of high school geometry students?” Descriptive statistics, and t-tests were used to analyze the data and the alternate hypothesis was accepted. This finding appeared to show that technology used during geometry instruction improved spatial visualization scores in high school geometry students.

Research question 3 stated, “To what extent do concrete, tangible models used during geometry instruction without technological tools increase the spatial visualization skills of high school geometry students?” Descriptive statistics and t-tests were used to analyze the data and the alternate hypothesis was accepted. This seemed to suggest that the use of manipulatives during geometry instruction improved spatial visualization scores of high school geometry students.

Research question 4 stated, “Which method of instruction has a more significant impact on students’ spatial visualization skills, instruction using technological tools or instruction that does not use those tools?” A repeated measures ANOVA was used to analyze the data and the null hypothesis was accepted. This suggested that using concrete, tangible models had a more significant impact on participants’ spatial visualization skills than using technology.

CHAPTER V – DISCUSSION

The purpose of this chapter is to interpret the results of the data analysis and discuss the results and conclusions for each research question. This chapter also includes a discussion of the limitations of the study and how the results fit into the current literature including implications to theory. This chapter also provides recommendations for future research.

Introduction

“Spatial visualization is an important aspect of geometric thinking... Geometric modeling and spatial reasoning offer ways to interpret and describe physical environments and can be important tools in problem solving” (NCTM, p. 41) From giving travel directions to driving a car, designing floor plans, or creating art, spatial visualization skills are important for completing all these tasks. Instruction in the high school geometry classroom can help improve students’ spatial visualization skills to prepare them for life beyond the classroom. The purpose of this study was to investigate and compare teaching methods aimed at increasing spatial visualization skills of high school geometry students using multiple interventions. This study also examined whether there was a gap in spatial visualization skills based on gender. Finally, the purpose of this study was to compare the use of technology enhanced instruction to instruction with concrete, tangible models to facilitate spatial visualization skills of high school students.

The 65 participants were administered the Revised Purdue Spatial Visualization Test: Rotations (Revised PSVT:R) to measure their spatial visualization skills on four distinct assessment dates. Geometry classes were randomly assigned to technology and manipulative groups. The Technology Group used technology during the intervention and

the Manipulatives Group used concrete, tangible models during the intervention. All classes participated in the same activities. Descriptive statistics revealed that participants improved from the first assessment of the Revised PSVT:R to the fourth assessment. A repeated measures ANOVA was used to determine which group had greater gains in their spatial visualization scores.

Interpretation of Results

Research question one asked: “Which learner characteristics predict spatial visualization as measured by the Revised Purdue Spatial Visualization: Rotations Test?” To address this question, participants were given a survey (Appendix A) at the beginning of the study that asked for their gender, and whether or not they played sports, musical instruments, or video games. The survey also asked if the participants played with building blocks as a child, if they preferred Algebra or Geometry, and the time of day they were enrolled in Geometry. Those results were coded and entered in Statistical Package for Social Sciences (SPSS) and analyzed using a correlation and multiple regression analyses. Gender and playing a musical instrument were the only learner characteristics that had a significant effect on the first assessment of the Revised PSVT:R.

Gender as a variable with significant impact on the model is supported by the literature. Research conducted by Fennema & Sherman (1977), Ben-Chaim et al. (1988), Quaiser-Pohl & Lehmann (2002), Logan & Lowrie (2017), Julia & Antoli (2018), and Ogunkola & Knight (2019) all showed males outperforming females in spatial visualization. Tzuriel & Egozi (2010) and Jackson et al. (2015) showed females outperforming males in their research. Previous research discussed spatial ability as an

innate skill causing many researchers to believe children were born with those skills. According to Fennema & Sherman (1977) “Mathematics traditionally has been regarded as a male subject” (p. 53). Over time, it was determined that spatial ability is malleable. The literature from the late 1970s on gender differences supported the idea that males outperformed females in spatial ability or spatial visualization skills, but some current research gives insight into potential reasons why. Brosnan (1998) and Wolfgang et al. (2003) found a relationship between spatial ability and children playing with blocks. Feng et al. (2007), Cherney (2008), and Terlecki et al. (2008) found the use of video games to be helpful to improve spatial visualization scores. These other methods of improving spatial visualization that were not taking place in the classroom could have gender bias which could potentially contribute to the research showing males outperform females in spatial visualization tasks. The research of Feng et al. (2007) discussed how active shooter type video games were more effective at improving spatial visualization. “Thus, boys may realize benefits in spatial attention that are largely denied to their female counterparts, who participate in such action games in much smaller numbers” (p. 851). A similar concept applies to building blocks as well. According to multiple studies, (Etaugh, 1983; Cherney & London 2006; Bradbard & Parkman, 1983; Fagot, 1974; and Miller, 1987) it is more common for boys to play with building blocks or Legos and to play video games as children than it is for girls which contributes to the early development of spatial ability and spatial visualization skills for boys. The interventions used in these studies improved the spatial visualization skills of boys and girls with girls showing greater gains because their spatial visualization skills were lower at the start.

Despite where they start, they can improve their spatial visualization skills through participation in these activities.

Playing an instrument was also a variable that significantly impacted the model of this study. The literature supported that playing instruments improved spatial visualization. According to Gaser & Schlaug (2003), “Musicians repetitively practice this visual-spatial to motor transformation by reading musical notation and translating it back into motor plans accompanied by simultaneous auditory feedback...” (p. 9243).

Research question two asked: “To what extent does geometry instruction using technological tools increase the spatial visualization skills of high school geometry students?” To address this question, descriptive statistics were used to determine if the mean scores showed continuous gains over time. The increase over time appeared to show that the intervention increased spatial visualization scores for students in the Technology Group. Students using technology were able to use the tools to help them manipulate objects in a variety of ways that helped increase spatial visualization skills. In terms of the extent to which the participants in the Technology Group improved their spatial visualization skills, the mean score on the first administration of the Revised PSVT was 14.0588 then on the fourth administration it was 15.3235 which is a difference of 1.26 points. This result aligned with the work of Baki et al. (2011) who claimed, “Our findings illustrate that 3D DGS environments enable students to construct their geometric structure through investigation by dragging and measuring their structure and helping them to learn solid geometry and developing their spatial visualization skills” (p. 304). It also aligned with the results of Idris (2007) who said, “Firstly, the significant differences in geometry achievement of the experimental groups as compared to the

control groups indicate that the GSP shows promising implications for the potential of using the GSP in teaching geometry at the secondary school level” (p. 177). Participants in the Technology Group engaged in five different activities that used technology. The first activity in the intervention involved an exploration of transformations (Appendix C) that allowed the participants to translate, reflect, or rotate figures on the graph. Using the computer decreased the potential for error because the activity in Desmos moved each figure, which allowed participants to discover consistent methods for translating, rotating, and reflecting images for each transformation. The second activity was a dilations exploration (Appendix E) that allowed participants to enlarge and reduce figures on the coordinate plane using Desmos. The tessellations activity (Appendix G) caused some frustration for the Technology Group. The activity was designed for GSP. The school-issued laptops used by the participants were not compatible with the updated version of GSP. Being unable to download GSP to use it for the activity forced the researcher to adapt the activity to be completed in GeoGebra. There was some user error and frustration because the directions did not fully align with GeoGebra. Participants had to create a figure and had trouble rotating or translating it to make the tessellation. Many asked if they could draw it instead of using the computer because they were frustrated with GeoGebra. Building a 3D Object (Appendix I) was done using Tinkercad to create and stack three-dimensional figures. The researcher originally planned to use GeoGebra for this activity, but after the frustration of using GeoGebra for the tessellations activity the switch to Tinkercad was made to avoid frustration. The last activity, Proportional Reasoning with a Pyramid (Appendix J) was a mixture of hands-on and technological tools. Participants used the three-dimensional figure and poured water into the figures to

measure the lengths of the sides of the cross-sectional polygon then used technology to create a representation of their dilated polygon and the proportion used to enlarge it. Each of these tasks created opportunities for participants to increase their spatial visualization skills. The results indicated spatial visualization skills did not go down throughout the four administrations of the Revised PSVT:R. While the means did not decrease over time during this study, the larger standard deviations indicated more variability among the data. The Revised PSVT:R had 30 items. Each item was worth one point. Scores recorded could range from 0-30. The mean scores ranged from 14-15 for the Technology Group. There were a few participants with outlier scores who scored high in the 25-30 range and a few scored on the low end, 0-5, as well. These outliers could be the contributing factor for the high standard deviations.

Research question three asked: “To what extent do concrete, tangible models used during geometry instruction without technological tools increase the spatial visualization skills of high school geometry students?” To address this question, descriptive statistics were used to determine if the mean scores continuously increased over time. The increase over time appeared to show that the intervention increased spatial visualization scores for students in the Manipulatives Group. In terms of the extent to which the participants in the Manipulatives Group improved their spatial visualization skills, the mean score on the first administration of the Revised PSVT was 14.2581 then on the fourth administration it was 16.7419 which is a difference of 2.48 points. This finding was consistent with the findings of Baki et al. (2011), “Results of the study showed that PSVT scores of the students working with physical manipulatives in Manipulatives Group were significantly higher at the end of the semester than at the beginning” (p. 304). Participants in the

Manipulatives Group engaged in the same five activities as the participants in the Technology Group, but without the technology. The first activity was the Transformation Exploration (Appendix D) in which participants used tracing paper to trace the figure and translate, reflect, and rotate the figure to complete the task to help them look for patterns in the coordinates of the figures after each transformation. The Dilations activity (Appendix F) required students to draw anything they wanted and then enlarge or reduce it by a scale factor they chose. They plotted all points for the pre-image and image and showed their dilations on a graph. The tessellations activity (Appendix H) required participants to create a tile by drawing and cutting out the edges of one of the included polygons. Participants then traced the tile on a sheet of copy paper and performed a translation or rotation to complete the tessellation. For activity 4, participants built three-dimensional figures using cardstock and tape to complete the Building a 3D Object (Appendix I) activity. They measured their figures to calculate composite volume for their solid figure. All work was shown on a separate sheet of paper. For activity 5, the Proportional Reasoning with a Pyramid (Appendix J), participants used the same solids and filled them with water as the Technology Group, but the Manipulatives group drew their polygons on paper using measurements they took to show the proportional relationships. All aspects of each activity were completed with physical manipulatives and drawing on paper. Throughout these activities, participants made comments about how fun the activities were. The results showed that spatial visualization skills did not decrease throughout the four administrations of the Revised PSVT:R. While the means increased over time during this study, the larger standard deviations indicated more variability among the data. The Revised PSVT:R had 30 items. Each item was worth one

point. Scores recorded could range from 0-30. For this study, the mean scores ranged from 14-16 for the Manipulatives Group. There were a few participants with outlier scores who scored high in the 25-30 range and a few scorings on the low end, 0-5, as well. These outliers could be the contributing factor for the high standard deviations.

Research question four asked: “Which method of instruction has a more significant impact on students’ spatial visualization skills, instruction using technological tools or instruction that does not use those tools?” To address this question, a repeated measures ANOVA was used to compare the mean scores of the Technology Group to the mean scores of the Manipulatives Group to determine which method of instruction produced the greatest gains. The Manipulatives Group showed greater increases over time, thus showing the instruction using concrete, tangible models seemed to be more effective than instruction with technology. This contradicts the findings of Baki et al. (2011) who reported, “The ANOVA results showed a statistically significant difference among the groups’ PSVT scores. These differences are in favour of computer-based instruction in both the computer group-manipulative group and the computer group-traditional group comparisons...” (p. 305). The difference in the findings of these studies could be due to implementation. The last task that was implemented as part of the intervention (Proportional Reasoning with a Pyramid) was created and designed to be completed in GSP. The participants all had school issued MacBook computers and GSP had not updated their software to be compatible with the newer operating systems of these laptops. This led to the researcher to adapting the task for GeoGebra which potentially made the task more difficult and the participants more frustrated. Throughout the five tasks that were implemented as part of the intervention, participants in the

Manipulatives Group who were using concrete, tangible, models made comments such as “This is fun!” or “I like this, can we do this more?” Participants in the Technology Group who were using technology were expected to have the same comments and feelings towards the intervention, but those participants had quite different responses. Often, they were frustrated with the technology and ask, "Do I have to make this in GeoGebra, can I just draw it?" The technology driven frustration could be a reason for the Technology Group demonstrating less growth over time than the Manipulatives Group, but without using different technology tools or implementing the intervention again it is difficult to know the actual cause for the difference in scores.

This study lacked a true control group because the researcher wanted to provide high, quality instruction based on research-based methods. Using technology and manipulatives are supported by research while traditional instruction in which the teacher talks while students write is not an instructional method supported by research. The researcher did not want a class to go without treatment because of the desire for all students to succeed. There was no way to justify to parents why other classes were receiving instruction using technology or manipulatives while their child received neither. This made it impossible to use a true control group for this study. Without a true control group, the researcher was not able to determine if the use of technology and/or the use of manipulatives were the cause of spatial visualization scores increasing. The constant in both groups was an effective teacher with strong content knowledge, eight years of experience teaching high school geometry, and someone who has spent an entire year building relationships with the participants in the study. The effectiveness of the teacher/researcher could also be a contributing factor to the increase in spatial

visualization scores. Without a group only receiving traditional instruction from the teacher/researcher it cannot be determined which of the variables or combination of variables truly caused the increase in spatial visualization scores.

Limitations

The limitations of this study included the number of participants and the generalizability of the study. The researcher had 130 students across six classes but only 65 students returned consent forms. This limited the number of participants included in the data. Having a larger sample could have impacted the results. Another limitation was that only the researcher's students were used for this study instead of all students enrolled in Geometry in the school or across multiple schools in the region. This limited the generalizability because the sample only included participants from one high school. Another limitation of this study was the various backgrounds of the participants. Some of them had prior experiences that affected their spatial visualization such as playing sports, participating in band, or playing video games. Participants who already had high levels of spatial visualization may have shown increases in their spatial visualization scores because their levels were already high. Another limitation of this study was the participants themselves being in high school. High school students maximize their efforts towards activities they want to do. After a couple of administrations the Revised PSVT:R, the participants were not interested in completing the test again. Some participants bubbled in answers without taking the test seriously. This impacted the scores and subsequently, the results of the data analysis.

Recommendations for Practice

The interventions used within this study improved students' spatial visualization scores over time. Thus, the researcher recommends that geometry teachers incorporate concrete, tangible models, or the use of technology during instruction to improve students' spatial visualization scores. The researcher also recommends that mathematics teachers use the warmup activities of Quick Image, Quick Draw, and Quick Blocks. Consistent use of these warm-up activities would provide students of any age with opportunities to enhance their spatial visualization skills. Finally, the researcher strongly recommends that the specific activities used within this study be implemented when teaching transformations in high school geometry.

Recommendations for Future Research

This study could be replicated using technology compatible with GSP making the technology more user friendly which could yield different results. It would be beneficial to determine if using the technological tool to complete the tessellation activity was designed for would increase spatial visualization scores. Additionally, multiple tessellation activities could be completed with appropriate technological tools, rather than just one tessellation activity so that any frustration that might occur during the completion of the first activity would lessen as additional tessellation activities were completed. This could remove possible negative impacts of frustration on the spatial visualization scores.

This study could also be replicated by having the teacher teach a Technology Group, a Manipulatives Group, and a true Control Group to see if the teacher's effectiveness impacts success. Teacher effectiveness can improve student learning.

Creating a group that completes activities that teach the same skills from each of the five activities in this study, but without technology or manipulatives by receiving quality instruction from an effective teacher could improve spatial visualization scores. Using a control group could also show the effectiveness of the manipulatives and technology when compared with the control group.

Additionally, this study could be replicated on a larger scale to gain generalizability. For example, using all geometry students in a school or using geometry teachers from multiple high schools in the same district to implement the same interventions could provide more data to support or reject the hypotheses from this study while also using a different timeline of assessments. Participants could be assessed to determine their spatial visualization scores at the end of the geometry course and then assessed again at the beginning of the next school year to ascertain if those gains remained over the summer.

A similar study could be conducted using different geometry content. Transformations and 3D figures were used during this study as the content focus. The study could be replicated using different geometry content such as the properties of quadrilaterals or similar triangles to determine if the transformations content contributed to the increase of spatial visualization scores or if a variety of content topic could be useful for increasing spatial visualization scores.

A follow-up study to this one could be conducted using only participants with musical backgrounds to determine if the use of technology improved their spatial visualization scores. Another group could be participants with no musical background to

determine if technology improves their spatial visualization scores. This could provide insight into the extent that musical backgrounds impact spatial visualization scores.

Conclusion

Spatial ability is crucial for every person's daily life. People are impacted by their spatial ability and spatial skills as they learn to navigate the three-dimensional world. Spatial visualization skills can be gained through life experiences and in the classroom. This study contributed to the body of knowledge focused on spatial ability in that the researcher found that gender and playing musical instruments had an impact on spatial visualization scores. This study's results further suggested that using technology as part of an instructional intervention improved spatial visualization scores. Finally, this study also suggested that using concrete, tangible models as part of an instructional intervention also improved spatial visualization skills. This study also found that participants using manipulatives showed greater gains in spatial visualization scores than participants using technology. This has implications for teachers of high school geometry supports the incorporation of concrete, tangible models, or the use of technology during instruction to improve students' spatial visualization scores. The researcher also recommends that geometry teachers use the warmup activities of Quick Image, Quick Draw, and Quick Blocks to provide student with different ways to increase their spatial visualization skills.

APPENDIX A – Figures

Figure 1. Example of Quick Draw Image (Wheatley, 2007)

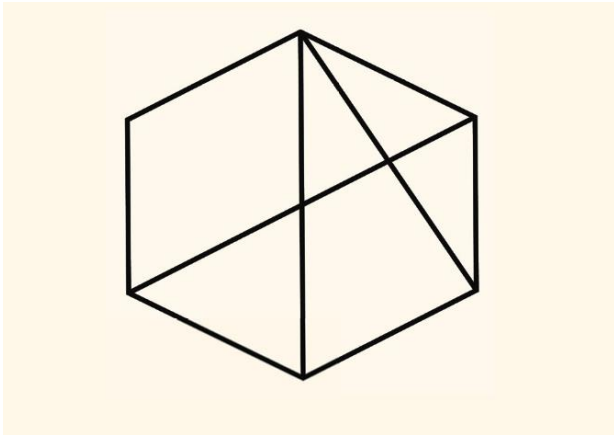


Figure 2. Example of Quick Images Image (Matney et al., 2020)

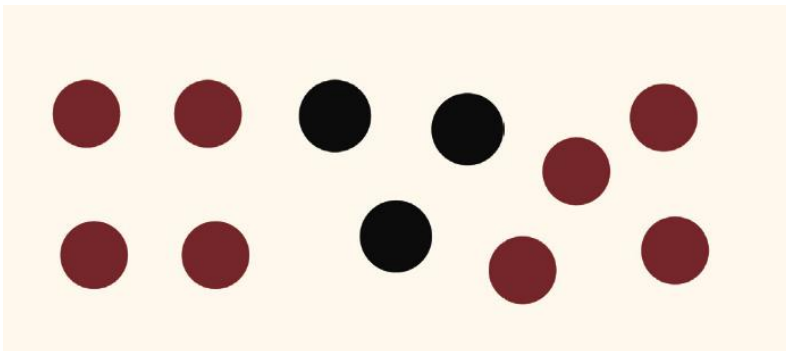


Figure 3. Example of Quick Block image (Matney et al., 2020)

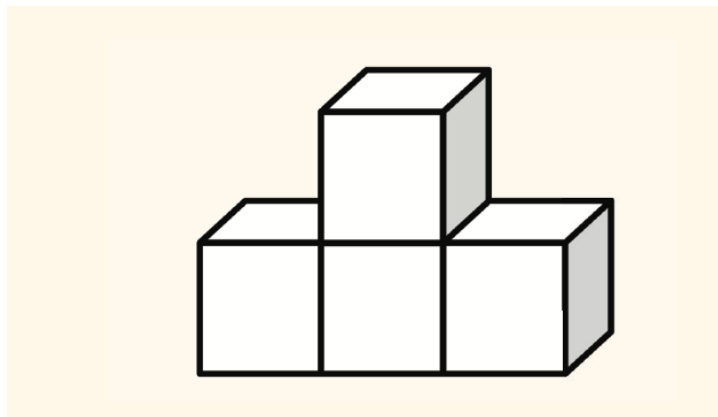


Figure 4. Differentiated images of Quick Blocks (Matney et al., 2020)

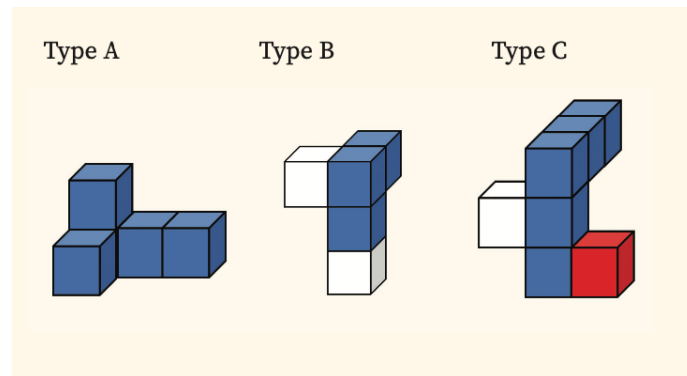


Figure 5. Proportional Reasoning with a Pyramid (Mamolo et al., 2011)



Base of the pyramid is parallel to the floor

Figure 6. Proportional Reasoning with a Pyramid (Mamolo et al., 2011)



Base of the pyramid is not parallel to the floor

APPENDIX B – Student Survey

ID# _____ Time of Geometry: ____ am ____ pm

Spatial Visualization Survey

Directions: Check the appropriate response and write your response to the open-ended questions on the blanks provided.

Gender:

- Female
- Male
- Nonbinary
- Prefer not to say

Do you play sports? Yes No

Have you ever played any sports? Yes No

If yes, which sports have you played?

Do you or have you ever played a musical instrument? Yes No

If yes, which instrument(s)?

Do you play video games? Yes No

List your past and present hobbies: (Try to be as specific as possible)

Past

Present

As a child, what was your favorite toy? _____

As a child, what was your favorite “play activity”? _____

While growing up, did you play with any type of “building blocks”? Yes No

Which do you prefer? Algebra Geometry

Revised Purdue Spatial Visualization Tests: Visualization of Rotations (Revised PSVT:R)

So Yoon Yoon

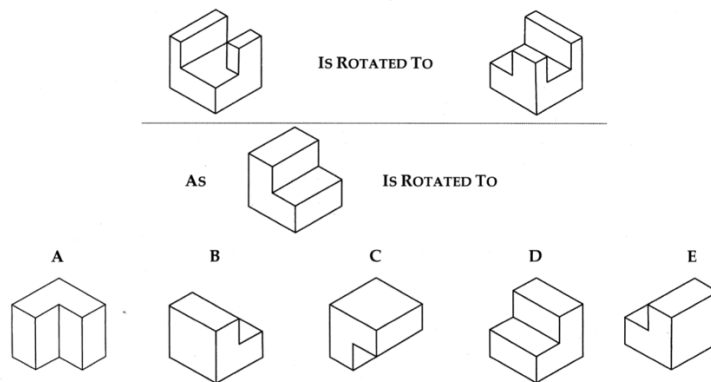
- DO NOT open this booklet until you are instructed to do so.
- DO NOT make any marks in this booklet.
- MARK your answers on the separate answer sheet.

Copyright © So Yoon Yoon 2011

(Permission to revise the test was granted by Roland B. Guay)

DIRECTIONS

This test consists of 30 questions designed to see how well you can visualize the rotation of three-dimensional objects. Shown below is an example of the type of question included in the second section.



You are to:

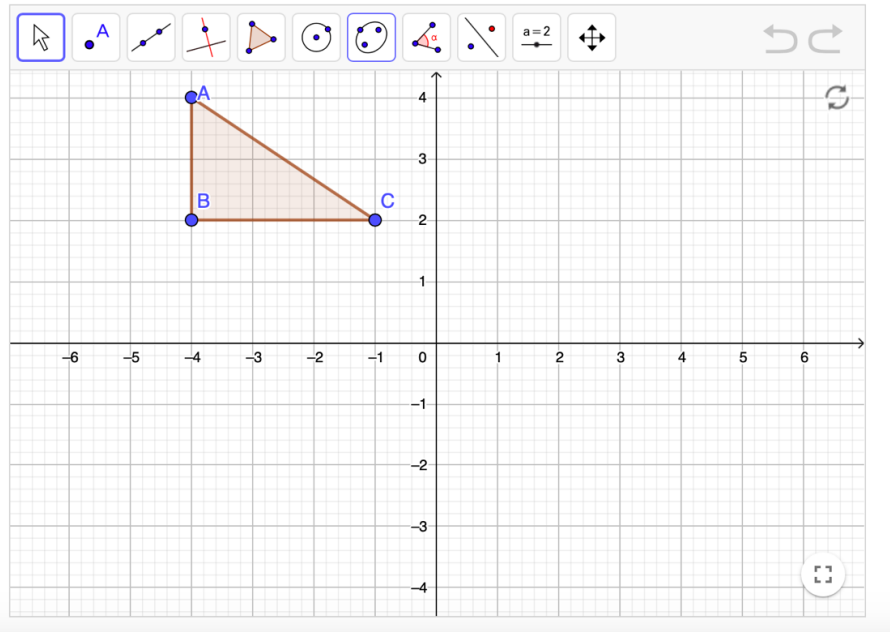
1. study how the object in the top line of the question is rotated;
2. picture in your mind what the object shown in the middle line of the question looks like when rotated in exactly the same manner;
3. select from among the five drawings (A, B, C, D, or E) given in the bottom line of the question the one that looks like the object rotated in the correct position.

What is the correct answer to the example shown above?

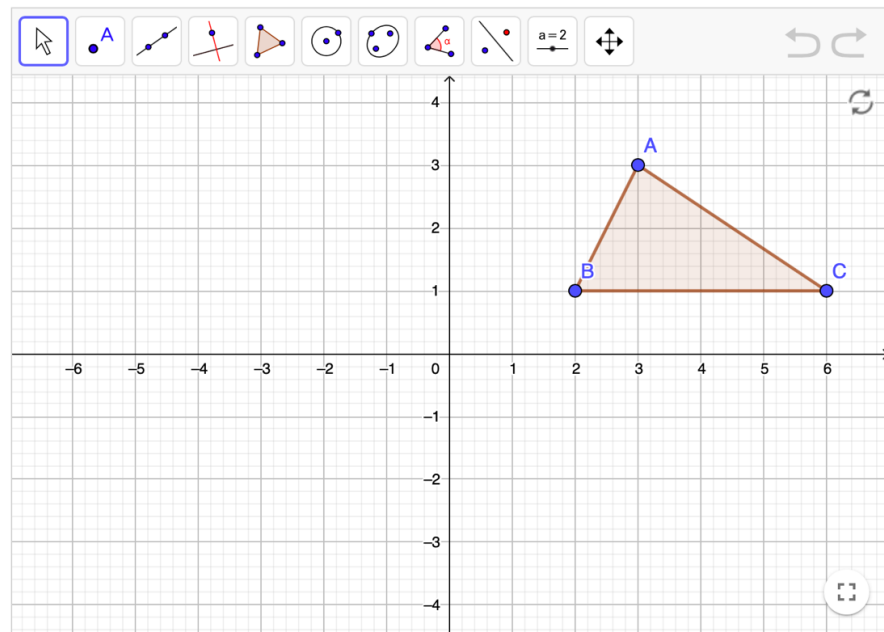
APPENDIX D – Sample of GeoGebra Activity: Transformation Practice

Perform the transformation as described.

Translate the object 5 units to the right and 3 units down.

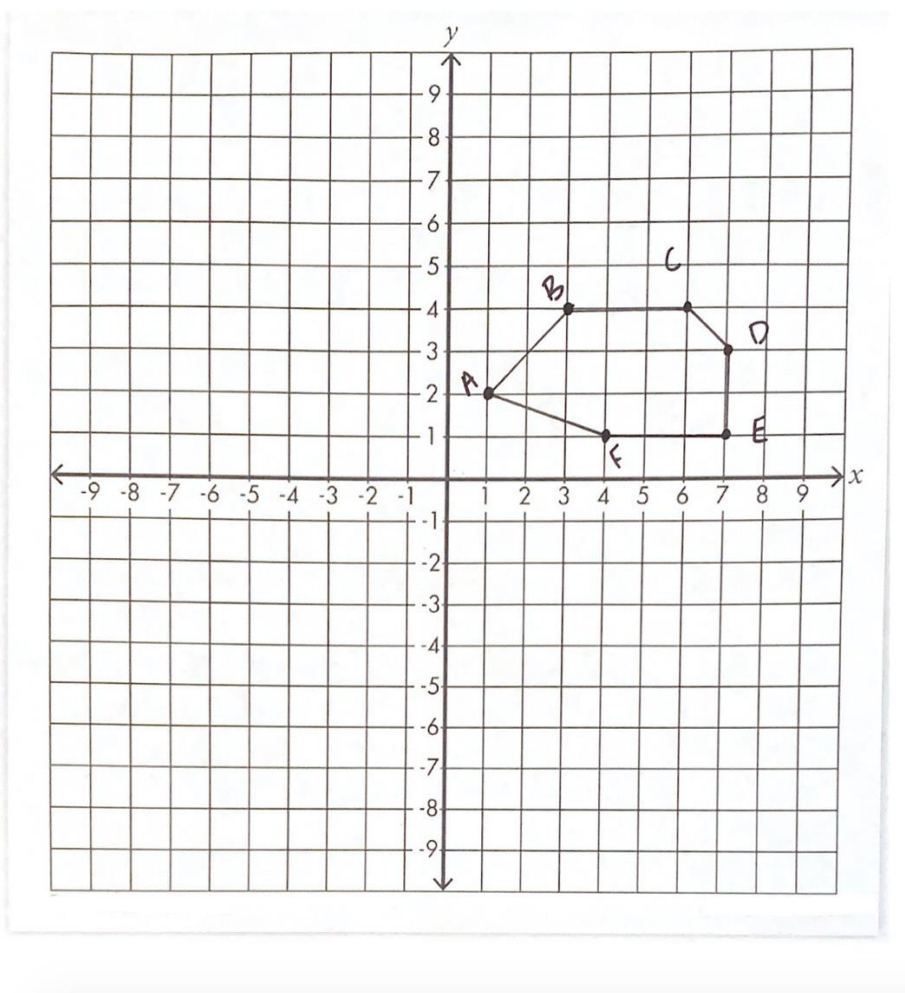


Rotate triangle ABC 180 degrees on point B



Full activity accessed: <https://www.geogebra.org/m/kgepiqsd>

Transformation Exploration



Transformation Exploration Questions

1. Give the ordered pair of each point you labeled on your **original** shape.

A _____ B _____ C _____ D _____ E _____ F _____

2. **Translate** your shape 10 units to the **left** and 1 unit **up**.
3. Give the ordered pair of each point on the **translated** shape.

A _____ B _____ C _____ D _____ E _____ F _____

4. Compare the ordered pairs of the original shape with the translated shape, and look for patterns. Write a rule for **translating** an ordered pair:

5. **Reflect** your original shape **over the x-axis**.
6. Give the ordered pair of each point on the **reflected** shape.

A _____ B _____ C _____ D _____ E _____ F _____

7. Compare the ordered pairs of the original shape with the **reflected** shape, and look for patterns. Write a rule for **reflecting** an ordered pair over the x-axis:

8. **Rotate** your original shape **180 degrees** around the origin.
9. Give the ordered pair of each point on the **rotated** shape.

A _____ B _____ C _____ D _____ E _____ F _____

10. Compare the ordered pairs of the original shape with the **rotated** shape, and look for patterns. Write a rule for **rotating** an ordered pair around the origin:

Bonus

1. What do you think a rule would be for translating 5 units right and 3 units down?
2. What do you think a rule would be for reflecting over the y-axis?
3. What do you notice about the last two shapes you drew?
4. Predict a rule for rotating 90 degrees counterclockwise around the origin.
5. Which transformation is tracing paper most helpful?

APPENDIX F – Exploring Dilations in the Coordinate Plane

Exploring Dilations in the Coordinate Plane

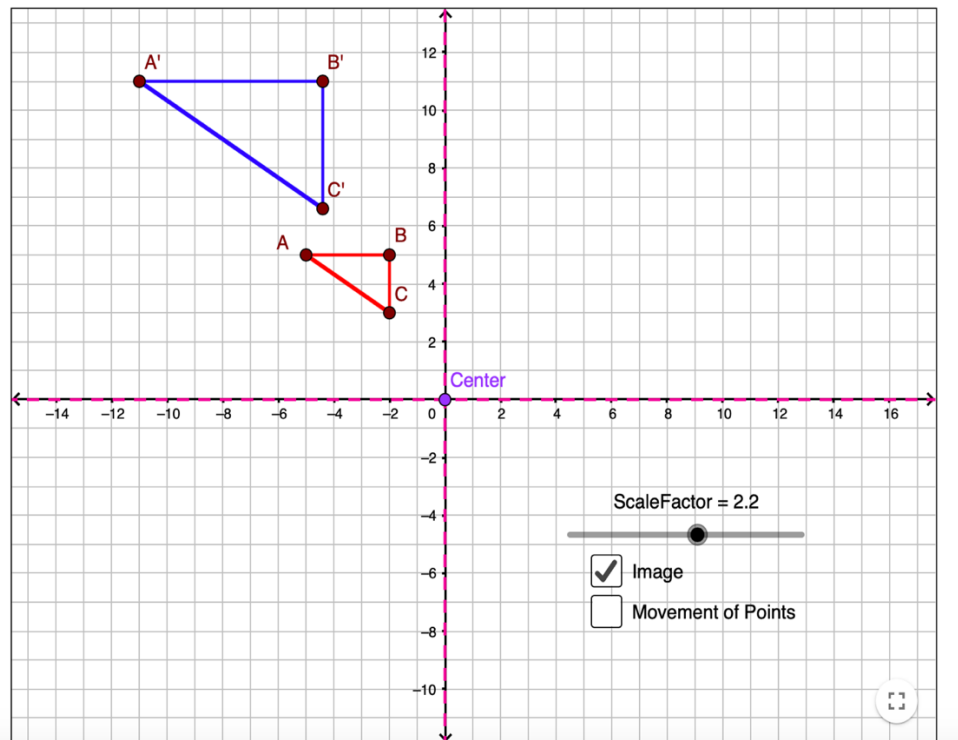
Author: Katrina Cushenberry, Rick Stuart, Kim Kembitzky

Topic: Dilation

Explore dilations on the following graphs. Be ready to discuss the following topics:

1. How does the scale factor influence the location of the pre-image and corresponding image points with regards to the center?
2. How does the scale factor and center influence the length of the pre-image sides and their corresponding image sides.
3. How does the scale factor and center influence the location of the image with regards to the pre-image.
4. Other relationships between the pre-image, image, scale factor and center.
5. Using the second graph find the relationship between the scale factor and the ratio of the area of the image to the area of pre-image.
6. Using the second graph find the relationship between the scale factor and the ratio of the perimeter of the image to the perimeter of pre-image.

Dilations



Interactive activity can be found: <https://www.geogebra.org/m/ksdmz4jh>

APPENDIX G – Instructions for Dilation Assignment

Directions:

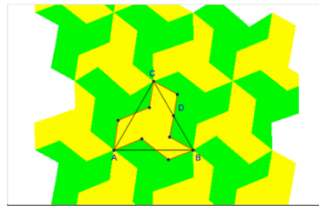
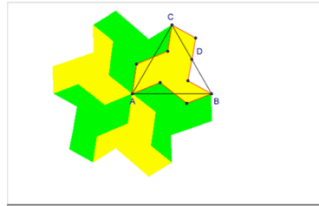
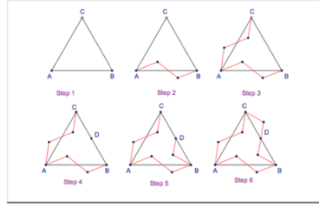
1. Decide on a sketch (what do you want to draw?). Your sketch must be hand sketched. It cannot be printed from computer.
2. Is your dilation going to be an enlargement or reduction?
Draw your original shape accordingly.
3. Must have at least 10 coordinate points labeled.
4. Decide on your CENTER OF DILATION and mark that as $(0,0)$.
5. Decide on your SCALE FACTOR and write that on the drawing.
6. Show your calculations on the back of the paper.
7. Draw the dilation using your calculated points.
8. Label the new coordinate points.
9. COLOR the dilation and pre-image.

APPENDIX H – Geometer’s Sketchpad: Tessellations that use Rotations Activity

Using the Sketch:

Students construct an equilateral triangle, labeled ABC , and use rotation to construct an irregularly-shaped tile based on the triangle. Students mark point A as the center of rotation and rotate the tile five times by the appropriate number of degrees to surround point A with tiles. They then mark point D as the center of rotation, where point D is the midpoint of segment BC , and rotate the group of six tiles by 180° .

Next, students examine the tiles around various points and determine the rotational symmetry about these points. Finally, they use the appropriate rotations to fill in tiles around points B and C and then experiment with their tessellation by dragging one of the original vertices.



Sketch Tips:

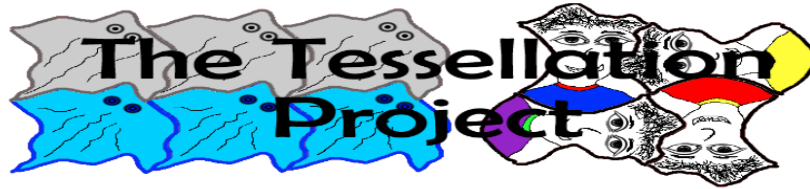
Sketch Tips show skills needed in this activity, and the step at which the skill is first used.

Sketch Tip	Tip Sheet or Tip Video
Step 1: Use a custom tool from a different document	Using Custom Tools
Step 3: Rotate an object using Transform Rotate	Rotating and Dilating
Step 4: Construct a midpoint using Construct Midpoint	Constructing Points
Step 9: Color an object using Display Color	Changing Color
Step 11: Undo actions using Edit Undo	Undoing and Redoing

APPENDIX I – Tessellations Activity from Geometry Common Core Curriculum (3 pages)

G.CO.A.5 ACTIVITY #3 – *geometrycommoncore*

6



OBJECTIVE:

To create a theme based tile and then to use that tile to tessellate an “8.5 by 11” piece of paper.

Use the six starter polygons provided to create six possible tiles. Don’t worry what they are until after you make them - just create them using the tips below and something will appear in at least one of them.

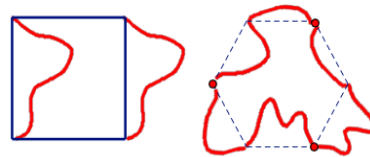
With the two regular hexagons (and the two squares) create one tile using translations and the other tile using rotations - that way you will have a little variety in the type of shapes you create. From these six shapes search for one that you see something in.... a theme (a frog, a flower, a mountain range, etc...). To find something, rotate the shape continually asking yourself - **WHAT IS THIS?** If you don’t find anything, give it to a friend to look over. The key is not to make something intentionally from the beginning (don’t start with the thought – “I want to make an elephant”). This will only bring frustration. Just design the shape using the tips below and the shape will dictate an appropriate meaning.

INSTRUCTIONS:

1. Cut out all six regular polygons (be as exact as you can when cutting).
2. Using a pencil to draw the cut lines on the polygon and then cut them out. When you cut a piece out immediately tape it to the side that it is being moved to using your transformation rules learned earlier. Use the vertices and the midpoint to help you line it up correctly and then tape it so that there are no gaps/holes.
3. When designing the cut lines follow these suggestions.

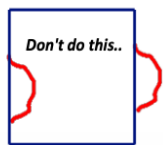
Design from vertex to adjacent vertex. (Start at a vertex and end at the adjacent vertex)

- This makes it easy to tape in the right place.
- This transitions sides smoothly.
- This eliminates straight edges from the shape.
- Use side midpoints as a way to make sure you are lining up the cuts correctly.



Use Soft Curves

- When trying to determine what the shape is... more things have soft curves.



4. Select one of the six shapes as your tracer shape. (A tracer shape on cardstock is easier to trace.)
5. Start Tracing - Place your first tile in the approximate center of the 8.5 by 11 sheet so a full shape is made and then build from there. Trace the exterior of the shape continuously until the entire paper is covered – even shapes that don't fully fit on the page. There will be small errors due to human error. Take a dark marker (probably black) and outline all of the shapes. This will fix any small errors formed when copying the shape. Then start working on details.
6. Tile the entire 8.5 by 11 sheet of paper.... Don't leave any spaces even though some shapes won't be complete.
7. Color and Design the Tessellation

DESIGN TIPS

Variety – Create 2 or 3 different versions of the same shape and then repeat a pattern with those. Too many variations... creates confusion... a few versions creates a nice variety to the diagram.

Coloring – Use strong deep colors that show up.

Details – Detail your shape so we know what it is. We don't want a blob... the shape must be something. The details of the shape give the tessellation greater emphasis.

8. When you are done... tape your tracer shape to the back of the tessellation and write your name on the back as well.

Project Rubric:

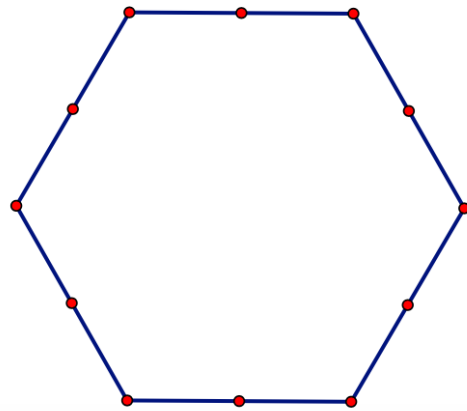
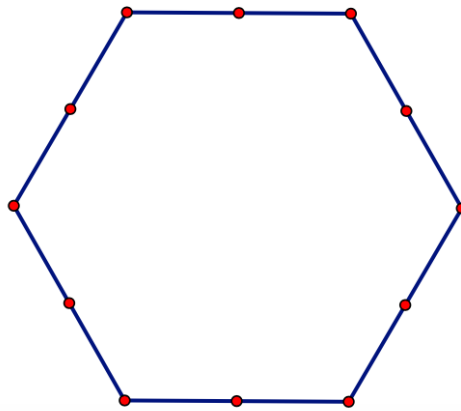
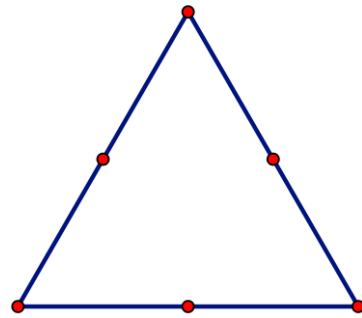
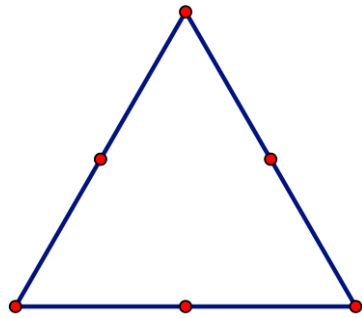
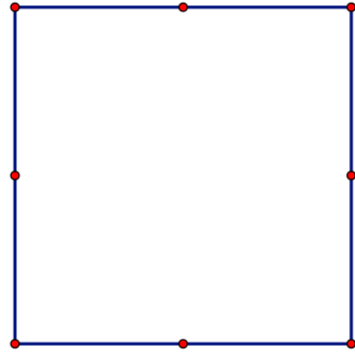
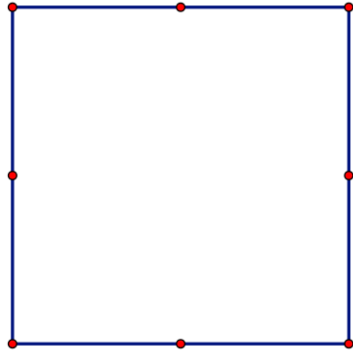
Tracer Shape & Name on Back	1 2
Shape is Themed (Has Meaning)	1 2 3
Shape Detail (Clear what it is)	1 2 3 4 5
Color Strength & Pattern	1 2 3 4 5
Entire Paper is Tiled	1 2
Shape is Mathematically Correct	1 2 3

TOTAL ----->

/2
/3
/5
/5
/2
/3
/20

Watch the Tessellation Project Video for more help.





APPENDIX J – Building a 3D Object (2 pages)

NAME: _____

PERIOD: _____

DATE : _____

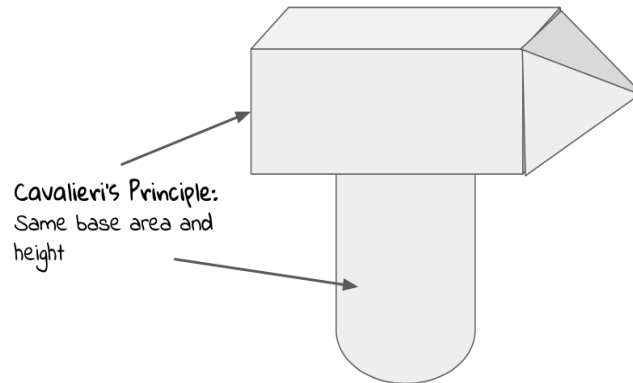
BUILDING A 3D OBJECT (CAVALIERI'S PRINCIPLE)

During this project, you will be grouped in groups of 3-4 and build a 3D object of your choice. As part of your project you will show the step-by-step process how you calculated the volume and show that two 3D figures have the same volume, based on Cavalieri's Principle.

Phase 1 : Design

- Your object must contain **at least 3 components** of which one is either a prism or a cylinder and another is either a pyramid or a cone.
- **Two solids** need to have the **same volume** which can be proven by Cavalieri's Principle (cannot be exactly the same object - different shapes or skewed).
- Creativity earns **extra points**. You can do all types of prisms, not just rectangular prisms. **BE CREATIVE**

Example of a 3D Design



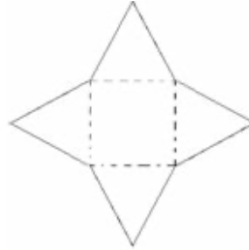
Phase 2 : Calculations

- You will draw a 3-dimensional diagram of your individual components and label all lengths including units.
- You will calculate the volume of each object based on your determined side lengths (make sure to include units)
- You will show that two objects have the same volume based on Cavalieri's Principle (write an explanation in full sentences which includes your values of the base area and height)
- Each person needs to submit their own calculations (but each group will only build one model).

The majority of your project grade will be based on clarity, accuracy, and organization of your math work!!!

Phase 3 : Building

- Use different colors of paper for different components. Make your calculations on paper first before cutting.
- Research the net of your 3D figure on the internet and then apply the net with the correct dimensions to the cardstock
- For example, to build a pyramid you will need the following net:



Grading Rubric

Calculations - 10

- 3-dimensional diagrams of your individual components and all lengths are labeled _____/5
- Volume calculations of each object are correct, state formulas, and include units _____/10
- Explanations of Cavalieri's Principle shows that two objects have the same volume _____/5

Design - 10

- Your design includes at least 3 objects _____/2
- Your design has two objects that have the same volume (Cavalieri's Principle) _____/2
- Each object uses a different cardstock color _____/3
- The final design object is neatly assembled _____/3

Extra Credit - 3

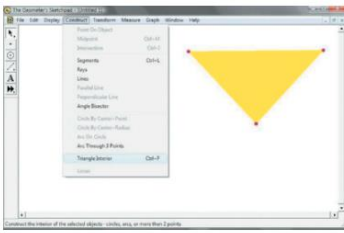
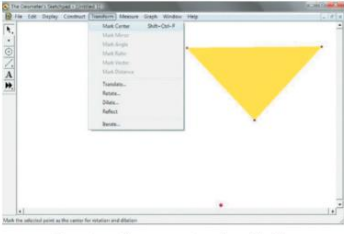
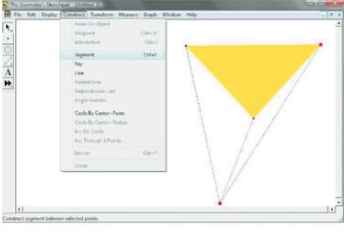
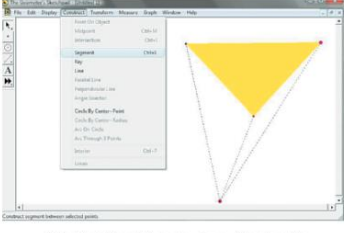
- Creative design, use of different prisms _____/3

Total Score _____/30

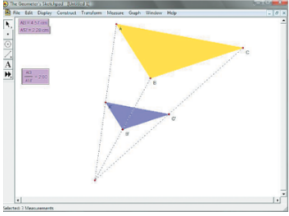
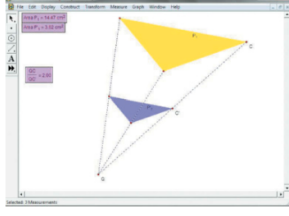
Design and Calculations are due by _____

APPENDIX K – Proportional Reasoning with a Pyramid (Mamolo, et al., 2011)

Instructions for Creating the Sketchpad Dilation Drawing

 <p style="text-align: center;">Creating a colored triangle</p>	<ol style="list-style-type: none"> 1. To create the triangle, select the Point Tool on the tool bar at the left-hand side of the sketch, and draw three points on the page. 2. Using the Arrow Tool, select all three points. 3. Go to the Construct tab in the menu, and select "Triangle Interior." 4. Specify colors of the triangle or the corner points using the "Color" item in the Display tab of the menu.
 <p style="text-align: center;">Constructing a center for dilation</p>	<ol style="list-style-type: none"> 5. Once the triangle interior is established, draw a fourth point on the page (not on the triangle). 6. Using the Arrow Tool, select this point (if it is not already selected). 7. Go to the Transform tab in the menu, and select "Mark Center." This establishes the fourth point as the center of dilation.
 <p style="text-align: center;">Connecting vertices to the center</p>	<ol style="list-style-type: none"> 8. Connect the center of dilation to the points of the triangle as follows: <ol style="list-style-type: none"> A. Use the Arrow Tool to select the center point and one of the corner points. B. Go to the Construct tab, and select "Segment." C. Repeat for the other two vertices. 9. Specify the line segment style or color by selecting the segment and using the "Line Width" or "Color" items, respectively, in the Display tab.
 <p style="text-align: center;">Dilating the triangle by a fixed ratio</p>	<ol style="list-style-type: none"> 10. Using the Arrow Tool, select the triangle, including its interior and vertices. 11. Go to the Transform tab in the display menu, and select "Dilate." 12. Input an appropriate ratio and click "Dilate." This will create a reduced or enlarged triangle (depending on the ratio), which is a dilation of the original.

Instructions on Including Measurements in the Sketchpad Drawing

 <p>Keeping track of lengths and ratios</p>	<ol style="list-style-type: none">1. To add distance measurements, select two vertices on one of the triangles, go to the Measure tab in the menu, and select "Distance."2. Labels can be tailored through the Edit tab by selecting and adjusting the "Properties."3. To add ratio measurements, select the two distance measurements, go to the Measure tab, and select "Calculate."4. The lengths can be input into the calculation by selecting them from the Values menu. Dividing the lengths will yield the ratio.
 <p>Keeping track of areas and ratios</p>	<ol style="list-style-type: none">5. To add area measurements, select both of the triangles, go to the Measure tab in the menu, and select "Area."6. Labels can be tailored through the Edit tab by selecting and adjusting the "Properties."7. To add ratio measurements, select the center point, a point on the dilated triangle, and its corresponding point on the original triangle. The order in which the points are selected will impact the ratio measured.8. Go to the Measure tab and select "Ratio."

APPENDIX L – Madison County Schools Approval Letter



SUPERINTENDENT OF EDUCATION
Charlotte A. Seals

BOARD OF EDUCATION
Dr. Pollia Griffin, President
Sam Kelly, Secretary
Dr. Jason Dean
Bill Grissett
Philip Huskey

September 14, 2022

To Whom It May Concern:

I give permission to allow Jennifer Crissey, graduate student in Mathematics Education at the University of Southern Mississippi, to conduct research for her dissertation during the 2022-23 school year as a requirement for the Doctor of Philosophy. Please reach out with questions.

Sincerely,

Greg Paczak, Ph.D.
Director of Research & Development
601-499-0800
gpaczak@madison-schools.com

APPENDIX M – IRB Approval Letter

Office of
Research Integrity



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NOTICE OF INSTITUTIONAL REVIEW BOARD ACTION

The project below has been reviewed by The University of Southern Mississippi Institutional Review Board in accordance with Federal Drug Administration regulations (21 CFR 26, 111), Department of Health and Human Services regulations (45 CFR Part 46), and University Policy to ensure:

- The risks to subjects are minimized and reasonable in relation to the anticipated benefits.
- The selection of subjects is equitable.
- Informed consent is adequate and appropriately documented.
- Where appropriate, the research plan makes adequate provisions for monitoring the data collected to ensure the safety of the subjects.
- Where appropriate, there are adequate provisions to protect the privacy of subjects and to maintain the confidentiality of all data.
- Appropriate additional safeguards have been included to protect vulnerable subjects.
- Any unanticipated, serious, or continuing problems encountered involving risks to subjects must be reported immediately. Problems should be reported to ORI via the Incident submission on InfoEd IRB.
- The period of approval is twelve months. An application for renewal must be submitted for projects exceeding twelve months.

PROTOCOL NUMBER: 22-1251
PROJECT TITLE: Spatial Visualization in High School Geometry Students
SCHOOL/PROGRAM Center for STEM Education
RESEARCHERS: PI: Jennifer Crissey
Investigators: Crissey, Jennifer-Wan, Anna~
IRB COMMITTEE ACTION: Approved
CATEGORY: Expedited Category
PERIOD OF APPROVAL: 28-Oct-2022 to 27-Oct-2023

Donald Sacco

Donald Sacco, Ph.D.
Institutional Review Board Chairperson

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