The Relationship Between Teachers’ Beliefs Concerning Mathematical Problem-Solving and the Strategies Used for Teaching Third - through Fifth - Grade Students

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The Relationship Between Teachers’ Beliefs Concerning Mathematical Problem-Solving and the Strategies Used for Teaching Third - through Fifth - Grade Students

by

Jasmine Thomas

A Thesis
Submitted to the Honors College of
The University of Southern Mississippi
in Partial Fulfillment
of the Requirements for the Degree of
Bachelor of Science
in the Department of Curriculum, Instruction, and Special Education

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ABSTRACT

Everyone uses problem-solving on a daily basis: paying bills, managing time, and making decisions. It is important to be able to solve problems effectively, especially in mathematics. In 2007, the Mississippi Department of Education produced a set of mathematical objectives for third-through fifth-grade students. These objectives use the phrase problem-solving, without defining its use in the mathematics classroom. To discover what mathematics teachers were actually executing in the classroom, this study used a valid online survey administrated through Qualtrics. The survey focused on determining third-through fifth-grade teachers’ definition of a problem, their beliefs about problem-solving, and the problem-solving strategies they used while instructing students. Additionally, teachers’ thoughts and opinions about mathematical problem-solving were examined by asking a variety of open-ended questions. The participants were third-through fifth-grade teachers in three school districts in the area of Hattiesburg, Mississippi.

Results indicated there are some differences among teachers concerning what problem-solving actually is and what problem-solving skills are necessary for students to have. Further, there were two distinct opinions on how many mathematical strategies a competent student should use for successful problem-solving. Interestingly, teachers could not distinguish a problem as abstract; instead, they viewed a problem as concrete. The results of this study demonstrated that problem-solving should certainly be further examined in schools across Mississippi and further implemented into the classroom.

**Key Words:** elementary education, Mississippi, mathematics, classroom implementation, educator opinion, Honors College, Southern Miss, South Mississippi.
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CHAPTER ONE: INTRODUCTION

Problem-solving

Everyone uses problem-solving, whether he or she is aware of it or not, on a daily basis. Problem-solving is the ability to carry out methodical procedures to reach a plausible solution to a mathematical problem. The most common type of problem encountered daily is decision-making. Examples of decision-making problems include “Should I move in order to take another job?” or “How am I going to pay this bill?” (Jonassen, 2000, p. 76). This type of problem requires thinkers to make a resolution on a matter they may or may not have come across before. Troubleshooting, a kind of problem-solving strategy most people encounter, happens when a person attempts to find a solution through a series of trial-and-error processes. Decision-making problems and the troubleshooting strategy are also used in answering mathematical problems.

National study

In a national education study, the Mississippi Educational System has ranked between 45th and 50th in academic achievement for the past few years (Miller, 2012, n.p.). On a grading scale from A+ to F-, Mississippi has scored a C-. Although the system is attempting to place more emphasis on mathematics and sciences in public funding, the improvement in ranking is minimal. Even if researchers and educators do not fully agree, it can be deduced from this study that the low scores in mathematics have a lot to do with
the way the students are taught to solve problems. This is because problem-solving is needed to accomplish all mathematical tasks students will encounter. Teachers’ beliefs and use of problem-solving strategies are critical to understanding the nature and effectiveness of mathematics curriculum and instruction. It is essential to understand teachers’ influence on the idea of a problem and problem-solving simply because “when teachers’ beliefs rely on the curriculum, it will be implemented within the classroom” (Bright and Vacc, 1999, p. 91). Teachers significantly impact the ideas and strategies that students use to unravel mathematical problems.

**Lack of research**

There are many studies on elementary and secondary school mathematics; however, most of those studies do not contain in-depth descriptions of problem-solving or teaching strategies for problem-solving as they relate to distinct grade levels. The lack of relevant research shows that the majority of those studies focus on teachers’ beliefs about what a mathematical problem and mathematical problem-solving is. Yet, those studies do not necessarily highlight strategies teachers use based on their beliefs. In essence, this study offers contributable insights into teachers’ beliefs and strategies used to solve problems.

**Statement of the problem**

Chapter Two will indicate that in mathematical literature there is a connection between the beliefs that teachers have in regard to the effectiveness of instructing students to use specific strategies. Nonetheless, the body of empirical knowledge on the effect of teacher beliefs on the strategies used particularly in third - through fifth - grade is limited. The purpose of this study is to examine teachers’ beliefs concerning
mathematical problem-solving definitions, mathematical strategic analysis, and implementation into their classrooms.
CHAPTER TWO: LITERATURE REVIEW

Introduction

The National Council of Teachers of Mathematics (NCTM) views problem-solving as a fundamental skill needed to succeed in mathematics. This national, professional organization for teachers of mathematics has four principles established to enable students to solve problems effectively:

1. Build new mathematical knowledge through problem-solving.
2. Solve problems that arise in mathematics and in other contexts.
3. Apply and adapt a variety of appropriate strategies to solve problems.

In addition to the NCTM’s standards, the Common Core State Standards Initiative was designed as a national curriculum. Common Core State Standards Initiative is a set of standards for kindergarten through twelfth grade students for mathematics and English, which are in place for most of the United States This curriculum has been adopted by forty-five states, including Mississippi. The mathematics section of the curriculum requires students to make sense of problems, monitor chosen processes for solving the problem, solve the problem, and reflect upon and evaluate the process used (Common Core State Standards Initiative, 2012, n.p.). The standards and curriculum, put forth by the NCTM and Common Core, are broad guidelines for what states across the country should incorporate into their mathematics curriculum. How each individual state actually applies these guidelines still needs to be identified. In 2007, the Mississippi Department of Education (MDE) produced a set of mathematics objectives for mathematics for third-through fifth-grade students. These objectives mention the phrase “problem-solving,”
without elaborating on its role in the classroom. Shedding light on the definition of problem-solving, and all its elements will aid Mississippi educators in understanding its importance and necessity in the mathematics classroom.

**Definitions and Important Distinctions**

To illustrate the definition of problem-solving, Lorain County Community College (LCC) notes that “problem-solving is a tool, a skill, and a process” (2012, n.p.). The college also advocates that problem-solving aids students in implementing a strategy and accomplishing a goal. More importantly, this calls attention to the fact that everyone will commonly encounter “barriers” or obstacles while attempting to solve problems; however, understanding and implementing strategies assists students in solving problems. These ideas will be supported later in the chapter.

M. E. Martinez of the Department of Education at the University of California (1998) defines problem-solving as a specific process that requires a solver to achieve a goal when the path is not clear. Martinez goes further to explain the level of problems range from simple functions such as finding lost keys, to more complex including taking the steps to be successful in life. Notably, Martinez offers a sound idea in problem-solving which most educators may argue against – there is no explicit method of solving a problem. Martinez suggests problem-solving can only take place when there is no clear method and requires the solver to go beyond traditional computation. To support this concept, Martinez demonstrates that problem-solving is not only an advanced function for reasonable thinkers, but also everyone must apply problem-solving to carry out daily tasks.
Jonassen (2000) agreed with Martinez, but elaborated further by emphasizing that it is important to highlight the difference between a problem and an exercise. Jonassen describes a problem as “an unknown entity in some situation (the difference between a goal state and a current state)” (Jonassen, 2000, p. 65). Within the teaching field of mathematics, some misinterpret the meaning of a problem versus an exercise. An exercise is given to build upon a skill already learned, whereas a problem is the opportunity to choose between multiple skills and strategies learned to decipher a solution. Thus, students solving mathematical problems are not just relying on skill to complete a problem. Instead, they are applying their knowledge of skills and strategies to unravel a problem.

To carry out any method of problem-solving, one must have the ability to critically think. Critical thinking, as defined by Indiana University Southeast (2011), is the work of a specific set of skills and attitudes that assist people in evaluating an argument based on rationalization. To some, critical thinking may be narrowly defined as a critic who only states dissenting comments about an element. Nevertheless, Indiana University Southeast mentions that critical thinking is creative, and summons people to use their imagination and take a different route to discover a plausible solution. Of great importance is the notion that problem-solving actually necessitates skills beyond critical thinking, and limited critical thinking skills can inhibit solvers to reach an answer.

Norman Webb (as cited in Professional Developments Associates, 2009) further illustrates this. Webb’s Depth of Knowledge Levels (DOK) is widely used in subject areas and offers its own definition of mathematical problem-solving. According to Webb, there are four distinct levels of knowledge: recall, skill/concept, strategic thinking, and
extended thinking. A mathematical exercise falls into level one because it is a routine procedure, whereas solving a mathematical problem fits into level four. It calls solvers to analyze, select, and apply concepts for successful solving.

Mathematical problem-solving is viewed mostly as a process with certain characteristics. For the most part, the process requires engaging in cognition and seeking a solution without an obvious method. Mathematical problem-solving should be appropriate for the level ability of solvers and promote flexibility in thinking (Chamberlin, 2012, p. 17). While determining a working definition of a problem and problem-solving is essential, it is even more serious to consider the role teachers play in students’ abilities of mastering problem-solving in mathematics.

**Teachers’ Effect on Mathematical Problem-Solving**

It is critical to call attention to the actual difference between the two terms “beliefs” and “knowledge.” Beliefs are typically a point of view that a person, or in this case a teacher, has about a specific entity. These beliefs do not have to be proven valid. Beliefs can stand alone without proof simply based on feelings, evaluations, and personal experiences. Be as it may, knowledge must be proved (Grouws, 1992, p.129-130). It is common that before prospective mathematics teachers go through a baccalaureate program, they tend to carry invalid beliefs about mathematical problem-solving. Perrenet and Taconis (2009) show a group of teacher candidates’ beliefs about problem-solving changed upon baccalaureate completion. Those students revealed that problem-solving requires special attention to metacognition, which calls for a solver to know themselves as a thinker and learner.
The study by Perrenet and Taconis also suggests that problem-solving is not a routine. Some teacher candidates or seasoned teachers may not fully be aware of this aspect of problem-solving. This may demonstrate that primary and secondary mathematics students do not have a solid foundation to unravel problems. Instead, they solve problems with a routine in mind, not adhering to the creativity that problem-solving ignites.

Additionally, teachers play a significant role in students’ learning of mathematical problem-solving skills. In stressing the teachers’ critical role, the NCTM encourages teachers to select engaging problems and to develop a classroom that fosters exploration and critical thinking (NCTM, 2012). Vacc and Bright’s study on *Elementary Preservice Teachers’ Changing Beliefs and Instructional Use of Children’s Mathematical Thinking* (1999) found that “teachers’ beliefs about teaching and learning mathematics significantly affect the form and type of instruction they deliver” (p. 91). Considering teachers impact on the skills and strategies that students learn and apply, it can be deduced that if teachers have limited beliefs about problem-solving, then students may have an inadequate ability to problem solve.

Teachers’ beliefs about mathematical problem-solving tend to vary based on experiences. As cited in Charles’ (Silver, NCTM, 1989) *The Teaching and Assessing of Mathematical Problem-solving*, a survey was conducted by the NCTM with sixteen elementary school teachers. Five of the sixteen teachers explained that a “problem” was a grouping of words to form a story that students must answer. The remaining eleven of the sixteen teachers had different views of problem-solving: puzzles, mazes, and optical illusions. Between the two groups of teachers, it was clear that five teachers held the
notion that a “problem” was concrete whereas the remaining eleven had an abstract perception of a problem. Those same five teachers articulated characteristics of problem-solving:

1. It is the answer that counts in mathematics, once one has an answer, the problem is done.
2. One must get an answer in the right way.
3. An answer to a mathematical question is usually a number.
4. Every context (problem statement) is associated with a unique procedure for “getting” answers.
5. The key to being successful in solving problems is knowing and remembering what to do.

(Charles, Silver, NCTM, 1989, p. 234-236).

While those five teachers held distinct ideas about the nature of problem-solving, the other eleven thought of problem-solving as challenging, mindboggling, and fun. In summation, these eleven teachers believed that problem-solving is “the processes of searching for and discovering new ideas” thus, it is not “dependent on learned skills as other mathematical activities” (Charles, Silver, NCTM, 1989, p. 236). These eleven teachers’ views of problem-solving held that creativity is required, whereas the other five teachers viewed problem-solving as clear-cut with no room for experimentation. This illustrated that there are divergent opinions about mathematical problem-solving between teachers of the same discipline.

Margaret I. Ford (1994) also examined fifth-grade teachers’ and students’ beliefs about mathematical problem-solving. Using qualitative research methods (interviews), the results showed “students’ beliefs about mathematical problem-solving are, for the most part, consistent with the beliefs held by the teachers.” Also, ten of the participating
fifth grade teachers thought students’ general abilities contribute to their success or failure in mathematics. In contrast, the student participants reported that their success or failure had more to do with how much effort they put forth. The problem-solving activities in the participating classrooms provided a base for students’ computational abilities, although the ten teachers discouraged the use of calculators. Finally, the study reported that teachers tend to overestimate the students’ abilities to do problems involving computation and underestimate students’ chances to do reasoning problems. These results indicate teachers and students have misconceptions about their potential problem-solving abilities. More importantly, the strongest and most definitive conclusion contradicts the finding that students acquire the problem-solving beliefs of their mathematics educator, as found in Charles’ study (Silver, NCTM, 1989).

It is evident that teachers must have knowledge in mathematics to successfully educate their students, but it is not clear just how much knowledge is needed for effective teaching. One set of educators deem that teachers must not only have an in-depth knowledge of the specific type of mathematics they are training students in, but also they should be aware of the mathematics their students will encounter in the future (Gouwers, 1992, n.p.). In this way, teachers can be sure to prepare students effectively so they will thrive and not fall behind in mathematics, which is a common problem in mathematics education today.

Another group of educators take a more social approach. They go as far as to suggest teachers must be fully aware of the ethnicity and cultural differences of their students. These educators contend that doing so will assist the teachers in understanding the various ways those students of divergent background learn and problem solve
The final two sets of educators assert that knowledge, concerning how students think and learn, is just as important pedagogical knowledge for teachers to obtain (Gouwers, 1992, n.p.). It must be noted that a teacher’s knowledge in mathematics is not narrowly defined in the subject itself; rather it requires a well-rounded awareness.

As noted by the National Council of Teachers of Mathematics (2012), teachers take on an important role in educating students in mathematics. The practices teachers utilize have weighty effects and are mostly seen on four distinct levels. Level I identifies with the characteristics of the teachers such as years of experience, number of mathematics courses taken and even enthusiasm. Level II measures the student-teacher interaction in which observation is key. Teachers’ behaviors including the depth of inquiry and length of time spent on lessons influence their students’ likelihood to exhibit success in mathematics. Level III quantifies student attention and the content students master. Finally, level IV highlights the complexity of teaching, which has an impact on teachers’ behaviors and beliefs during instruction (Grouws, 1992, n.p.).

As reported in NCTM’s Problem-solving in School Mathematics (2012), two surveys were conducted to discover preferences and priorities in mathematical problem-solving. Seven conclusions were drawn from the findings. The majority of participants indicated that problem-solving should be given more emphasis in the mathematics classroom in the years to come. For the most part, participants agreed on the role problem-solving serves in mathematical instruction. For elementary and secondary school levels, four strategies of problem-solving were deemed acceptable:

1. translating a problem to an equation
2. constructing a table and searching for patterns
3. drawing pictures or diagrams to represent a problem
(4) solving a simple problem first and extending the solution to the original 
(NCTM, 2012).

It was reported there are no particular teaching methods to conduct problem-
solving, but “using a mathematical problem as the means or the vehicle to develop and 
introduce mathematical topics” was largely agreed upon as the best method for teaching 
(NCTM, 2012). A majority of participants counted problem-solving as essential, and it 
should be introduced to students at the beginning of mathematics courses. Participants 
came to a consensus that modifying mathematics curriculum for certain groups of 
students is not suitable or beneficial for those students to succeed. Considering these 
seven conclusions, it is clear that problem-solving is viewed as important and can be 
implemented by instructors who adhere to the most effective strategies and teaching 
methods for their students.

General Problem-Solving Strategies

Lorain County Community College (2012) sheds light on a problem-solving six-
step process, which is quite useful to its students. The first step is defining the problem by 
asking a series of questions:

- How is the current situation different from what I actually want it to be?
- What do I actually want, or how do I actually want things to be?
- What is preventing me from achieving my goals, or from things being the way 
  I want them to be?

(Lorain County Community College, 2012, n.p.).

The method even advocates writing the answers, for sample questions, to avoid any 
forms of confusion. If there are multiple problems to be determined, then the solver 
should prioritize them. Once problems are defined, the next step of the process
recommends that solvers take on multiple perspectives in viewing the problem. In the third step, solvers are required to set specific, explicit goals. Step four requires the solver to list as many solutions as possible without being concerned with practicality. Also, step four suggests that it may be beneficial for students seeking solutions to ask others that may have faced a similar problem. After seeking solutions, step five advocates that problem solvers critically analyze the solution by asking specific questions:

- Is it relevant to my situation?
- Is it realistic?
- Is it manageable?
- What are the consequences – both good and bad?
- What is the likelihood that it is going to help me reach my goal?

(Lorain County Community College, 2012, n.p.).

The last step in this process is to implement the chosen explanation. If a solver feels secure about the solution, then he or she can continue to benefit from their results; however, if the solution does not fit, then the solver should modify the solution. Generating solutions is an on-going process, and it may take several steps to finally come to a plausible answer. These general steps lead the way into subject matter strategies that help students to solve a variety of problems. Interestingly, problem-solving is not just a critical component in mathematics.

**Problem-Solving Strategies in Reading**

While elementary students are being taught in mathematics to do simple computations that lead up to complex problem-solving, they should also be doing the same in the subject of reading. Garner (1984) suggests this idea and further elaborates on it by stating, “Problem-solving processes are the skills involved in producing alternatives
for a common situation and reducing those alternatives to a solution” (Garner, 1984, p. 36). This statement suggests that problem-solving, just like in mathematics, is needed in the subject of reading for one main reason – it increases elementary students’ achievement level. The two main strategies that Garner recommends are brainstorming and evaluating alternatives. Brainstorming appears to be an effortless strategy for most. However, Peck, as cited in Garner (1984, p. 37) proposes that brainstorming is:

“A conference technique by which a group attempts to find a solution for a specific problem by amassing all the ideas spontaneously contributed by its members…participants quickly bring forth a multitude of ideas and soon run out of conventional ideas. When this happens, the ideas previously suggested can be adapted, combined and rearranged to form new ideas. Thus, the participants begin to perceive new and unusual relationships among their thoughts and experiences.”

The second strategy that Garner mentions is evaluating alternatives, which requires established criteria that can be applied to the alternatives that can be eliminated with further inspection. When young learners apply these strategies, reading becomes alive, and is not just a system of stating words that make sentences and paragraphs.

Overall, Garner’s suggestion of reading as a subject that uses problem-solving for ultimate success offers a sound idea – problem-solving is needed to put together the meaning of ideas in words, sentences, paragraphs, or reading. Then it is necessary to be used in the study of mathematics. Solving mathematical problems and reading are complex skills to tackle; therefore, problem-solving is necessary to bring about understanding to the young learner.

**Strategies Advocated by Researchers**

Krulik (1980) maintains there are two types of problems: standard textbook problems and process problems. For the purpose of this study, process problems will be considered.
Process problems either require more than one strategy or no algorithm at all. Many teachers find George Polya’s (1980) four-step problem-solving process. Polya’s method calls for solvers to understand the problem and to ask, “what is the unknown?” Then, the solver must devise a plan that is sufficient to find the unknown. Plans can involve making an orderly list, working backwards, or even drawing a picture. Once the plan is chosen, the solver should carry out the plan. Finally, Polya’s method suggests that the solver reflect over the process and results.

In addition to Polya’s problem-solving process, mathematics educators find the problem-solving strategy “the anatomy of a problem”, theorized by Jesse Rudnick (Krulik, 1998), very beneficial when teaching. This method requires students to separate a problem into facts, distracters, a setting, and a question before attempting to seek a solution. For example, consider the following problem:

One night, the king could not sleep, so he went down into the royal kitchen, where he found a bowl full of mangoes. Being hungry, he took 1/6 of the mangoes. Later that same night, the queen was hungry and could not sleep. She too found the mangoes and took 1/5 of what the King had left. Still later, the first prince awoke, went to the kitchen, and ate 1/4 of the remaining mangoes. Even later, his brother the second prince ate 1/3 of what was then left. Finally, the third prince ate 1/2 of what was left, leaving only three mangoes for the servants. How many mangoes were originally in the bowl? (Peters, 2012, n.p.).

In the anatomy of a problem, facts are defined as true, explicit statements. For this problem, two examples of facts would be the king taking 1/6 of the mangoes and the queen taking 1/5 of the remaining mangoes. The setting is implied information, which in this problem is 1/6 of the mangoes were eaten and 5/6 of the mangoes are remaining. Adding those two fractions together will make a whole: 6/6 is equal to 1. Distracters are words that can be changed and will not affect the outcome of the problem. In the problem
above, distracters include mangoes, king, servants, and bowls, among others. The final piece of anatomy in a problem is the question that simply asks the solver to find the unknown.

The general and procedural strategic plans above explain specific steps to fully gain understanding of a problem, deciding which method to use, puzzling out a solution to a problem, and finally reflecting. In particular, Long (2009) provides several example problems and strategies that mathematics teachers find beneficial when instructing third-through fifth-grade students on disentangling problems. *Mathematical Reasoning for Elementary Teachers* is an adopted math textbook/curriculum for teacher candidates at The University of Southern Mississippi. The *Houghton Mifflin Mathematics* textbook implements problem-solving throughout the entire text and suggests problem-solving gives students a chance to apply previously learned skills that they may encounter in standardized tests (Greenes, 2005). In addition to mentioning the importance of problem-solving, the textbook supplies a variety of mathematical problem-solving strategies. It also places emphasis on those that are the most beneficial to teachers, while instructing students in problem-solving.

In *Mathematical Reasoning for Elementary Teachers*, teacher candidates are instructed on this type of problem: “Old MacDonald had a total of 37 chickens and pigs on his farm. All together, they had 98 feet. How many chickens were there and how many pigs?” This is similar to one of the problems third-through fifth-graders may face, which requires them to engage in critical thinking (Long, 2009). The problem cannot be unraveled with simple addition or multiplication. Instead, “Guess and Check” would be the best strategy to use. To carry out this strategy, students must first figure out
the number of legs that each individual animal has (chickens have 2 and pigs have 4). Then, they can speculate the total number of each animal on the farm. Let us say that there were 20 chickens and 17 pigs. Now, we would count the number of feet to see if it adds to our total of 98 feet – we will then have 108 feet, which is too many. Students would continue this process until they chose the correct number of animals that aligned with total number of feet, 98.

Because “Guess and Check” can seem to be a “do it in your mind” strategy, Long advocates that students “make a table.” This is another useful mathematical teaching strategy. This table includes their conjectures and final solutions in an organized manner, with appropriate headings and number placement. Constructing the table allows students the opportunity to “Find a Pattern” within the numbers guessed. Finally, if students are visual learners, then “Drawing a Picture” may be the best strategy to decipher the solution. The picture can be as simple as circles with 2 or 4 stick legs, to represent the number of chicks and pigs.

Estimation or Exact Answer is another strategy that Houghton Mifflin Math Third Grade highlights (Greenes, 2005, p. 99). This strategy asks solvers to simply round numbers to the nearest value as directed or to give the most precise answer when determining an answer. An example of a problem that would require this strategy is as follows: “An Asian male elephant weights about 8,000 pounds more than a female giraffe, which weighs 2,790 pounds. About how much does the male elephant weigh?” (Greenes, 2005, p.102). It is evident that the values should be added together through estimation. Since the male elephant’s weight is estimated as 8,000 more pounds than the female giraffe, it is best to round the giraffe’s weight, 2,790 pounds, to 3,000 pounds.
Utilizing this strategy makes it easier for students to come up with a plausible solution without generating a solution through simple computation.

Another successful strategy suggested in the *Houghton Mifflin Math Third Grade*, is working backwards (Greenes, 2005, p. 387). Working backwards asks students to start from the end of the problem and work their way to the beginning of the problem. For example, “Kareem weighed a bag of boat screws. He added 2 pounds of screws to the bag. After he took out 3 pounds of screws, the bag weighed 4 pounds. How much did the bag weigh at the start?” (Greenes, 2005, p. 390). Because the weight of the bag is not known before Kareem began adding and taking away boat screws, it is best to start with what is given – the weight of the bag once Kareem completed his manipulations, four pounds. From there, he took out three pounds of screws, which may seem like subtraction. However, because the strategy of working backward is being used, it is clear that the computations should be done “backwards.” Add four pounds to three pounds, and that will make the bag weigh seven pounds. Subtract two pounds from the weight, and the bag will have a beginning weight of five pounds. To check the problem, teachers demonstrate to students by working the problem forward to hopefully retrieve the same answer.

The *Houghton Mifflin Math Fourth Grade* book advocates the use of formulas and equations are just as effective strategies for solving mathematical problems. Of course formulas only require students to find the missing values to find a solution. For example, the area formula for a square and rectangle is equal to the length multiplied by the width. Equations summon students to set up a number/variable sentence that must be proven as true. Glancing at this problem, “Nate is paid by the hour. He works 6 hours and
earns $48. How much does Nate earn each hour?”, it is evident that there is a missing variable (Greenes, 2005, p.573). The variable must be found by setting up an equation utilizing the facts that are already presented: 6\times x (Nate’s earnings per hour) = $48. Once the equation is set up, solvers must carry out simple computations to arrive at the answer.

From all of the problems and strategies that have been shared, it is clear that there are various ways to carry out particular mathematical problems, but making use of the most effective strategy is critical for students to truly grasp mathematics as a whole. Not comprehending “why” an approach works places a student at a slight disadvantage in gaining a conceptual understanding of problem-solving, the process of trial-and-error. The key to success in mathematics is work and rework problem for clarity – not just to have an answer. In another light, it can be deduced that when students are made aware of the variety of strategies, they will have a better chance at solving a problem correctly. Teachers should be well rounded in their knowledge of problem-solving strategies, which is necessary for students to reach a prominent level of achievement in mathematics.

The achievement students make in mathematics is partly due to their teacher’s understanding of mathematics in application. To create a situation where mathematics education rankings increase for Mississippi, educators must take a look at the entire picture. That picture encompasses identifying problem-solving strategies as well as teacher efficacy. For this study, initial research questions should be considered based on the literature reviewed: How do teachers define a mathematical problem? What are teachers’ beliefs on mathematical problem-solving? Which problem-solving strategies do teachers utilize most when tackling mathematical problems? What thoughts and opinions
do teachers have on mathematical problem-solving? Chapter Three will explain the methodology that will be used to answer these four questions.

**Conclusion**

This chapter has provided an overview of research, goals of professional organizations, and research on strategies for teachers to use while instructing problem-solving. Chapter Three will explain the methodology that will be used to answer the research questions.
CHAPTER THREE: METHODOLOGY

Introduction

As discussed in Chapter Two, there are recommended strategies to put problem-solving into practice, and there is research to support those strategies. The literature review indicated a connection between the attitudes of teachers in regard to the effectiveness of executing problem-solving strategies. However, the body of empirical knowledge surrounding the effect of teacher beliefs on the use of strategies, in elementary schools, is limited.

Participants

Data was collected from third - through fifth - grade teachers in public schools in Hattiesburg, Forrest and Jones Counties. After gaining The University of Southern Mississippi Institutional Review Board’s approval (Appendix A), I distributed a letter of participation consent to the superintendents of each district. Those superintendents gave permission for their teachers to participate on a voluntary basis. The link for teacher access to the survey in Qualtrics, an accessible online survey which can be used to process data for statistical analysis, was sent to the superintendents and principals.

Survey Development

A Likert-type scale was chosen as the format for teachers to grasp easily and to make data clear and concise. Once the scale was selected, several problems were taken from textbooks such as Mathematical Reasoning for Elementary Teachers 6th Edition as well as Houghton Mifflin Math. There are four parts to the survey. Each of the survey parts were designed to access data for each research question. Part One was designed to
examine how teachers define a mathematical problem. In this section, various definitions of a mathematical problem were listed in a Likert-type scale format. The first portion of Part Two was constructed to find how teachers defined problem-solving. To collect data about teachers’ beliefs regarding problem-solving, with permission, the researcher developed a table format survey similar to Dr. Scott A. Chamberlin’s survey tables in his paper *What is Mathematical Problem-solving?* (Appendix B, C, and D). Part Three was characterized with a variety of third - through fifth - grade problems and problem-solving strategies. The participants were asked to identify the ways problems could be solved, adhering to the listed strategies. Finally, Part Four had several open-ended questions on the level of importance that problem-solving plays in their classroom and also their own particular beliefs.

**Procedures**

Participants who gave their voluntary consent were allowed to participate in the survey. The survey had four distinct sections based on Part I, defining a problem, Part II, beliefs about problem-solving, Part III, strategies teachers use to instruct students on solving problems, and Part IV, an open-ended section to collect teachers’ thoughts, opinions, and practices. Teachers had approximately six weeks to complete the survey.

Because state testing preparation was a priority for teachers in those districts, only five had taken the survey. The six week deadline was extended, and two additional weeks were given for teachers to take the survey. Two more participants submitted surveys.
Data Analysis

The data was compiled into a Microsoft Spreadsheet document and graphs by the Qualtrics online survey system. Data was reviewed and examined by Dr. J. T. Johnson and the researcher. Part I and Part II’s purpose were to discover hard, concrete data about problem-solving. In these sections, the results were measured by the highest percentages on a scale of agreement in relation to the stem.

Part Three gave participants a chance to have more flexibility in their answer choices. Analyzing results of Part Three took place by selecting the top percentages of strategies chosen by participants. The top percentages of strategies, as chosen by participants, were recorded. For further analysis, questions and selected strategies were broken down into distinct grade level categories (third, fourth, and fifth grade).

Part Four was measured by the comments that participants made on open-ended questions. Those comments speak to the research itself and stand as a reflection of those teachers’ overall perception of problem-solving. Their responses are recorded for clarification. It also serves as a reference to shed light on teachers’ thoughts and opinions of mathematical problem-solving.

Summary

This chapter discussed the methodology of the research from survey design to data analysis. The results of the data analysis are discussed in the following chapter.
CHAPTER FOUR: RESULTS

Purpose

The purpose of this study was to answer four questions:

Question I – How do teachers define a mathematical problem?

Question II – What are teachers’ beliefs about mathematical problem-solving?

Question III – What strategies do teachers use while instructing third - through fifth - grade students in mathematical problem-solving?

Question IV – What opinions or thoughts do teachers have on mathematical problem-solving?

Seven teachers participated in the survey, and the results were analyzed and recorded using Qualtrics. Each question is discussed and illustrated using tables in the chapter. The first two questions’ results will be shown below in graphs and tables; however, for simplicity and understanding the last two questions’ findings will be shown in this section, but will be further discussed in the discussion section.

Question I - How do Teachers Define a Mathematical Problem?

Data gathered for Question One, the definition of a mathematical problem in third - through fifth - grade mathematics, indicates that four of the participants agreed that a mathematical problem was an unknown entity. Table I shows a difference in teachers’ attitude toward a mathematical problem – “the difference between a goal state and a current state.” Participant response demonstrated two neither agreed nor disagreed and one disagreed. As denoted in Table I, five concurred that a mathematical problem was “given to apply one skill learned to decipher a solution.” All seven participants were unified in agreement that a mathematical problem was also “given to choose between
multiple skills learned to decipher a solution” and “given to apply one strategy learned to decipher a solution.” The y-axis shows that there was not a unanimous decision in other ways of defining a mathematical problem. However, four agreed that a problem is similar to a story that must be solved, abstract, and like a puzzle, maze, or optical illusion. Further, a large number of participants, five, deemed that a problem is concrete.

Table I: How do Teachers Define a Mathematical Problem?

<table>
<thead>
<tr>
<th>Description</th>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>Neither Agree Nor Disagree</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>an unknown entity.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>the difference between a goal state and a current state.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>given to apply one skill learned to decipher a solution.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>given to choose between multiple skills learned to decipher a solution.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>given to apply one strategy learned to decipher a solution.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>given to choose between multiple strategies learned to decipher a...</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a group of words to form a story that solvers must answer.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>very concrete with no room for experimentation.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>abstract.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>like a puzzle, maze, or optical illusion.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The answer to Question One is that while many of the participants agreed that students work mathematical problems to find a solution, there is no common definition among teachers concerning what a problem actually is.
Question II – What are Teachers’ Beliefs on Mathematical Problem-Solving?

Table II demonstrates teachers’ beliefs about problem-solving were not as strong in agreement. Data in Table II reports that five participants indicated that problem-solving “builds upon mathematical skills.” In total unanimity, all seven of the participants considered problem-solving as a process that helps to develop critical thinking skills. Although a majority of the participants viewed mathematical problem-solving as challenging, creative, and a process of searching for and discovering new ideas, there was some differentiation when considering that problem-solving “is not dependent on learned skills as other mathematical activities.” Three participants strongly agreed, while three others disagreed.

Table II: Teachers’ Beliefs About Problem-solving

<table>
<thead>
<tr>
<th>Belief</th>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>Neither Agree Nor Disagree</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>builds upon mathematical skills.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>develops critical thinking skills.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>is a process.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>is a process of searching for and discovering new ideas.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>is not dependent on learned skills as other mathematical...</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>is challenging.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>is creative.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

26
The answer to Question II, concerning teacher beliefs about problem-solving, is that teachers see problem-solving as a process that involves using not only standard procedures but some level of creativity. Also, some teachers felt that problem-solving was not based on prior skills.

Question III – What Problem-Solving Strategies do Teachers Use while Instructing Third - through Fifth - grade Students?

Question III focused on what teachers thought a student needed to be able to do, in order to successfully solve a mathematical problem. Table III shows all seven participants consider that solvers must “plan a solution.” Adding to that, six participants perceived that successful problem solvers should “engage in higher level thinking such as analysis, synthesis, and evaluation, which may result in abstraction or generalization,” while only one participant disagreed. As indicated in Table III, five of the teacher participants reported that victorious solvers must “seek multiple solutions,” “create a solution through adapting or revising current knowledge,” and “create new techniques to solve a problem.” With a smaller level of agreement, four judged that problem solvers are obliged to look for more ways to decode a problem than what they may know. That same number of participants also indicated that in order for students to have a chance to achieve problem-solving, they should “be challenged” and in that times “seek a solution to a mathematical situation for which they have no immediately accessible/obvious process or method.” Further, four participants disagreed that students must “not implement a pre-learned or standard algorithm to solve it [a problem].”
Table III: Successful Problem-Solving

<table>
<thead>
<tr>
<th>#</th>
<th>Question – For problem solvers to successfully complete a mathematical problem-solving task, they must:</th>
<th>Strongly Agree</th>
<th>Agree</th>
<th>Neither Agree nor Disagree</th>
<th>Disagree</th>
<th>Strongly Disagree</th>
<th>Total Responses</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>seek a solution to a mathematical situation for which they have no immediately accessible/obvious process or method.</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>7</td>
<td>2.71</td>
</tr>
<tr>
<td>2</td>
<td>plan a solution.</td>
<td>1</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>7</td>
<td>1.86</td>
</tr>
<tr>
<td>3</td>
<td>seek multiple solutions.</td>
<td>1</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>7</td>
<td>2.14</td>
</tr>
<tr>
<td>4</td>
<td>create a solution through adapting or revising current knowledge.</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>7</td>
<td>2.29</td>
</tr>
<tr>
<td>5</td>
<td>seek a more efficient way to solve a problem than they currently have.</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>7</td>
<td>2.71</td>
</tr>
<tr>
<td>6</td>
<td>be challenged.</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>7</td>
<td>2.71</td>
</tr>
<tr>
<td>7</td>
<td>create new techniques to solve a problem.</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>7</td>
<td>2.86</td>
</tr>
<tr>
<td>8</td>
<td>NOT implement a pre-learnt or standard algorithm to solve it.</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>0</td>
<td>7</td>
<td>3.29</td>
</tr>
<tr>
<td>9</td>
<td>engage in higher level thinking such as analysis, synthesis, evaluation which may result in abstraction or generalization.</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>7</td>
<td>1.86</td>
</tr>
</tbody>
</table>

The answer to Question III centers around the skills involved in problem-solving. Because problem-solving is a process which involves using higher level thinking skills, multiple approaches, and judgment, it often requires incorporating strategies that
encourage the use of learned problem-solving strategies. Some teachers felt problem-solving did depend on the pre-learned skills.

Table IV: Teachers’ Beliefs on Mathematical Problem Solving

Table IV reports that all participants judged that the activities “can be solved with more than one tool”, and those activities could also be puzzles. The data reports five participants viewed the activities utilized should be “developmentally appropriate.” There was a general agreement between participants’ attitudes toward the ideological principles involved in mathematical problem-solving activities. The data reported that six participants agreed that problem-solving activities were not rigid, but instead activities “promote flexibility in thinking,” “do not lend themselves to automatic responses,” and “involve non-routine, open-ended, or unique situation.” With the flexibility aspect of
problem-solving, teachers considered activities should be solved with multiple algorithms and steps.

The answer to Question IV lies in the fact that teachers felt mathematical problems should be appropriately chosen. To obtain solutions, multiple steps must be applied, time should be taken to determine solutions, and flexibility is promoted by the teacher as each student may solve the problem using a different approach.

**Identifying Strategies**

To determine the most utilized mathematical problem-solving strategies, the state adopted mathematical textbooks *Houghton Mifflin Math* and *Mathematical Reasoning for Elementary Teachers* (6th ed.) were examined for third - through fifth - grade mathematical problems. The most frequently used strategies were determined and are as follows:

<table>
<thead>
<tr>
<th>Guess and Check</th>
<th>Make a List</th>
<th>Use an Equation</th>
<th>Make a Diagram</th>
<th>Make a Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>Organized List</td>
<td>Compute</td>
<td>Simplify</td>
<td>Find a Pattern</td>
<td>Act the Problem Out</td>
</tr>
<tr>
<td>Experiment</td>
<td>Work Backwards</td>
<td>Use a Variable</td>
<td>Solve a Simpler Problem</td>
<td>Draw a Picture</td>
</tr>
<tr>
<td>Use Direct Reasoning</td>
<td>Use Indirect Reasoning</td>
<td>Use Deductive Reasoning</td>
<td>Use Inductive Reasoning</td>
<td>Use Properties of a Number</td>
</tr>
<tr>
<td>Look for a Formula</td>
<td>Eliminate Possibilities</td>
<td>Using the Pigeonhole Principle</td>
<td>Do a Simulation</td>
<td>Use Dimensional Analysis</td>
</tr>
<tr>
<td>Identify Sub-Goals</td>
<td>Use Coordinates</td>
<td>Use Symmetry</td>
<td>Use Probability</td>
<td>Estimation</td>
</tr>
<tr>
<td>Choose the Operation</td>
<td>Use Integers</td>
<td>Use Models</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Participating teachers were asked to select which strategy and/or strategies they would choose to use when instructing their students based on certain questions. Only a few of the questions with chosen strategies will be noted in this chapter for clarity and conciseness.

Participating teachers viewed the following questions:
Third Grade Math Question 1: “Who am I? If you multiply me by 15 and add 28, the result is 103” (Mathematical Reasoning for Elementary Teachers, 2009). Four participants chose the strategies Compute and Work Backwards. Both of these strategies are effective in solving the problem. The likelihood of a calculator being utilized in a third grade classroom must be considered.

Third Grade Math Question 2: “A small drink at the Lunar Diner is $0.54. A medium drink is $0.60 and a large drink is $0.66. If the pattern continues, how much is an extra-large drink likely to be?” (Houghton Mifflin Math Grade 3, 2005). All participants deemed that Find a Pattern was the best strategy to teach their students. This strategy was considered the easiest and most workable. However, two participants chose Simplify as a plausible strategy. To be clear, Simplify means to reduce in size and is typically used for fractions. Making the prices of drinks into simplified fractions will not help students to obtain the answer, but it may aid solvers in discovering the pattern. For example, 54/100, 60/100, 66/100 is equivalent to 27/50, 3/5, 33/100. These simplified fractions in no way aid in getting closer to the actual answer of $0.72; therefore, Find a Pattern is the best strategy.

Third Grade Math Question 3: “Emily's birthday is in 37 days. Tan's birthday is 16 days after Emily's. In how many days is Tan's birthday?” (Houghton Mifflin Math Grade 3, 2005). Five participants chose Compute as the strategy they would instruct their students to use.

Third Grade Math Question 4: “Levison's Hardware has a number of bikes and trikes for sale. There are 27 seats and 60 wheels, all told. Determine how many bikes and how many trikes there are” (Mathematical Reasoning for Elementary Teachers, 2009).
Five participants chose the strategy *Make a Table*. Three participants chose *Guess and Check, Make a List, Compute, and Draw a Picture*.

**Third Grade Math Question 5**: “Mr. Akika has 32 18-cent and 29-cent stamps, all told. The stamps are worth $8.07. How many of each kind of stamp does he have?” (Mathematical Reasoning for Elementary Teachers, 2009). There were five participants who selected *Make a List* as a logical strategy to use, while three participants would use the strategies *Make a Table* and *Use a Variable*. *Using a Variable* would suggest that students are expected to carry out simple algebraic equations.

**Fourth Grade Math Question 1**: “Ian's assistants can each juggle 3 rings or 5 balls at a time. Ian wants them to juggle a total of 15 balls. How many assistants does he need?” (Houghton Mifflin Math Grade 4, 2005). A majority of the participants, five, selected *Compute* as the prime strategy to show their students to solve the problem above. *Compute* would be reasonable, but only after the solver has clear understanding of how many assistants are needed for a certain amount of rings or balls.

**Fourth Grade Math Question 2**: “At 6:00 A.M., the temperature was -5 degrees F. By noon of the same day, the temperature was 6 degrees F. How many degrees did the temperature change in 6 hours?” (Houghton Mifflin Math Grade 5, 2005). Interestingly, four participants chose *Compute* as the most plausible strategy. This particular problem requires careful consideration. Most assume that adding (*Compute*) the two temperatures together will attain the answer (-5 + 6 = 1), which is a misconception. The application of the integer system would make the change of eleven degrees more clear.

**Fourth Grade Math Question 3**: “Carol has 8 booklets about birds. Some of the booklets have 26 pages and the others have 41 pages. How many of each size booklet
does she have if the pages total 253?” (Houghton Mifflin Math Grade 4, 2005). Five participants chose Use an Equation, while 4 selected Compute. Use an Equation would be an excellent strategy, but it would require students to set up a system of equations, which they may not be familiar.

Fifth Grade Math Question 1: The strategies Use an Equation and Compute were the selected strategies often in this section of data gathering. These were the selected strategies for this question, as well. “Benita gets a raise of $315 per month. Her new monthly salary is $3,425. What was Benita's monthly salary before she got a raise?” (Houghton Mifflin Math Grade 5, 2005). Five of the participants chose Use an Equation with four selecting Compute as the best strategy to demonstrate solving this problem. Both of these strategies seem cogent for this particular problem.

Fifth Grade Math Question 2: “A solid figure is made up of blue cubes and white cubes. It is 4 cubes wide, 4 cubes long, and 4 cubes high. There are 3 times as many blue cubes as white cubes. How many blue cubes are there?” (Houghton Mifflin Math Grade 5, 2005). Four participants chose Use an Equation as a tenable strategic plan to find the solution of this problem. Setting up an equation of $3x$ (blue cubes) + $x$ (white cubes) = 64 (Area of the solid figure = length * width * height). Solving it will show that $x = 4$. When multiplying 4 by 3, the number of blue cubes, solvers will find that there are 48 blue cubes. As shown above, using an equation is not the only strategy that students will need to solve this problem. They must have a sense of algebraic terms, along with familiarity with formulas of solid figures. Although Use an Equation is a part of solving a problem, it is not sufficient enough to attain the answer.
The responses to this part of the data collection demonstrate the instruction that is taking place in the mathematical classroom. This shows just how much emphasis is placed on particular mathematical problem-solving strategies and which strategies are cogent enough to utilize for specific problems. The data collection responses display mathematical problem-solving strategies are necessary for solving problems. Further, teachers should instruct their students on how to utilize these strategies for greater success in the study of mathematics.

**Question IV – What Thoughts and Opinions do Teachers have on Mathematical Problem-Solving?**

The empirical data reported is enhanced against the backdrop of the qualitative data acquired in the survey. The interview responses are taken as they are and considered as a reflection of teachers in the third - through fifth - grade classrooms in Hattiesburg, Forrest, and Jones County School Districts. The questions were as follows:

1. *What is the difference between a mathematical problem and a mathematical exercise?*

   Each response to this question was somewhat similar. The comment that was an exception was noted.

   **Example:** “a mathematical problem is one that needs to be solved and an exercise needs to be worked out in an equation.” This is a huge misconception of an exercise, although it is true that a problem must be solved. Exercises do not have to be worked out just in an equation.

   **Example:** Exercise is the practice of a particular skill that helps a solver to prepare for a problem. An example of a mathematical exercise would be solving a division problem to then find a remainder – $46/3=\_ \_ R \_ \_$. To complete this exercise, solvers carry out simple
division and find the remainder. In an exercise, students are given the chance to use the selected skills they have learned, with no room for experimentation.

Along with that, participants acknowledged that a problem is the application of multiple skills and/or strategies.

Example: “There are $n$ people in a room, and each of them will shake hands with every other person once and only once. If there are 3 people in the room, how many handshakes are there?” (Long, 2009, n.p.). As you can see from above, this mathematical problem is a bit more convoluted. The problem above does not require one specific skill to be applied, like an exercise. Instead, it calls for multiple skills and strategies such as multiplication, addition, making an orderly list, and/or drawing a picture. All of these skills and strategies do not have to be applied simultaneously. A mathematical problem gives a solver room to experiment and select which method would work best for them as solvers and to complete the problem.

2. How do you define a mathematical problem?

What is a mathematical problem?

Example: “something that needed to be solved.” The presented statement does not take into account the complexities of a mathematical problem. Two particular responses encompassed those complexities, recording that “it is a process that involves critical thinking and experimentation to solve it” and that “a problem is given as an unknown, and it requires critical thinking to be solved.” In both of these responses, critical thinking is highlighted. Critical thinking is a mental process of actively applying, examining, and synthesizing information to reach a conclusion. The elements of critical thinking are embedded in the process of problem-solving. It should be brought out that a mathematical
problem is a process, meaning that it must be solved in a methodical manner. Additionally, a mathematical problem does not lend itself to one specific skill or strategy. Experimentation is key.

3. In what ways do you implement mathematical problem-solving in your classroom?

The responses to this question show that mathematical problem-solving is not fully implemented in the classroom. Some participants shared that mathematical problems are included in their students’ homework, class work, and tests; yet, they did not elaborate on how those problems were put into action. To be clear, implementation is more than just exposure – it is application and usage. One participant noted that they enacted problem-solving through the use of manipulatives. The participant shed light on the fact that students must apply problem-solving to understand how to utilize the manipulatives. In addition, the same participant shared that they asked questions aloud in discussion and written on tests that required problem-solving. This particular participant seems to have a conceptual understanding of problem-solving.

4. In your opinion, is mathematical problem-solving important in elementary mathematics? If yes or no, why? What benefits does it have for students?

All participants agreed that problem-solving was very significant in elementary mathematics, and they entailed a variety of benefits that problem-solving has. Participants suggested that problem-solving shows students how to work their way through a problem. When students learn to work their way through a problem, they are more inclined to solve a problem correctly, are ready for the next level of mathematics, and have a better understanding of the overall study of mathematics. In foresight,
problem-solving helps students to make better decisions based on the experience and confidence of solving problems.

5. Which mathematical strategies do you tend to teach most? Why?

Again, mathematical problem-solving is not being put into action in the classrooms of the participants of this study. When presented with this question, few answered, but one participant wrote that they focus on George Polya’s Four Step Process. This process includes understanding the problem, devising a plan, carrying out the plan, and reflection over the process. This method is very effective in solving problems. Adding to that, a participant reported they try to utilize all of the strategies listed in the survey, simply because students have different ways of learning. Sticking to a rigid strategy places limitations on a student’s ability and likelihood of successfully solving a mathematical problem.

6. Do you think the current curriculum makes enough room for students to grasp mathematical problem-solving? If so, how? If no, how could the curriculum be improved?

Most participants agreed the curriculum is not flexible enough to fully comprehend mathematical problem-solving. A teacher participant thought the curriculum was “test driven”, which limits opportunities for experimentation. A participant also shared that the elementary years typically require students to carry out computations and that by the time they enter middle school mathematics, they are immediately asked to engage in problem-solving. Not having the proper experience or experimentation makes it difficult to understand higher-level mathematics and what will benefit those students most. One participant deemed that “teachers have all the materials for students to be
successful”. This response does not fully consider if the curriculum supports students in understanding mathematical problem-solving nor does it state how so for evidence.

7. **At which grade level do you think it is appropriate to instruct students in mathematical problem-solving?**

   This study was in the elementary grades of third through fifth. Interestingly, all participants agreed that mathematical problem-solving should be taught to students as early as kindergarten, but also as late as second grade. With this consensus, it can be inferred that mathematical problem-solving is not as complex as theorists imply. Strategies like *Drawing a Picture* or skills such as simple addition can be taught to a preschool student.

**Summary**

A teacher’s definition of a mathematical problem is crucial to understand since their interpretation will be taught and emphasized to their students. The teacher’s comprehension of a mathematical problem will have a large impact on a student’s success. The results in this section were contradictory because some participants seemed unsure of what a mathematical problem actually was. The participants agreed particularly on strategies for solving problems. The data from the survey and questionnaire have been reported in this chapter. Chapter Five will include a discussion of this data, implications, and recommendations for further research.
CHAPTER FIVE: DISCUSSION

Teachers’ Definitions

A teacher’s definition of a mathematical problem is crucial to understand since their interpretation will be taught and emphasized to their students. The teacher’s comprehension of a mathematical problem more than likely determines a student’s success in mathematical problem-solving. The results in this section were contradictory, due to the fact that some participants seemed unsure of how to define a mathematical problem. Divertingly, some participants agreed wholly on matters of solving a problem.

It has been stated that teachers define a mathematical problem in a variety of ways, based on experience and expertise. When examining the teachers in Hattiesburg, Forrest and Jones County School Districts, it was found that a majority of the participants deemed a mathematical problem as an unknown entity. However, they were either neutral or disagreed that a problem is “the difference between a goal state and a current state”. If the participants agreed a problem was an unknown entity, then why would a problem not have a goal state? From a synthesis of literature, it can be inferred that the unknown entity is the current state, whereas the answer is the goal state. This shows that teachers may have not fully grasped the terms “current state” and “goal state”. Being familiar with the terms would certainly help teachers to better assist their students in problem-solving.

For the most part, participants in this study wholly agreed that a mathematical problem is - “given to choose between multiple skills learned to decipher a solution” and “given to apply one strategy learned to decipher a solution”. As a result, participants conferred that the use of multiple mathematical skills is a logical way to problem-solve as opposed to using a single skill. Interestingly, there were participants who considered one
a mathematical strategy adequate enough to unravel a problem. This indicates that teachers may be showing students only one way to decode a problem. All students do not learn the same. For example, students may learn by example, listening to a speaker while writing, drawing to conceptualize, or learning by trial-and-error. When a student is only exposed to a rigid method in carrying out mathematical problems, they tend to have a narrow view of the subject, and they are even more susceptible to not succeeding in problem-solving.

When accessing the views that teachers have about a mathematical problem, the data demonstrated that those views can be quite contradictory. Most of the participants judged that a mathematical problem was abstract and similar to an optical illusion, puzzle, or maze. However, participants also agreed that a mathematical problem was “a group of words to form a story that solvers must answer” and “very concrete with no room for experimentation”. The idea of “no room for experimentation” is in direct contrast to a puzzle. When putting a puzzle together, there are numerous of ways to put the puzzle together. If there were only one way to put a puzzle together, then it would not be problem-solving. Instead, it would be a monotonous exercise.

Certainly a mathematical problem cannot be abstract while being concrete at the same time. In the same manner, a problem does not have to be an actual story - it can have simple directions that call solvers to do a great deal of critical thinking to find solutions to graphs, charts, numbers, and the like.

Of all the hard results found in this portion of the study, one of the most interesting findings was that participants did not come to a consensus on whether or not a mathematical problem was abstract or concrete. Just for clarity, a mathematical problem is given as an opportunity for a learner to apply and utilize the skills that they have
grasped. A mathematical problem is abstract because a student is able to use different techniques or a combination of skills to arrive at the answer of a problem; therefore, a problem cannot be concrete.

Some participating teachers indicated that a mathematical problem is abstract, while still noting that a concrete grasp is still an advantage. With an abstract view of a mathematical problem, students have a chance to further use their different skill sets, which requires them to critically think in an individualized manner. A concrete perception does teach students a specific strategy and makes them familiar with that particular mathematical strategy. However, this may lead students to using the strategy blindly because it is a routine and does not render a chance for that student to conceptualize or critically think.

**Teachers’ Beliefs Concerning Problem-Solving**

The beliefs that teachers have of mathematical problem-solving are very significant because they are likely to be translated to the mind of students. If these beliefs are twisted from what they should be, then students are placed at a disadvantage. They may even find themselves unsuccessful in solving mathematical problems. This was illustrated as teachers considered strategies for specific problems from the state adopted textbook.

**Teachers’ Beliefs Concerning General Problem-Solving.**

Teachers’ beliefs on mathematical problem-solving encompass a wide scope of perspectives. In examining how a teacher regards problem-solving, participants wholly agreed it is a process, which implies problem-solving is a methodical procedure. Participants unanimously agreed that problem-solving aids in developing critical thinking
skills. Critical thinking itself is the process of evaluating, synthesizing, and analyzing a particular entity to discover an answer or solution. Both of the ideas that problem-solving is a process and it helps develop critical thinking skills work together to demonstrate problem-solving is not something that can be done in haste. It must be done in a systematic way, not lending itself to a specific skill or strategy. It can also be deduced that problem-solving does have a benefit to more than just the study of mathematics. These same transferrable skills can assist students in evolving their critical thinking skills, which are needed for almost all subject matter.

A majority of the participants agreed that problem-solving was creative. To postulate that problem-solving is creative neither speaks to the idea that it is not concrete, nor does it cling to the idea of rigid algorithms. Participants also conjectured that problem-solving was a process of searching for and discovering new ideas. The word “process” appears again as an idea teachers link to problem-solving, and solvers are given the opportunity to look for something that is not known. Other participants deemed that problem-solving was challenging and it builds upon mathematical skills. This further proves literature noted in Chapter Two.

Fewer participants agreed that problem-solving was “not dependent on learned skills as other mathematical activities,” with some disagreeing. This small difference certainly deserves some attention. Problem-solving is not dependent or linked to learned skills or specific strategies. Rather, it is an opportunity for solvers to recall strategies or skills that may be used individually or in a combination to decipher a solution.
**Problem-Solving Activities**

In addition to examining the beliefs participants have on problem-solving, there must be consideration of the problem-solving activities used. Unanimously, participants indicated that problem-solving activities could be puzzles. Participants also thought those activities could be solved with more than one tool, meaning materials help a student solve the problem such as manipulatives.

Participants viewed that problem-solving activity promoted flexibility in thinking. This speaks to the idea of the non-rigid fashion that problem-solving lends itself to. Along with that, a majority of participants thought that activities “require the use of multiple steps for a successful solution” and “require the implementation of multiple algorithms for a successful solution.” This means problem-solving is a methodical process, where several steps or algorithms may need to be utilized in puzzling out a solution. Majority of participants believed that activities “involve non-routine, open-ended, or unique situations” and “do not lend themselves to automatic responses”, which also demonstrates the abstract, flexibility that problem-solving promotes.

**Successful Problem-Solving**

Not only is it important to understand the beliefs of teachers on the idea of mathematical problem-solving, but also it is just as significant to contemplate what teachers deem a student needs to successfully solve a mathematical problem. A large number of students believed that they must engage in higher level thinking such as analysis, synthesis, and evaluation, which may result in abstraction or generalization. Again this shows just how much problem-solving contributes to necessary critical
thinking skills. Adding to that, a majority of participants disagreed that solvers must
“NOT implement a pre-learned or standard algorithm to solve it [a problem]”. To find a
solution, students must be familiar with skills, strategies, and algorithms that will aid
them in Successful Problem-Solving.

**Strategies Most Used by Teachers**

Problems from *Houghton Mifflin Mathematics* and *Mathematics Reasoning for
Elementary Teachers* were presented to participants to determine the strategies they
would find most useful in solving the problems. Those strategies were **Guess and Check,**
**Compute, Draw a Picture, Make a List, Make a Table, Use a Variable, Work Backward,**
**Find a Patter, Choose the Operation,** and **Use a Formula.**

**Conclusions Concerning Mathematical Problem-Solving**

Concerning the definition of a mathematical problem, there are several
contradictions involved in deciding a problem’s meaning in a classroom. While most
teachers decided a problem was an unknown entity, some also thought a problem did not
encompass a goal state and current state. With this discrepancy, students may not
understand how to solve a problem, and most importantly, finding what the problem calls
for. This may not permit students to discern information given in a problem needed to
solve it. Not understanding the components of a mathematical problem places students at
a lower probability of successfully solving the problem. Therefore, teachers should
understand the idea that a problem is an unknown entity and the difference between a
current state and a goal state. Once teachers master this, they will need to demonstrate to
their students how this idea is applicable when solving a problem.
Teachers may instruct their students to use several different skills to solve a problem. However, it is somewhat clear that teachers in the participating schools teach very few strategies to solve a particular problem. This method does not facilitate critically thinking about the problem at hand. Solving the problem may become a perfunctory effort in which they may be limited to solve facile problems, as opposed to more convoluted ones. This routine is not successful for problem-solving, and it will not benefits mathematics students when application is necessary to unravel a problem. Moreover, teachers must put more effort in presenting students a variety of skills and strategies that will help them to answer mathematical problems, while not hindering their abilities to apply the learned tactics in a wide range of problems based on each grade level.

Participants considered a mathematical problem carried both concrete and abstract characteristics. It is not possible for a mathematical problem to be concrete. Numbers alone are abstract, simply because they represent something that is not physically seen. Consequently, serpentine mathematical problems are abstract, and teachers should grasp this idea to then incorporate into their classroom. This will require teachers to exhibit more than one way to approach a problem. Instead of only instructing students to utilize the teacher’s preferred strategy, the students would greatly benefit by becoming familiar with a variety of mathematic strategies to meet the needs of all students.

Although teachers carry a wide range of beliefs when ruminating over mathematical problem-solving, participants agreed substantially on certain characteristics of problem-solving. Primarily, problem-solving is a methodical process, which does not always use specific strategies or skills for answer completion. Problem-solving helps
students develop critical thinking skills. Knowing that mathematical problem-solving is a process and that it develops critical thinking, teachers can understand that problem-solving plays a critical role in preparing students for the overall study of mathematics as well as in other content areas. Finding opportunities to demonstrate this methodical procedure to students, by thorough step-by-step explanation (Think Aloud Model) and a plethora of examples, does more than train them in general subjects. This type of teaching prepares them for challenges they may encounter in their lives.

Having the expertise in multiple mathematical problem-solving strategies gives students a higher chance at succeeding in solving problems. This study demonstrated there are certain strategies that work best in discovering answers to specific problems. The strategies that were most used for the third - through fifth - grade problems include Guess and Check, Compute, Draw a Picture, Make a List, Make a Table, Use a Variable, Work Backward, Find a Pattern, Choose the Operation, and Use a Formula. The use of each strategy varies by grade level appropriateness and mathematical content. Of all the strategies brought selected, Compute received the most interest and comprehensive use. Participants chose Compute for a third grade problem. Computation is not necessarily doable for a third grade student. This implies that teachers could possibly be thinking of what strategies they would carry out, not necessarily the ones best for students. It is important that teachers take the initiative to find what strategies and skills work best for their students based on learning styles and abilities.

The thoughts and opinions of teachers are important and should be considered because they offer insight into the mathematical classroom. Primarily, as indicated by the results of this survey, problem-solving is not carried out enough in the mathematics
classroom. Participants were asked for the ways in which they implement mathematical problem-solving. Almost all participants noted they utilize problem-solving, but very few were specific in how it was used in their curriculum and instruction. For example, one teacher shared that she “Have examples of different problems with items in the classroom”. This statement is not lucid, and can be interpreted in several different ways. However, another teacher was more clear: “In my classroom, manipulatives are almost always used to discuss mathematical concepts. The manipulatives are novel to learners, so they must discover how to use them. I tend to ask various questions aloud, on homework, and tests that require more than just a number”. Teachers were not explicit in noting which strategies they implemented most in their classroom as relating to specific problems.

Summary

All in all, mathematics teachers must find ways to implement problem-solving in their classrooms, curriculum, and instructions. Most importantly, teachers and students need to be aware of when they are making use of problem-solving strategies in the mathematics classrooms. Teachers and students can learn how to become better at tackling through methodical procedures of mathematical problem-solving, while teachers and administrators can find flaws in the curriculum and/or instruction that will ultimately enhance the quality of mathematical student achievement.

Teacher’s definitions and beliefs carry a substantial amount of weight in the mathematics classroom. It is clear that a mathematical problem-solving is utilized to build the skill set to solve problems. Solving those problems will require the use of
multiple skills and strategies. These strategies may not necessarily be easily recognized as the “go to” method. Looking at a more explicit definition of a problem, teachers seemed to agree that it must be solved. This implication has no depth, and can indicate that teachers may not be well versed in the definition of a mathematical problem. However, other participants revealed that a problem involves critical thinking, application of skills, and experimentation. These answer stems are plausible, and they are sufficient enough to define a mathematical problem.

When participants were asked “At which grade level do you think it is appropriate to instruct students in mathematical problem-solving?” they responded that second grade was fitting. However, some teachers even thought students should be introduced to mathematical problem-solving as early as kindergarten. From the results of this study, it can be inferred that third - through fifth - grade teachers may not be as knowledgeable on the subject of problem-solving as they should be. How can pre-kindergarten teachers be expected to instruct young students in mathematical problem-solving if third - through fifth - grade teachers are not at a level of understanding to do it themselves, as teachers expected to instruct higher level students? Essentially, teachers are aware that students need to be familiar with problem-solving, but the job is not being done because teachers are expecting it to be done by a previous or future mathematics teacher. Therefore, there should be a consensus on which grade level teachers are expected to introduce mathematical problem-solving to students.

Limitations and Further Recommendations

This study did have its limitations. First and foremost, there were not enough participants to generalize the results on a wide scale. Future researchers must consider
obtaining more participants to enhance the effectiveness of a study on mathematical problem-solving. For a higher level of validity, future researchers would greatly benefit from examining participants’ classroom documents to discover if they exemplify the ideas that are brought forth in the results.

Finally, the results of this study demonstrated there is no clear-cut consensus on mathematical problem-solving among teachers from the Hattiesburg, Forrest, and Jones County School Districts. In addition, it was found that teachers are not as knowledgeable about problem-solving as they should be. Since there are so many uncertainties, educators should do more studying to find solidarity. Future researchers should also consider the new curriculum and guidelines through Common Core State Standards Initiative for as they relate to mathematical problem-solving.
REFERENCES


http://www.mde.k12.ms.us/curriculum-and-instruction/mathematics


Lorrain County Community College (2012). *Problem-solving Strategies*. Retrieved 2013,


APPENDICES

Appendix A

INSTITUTIONAL REVIEW BOARD
118 College Drive #5147 | Hattiesburg, MS 39406-0001
Phone: 601.266.6820 | Fax: 601.266.4377 | www.usm.edu/irb

NOTICE OF COMMITTEE ACTION

The project has been reviewed by The University of Southern Mississippi Institutional Review Board in accordance with Federal Drug Administration regulations (21 CFR 26, 111), Department of Health and Human Services (45 CFR Part 46), and university guidelines to ensure adherence to the following criteria:
The risks to subjects are minimized.
The risks to subjects are reasonable in relation to the anticipated benefits.
The selection of subjects is equitable.
Informed consent is adequate and appropriately documented.
Where appropriate, the research plan makes adequate provisions for monitoring the data collected to ensure the safety of the subjects.
Where appropriate, there are adequate provisions to protect the privacy of subjects and to maintain the confidentiality of all data.
Appropriate additional safeguards have been included to protect vulnerable subjects.
Any unanticipated, serious, or continuing problems encountered regarding risks to subjects must be reported immediately, but not later than 10 days following the event. This should be reported to the IRB Office via the “Adverse Effect Report Form”.

If approved, the maximum period of approval is limited to twelve months. Projects that exceed this period must submit an application for renewal or continuation.

PROTOCOL NUMBER: 13022001
PROJECT TITLE: The Relation between Teachers' Beliefs about Mathematical Problem Solving and the Strategies They use for Teaching Third through Fifth Grade Students
PROJECT TYPE: New Project
RESEARCHER(S): Jasmine L. Thomas
COLLEGE/DIVISION: Student Affairs
DEPARTMENT: McNair Scholars Program
FUNDING AGENCY/SPONSOR: N/A
IRB COMMITTEE ACTION: Expedited Review Approval
PERIOD OF APPROVAL: 02/26/2013 to 02/25/2014
Lawrence A. Hosman, Ph.D.
Institutional Review Board
Appendix B

Hi Dr. Chamberlin,

I am a student at the University of Southern Mississippi, and I am in the Honors College as well as the Dr. Ronald E. McNair Scholar Program. To complete the requirements provided by both programs, I am writing a prospectus. My proposed title and study is “The Relation between Teachers’ Beliefs about Mathematical Problem-solving and the Strategies They Use for Teaching Third - through fifth - grade Students”. This study will be examined through a survey that will be completed by teachers in the surrounding area of Hattiesburg, MS. I made use of your thesis, "What is Mathematical Problem-solving", in my literature review with proper citation. With your consent, I would like to utilize certain information from the tables that you created for your study. It has valid information that would not only help my study, but also enable me to develop my own table and survey. Please let me know if this is acceptable.

Thank you,

Jasmine Thomas

Jasmine,

Feel free to use the tables that I created. That wasn’t a thesis. It was a paper accepted by Philosophy of Mathematics Education (PME).

Scott

Hi Dr. Chamberlin,

Thank you for your assistance.

Jasmine Thomas
### Appendix C

Table V: Mathematical Problem-solving as a Process

<table>
<thead>
<tr>
<th>STEM: For problem solvers to successfully complete a problem-solving task, they must</th>
<th>Grouped Median Round 2</th>
<th>Interquartile Deviation Round 2</th>
<th>Consensus Reached?</th>
<th>Grouped Median Round 3</th>
<th>Interquartile Deviation Round 3</th>
<th>Consensus Reached?</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. engage in cognition</td>
<td>4.00</td>
<td>0</td>
<td>YES</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. engage in metacognition</td>
<td>3.24</td>
<td>0.5</td>
<td>NO</td>
<td>3.33</td>
<td>0.5</td>
<td>NO</td>
</tr>
<tr>
<td>c. seek a solution to a mathematical situation for which they have no immediately accessible/obvious process or method</td>
<td>3.78</td>
<td>0.25</td>
<td>YES</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. self-monitor</td>
<td>3.41</td>
<td>0.5</td>
<td>NO</td>
<td>3.5</td>
<td>0.5</td>
<td>NO</td>
</tr>
<tr>
<td>e. plan a solution</td>
<td>3.29</td>
<td>0.5</td>
<td>NO</td>
<td>3.36</td>
<td>0.5</td>
<td>NO</td>
</tr>
<tr>
<td>f. communicate ideas to peers</td>
<td>2.73</td>
<td>0.25</td>
<td>YES</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>g. engage in iterative cycles</td>
<td>2.94</td>
<td>0</td>
<td>YES</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>h. create a written record of their thinking</td>
<td>2.88</td>
<td>0</td>
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<td></td>
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<tr>
<td>i. seek multiple solutions</td>
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<td>0.5</td>
<td>NO</td>
<td>2.63</td>
<td>0.5</td>
<td>NO</td>
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<tr>
<td>j. create a solution through adapting or revising current knowledge</td>
<td>3.5</td>
<td>0.5</td>
<td>NO</td>
<td>3.6</td>
<td>0.5</td>
<td>NO</td>
</tr>
<tr>
<td>k. seek a more efficient way to solve a problem than they currently have</td>
<td>2.65</td>
<td>0.5</td>
<td>NO</td>
<td>2.7</td>
<td>0.5</td>
<td>NO</td>
</tr>
<tr>
<td>l. mathematise a situation to solve it</td>
<td>3.24</td>
<td>0.25</td>
<td>YES</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table V: Mathematical Problem-solving as a Process continued

<table>
<thead>
<tr>
<th>STEM: For problem solvers to successfully complete a problem-solving task, they must</th>
<th>Grouped Median Round 2</th>
<th>Interquartile Deviation Round 2</th>
<th>Consensus Reached?</th>
<th>Grouped Median Round 3</th>
<th>Interquartile Deviation Round 3</th>
<th>Consensus Reached?</th>
</tr>
</thead>
<tbody>
<tr>
<td>m. create assumptions and consider those assumptions in relation to the final solution</td>
<td>3.06</td>
<td>0</td>
<td>YES</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n. revise current knowledge to solve a problem</td>
<td>3</td>
<td>0</td>
<td>YES</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>o. be challenged</td>
<td>3.44</td>
<td>0.5</td>
<td>NO</td>
<td>3.56</td>
<td>0.5</td>
<td>NO</td>
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<tr>
<td>p. create new techniques to solve a problem</td>
<td>2.78</td>
<td>0.25</td>
<td>YES</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>q. NOT implement a pre-learnt or standard algorithm to solve it</td>
<td>3.2</td>
<td>0.5</td>
<td>NO</td>
<td>3.3</td>
<td>0.5</td>
<td>NO</td>
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<tr>
<td>r. analyse relevant data and processes to identify a potential solution(s)</td>
<td>3.39</td>
<td>0.5</td>
<td>NO</td>
<td>3.67</td>
<td>0.5</td>
<td>NO</td>
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<tr>
<td>s. create mathematical models</td>
<td>2.83</td>
<td>0</td>
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<td></td>
<td></td>
<td></td>
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<td>t. define a mathematical goal or situation</td>
<td>3.13</td>
<td>0.25</td>
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<td></td>
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<td>u. seek a goal</td>
<td>3.47</td>
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<td>NO</td>
<td>3.75</td>
<td>0</td>
<td>YES</td>
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<td>v. engage in higher level thinking such as analysis, synthesis, evaluation which may result in abstraction or generalization</td>
<td>3.28</td>
<td>0.5</td>
<td>NO</td>
<td>3.69</td>
<td>0.5</td>
<td>NO</td>
</tr>
</tbody>
</table>

(Chamberlin, 16).
## Appendix D

### Table VI: Mathematical Problem-solving as Characteristics

<table>
<thead>
<tr>
<th>STEM: Problem-solving activities</th>
<th>Grouped Median Round 2</th>
<th>Interquartile Deviation Round 2</th>
<th>Consensus Reached?</th>
<th>Grouped Median Round 3</th>
<th>Interquartile Deviation Round 3</th>
<th>Consensus Reached?</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. have realistic contexts</td>
<td>2.89</td>
<td>0</td>
<td>YES</td>
<td></td>
<td></td>
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<td>b. require the use of logic</td>
<td>3.53</td>
<td>0.5</td>
<td>NO</td>
<td>3.38</td>
<td>0.5</td>
<td>NO</td>
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<tr>
<td>c. are developmentally</td>
<td></td>
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<td></td>
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<td>appropriate (e.g. what may</td>
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<td>be a task for one problem</td>
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<td>solver may not be for</td>
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<td></td>
<td></td>
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<td>another problem solver)</td>
<td>3.5</td>
<td>0.5</td>
<td>NO</td>
<td>3.6</td>
<td>0.5</td>
<td>NO</td>
</tr>
<tr>
<td>d. can be solved with more than</td>
<td>3</td>
<td>0</td>
<td>YES</td>
<td></td>
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</tr>
<tr>
<td>one tool</td>
<td></td>
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<tr>
<td>e. can be solved with more than</td>
<td>3.18</td>
<td>0</td>
<td>YES</td>
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<td>one approach</td>
<td></td>
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<td>f. are novel situations to</td>
<td>3.53</td>
<td>0.5</td>
<td>NO</td>
<td>3.53</td>
<td>0.5</td>
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<td>solvers</td>
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<td>g. can be used to assess level</td>
<td>3.06</td>
<td>0</td>
<td>YES</td>
<td></td>
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<td>of understanding</td>
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<td>h. require the</td>
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<td>YES</td>
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<td>implementation of multiple</td>
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<td>algorithms for a successful</td>
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<td>solution</td>
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<td></td>
</tr>
<tr>
<td>i. DO NOT lend themselves to</td>
<td>3.65</td>
<td>0.5</td>
<td>NO</td>
<td>3.9</td>
<td>0</td>
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<td>automatic responses</td>
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<td>j. promote flexibility in</td>
<td>3.18</td>
<td>0.25</td>
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<tr>
<td>thinking</td>
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Table VI: Mathematical Problem-solving as Characteristics continued

<table>
<thead>
<tr>
<th>STEM: Problem-solving activities</th>
<th>Grouped Median Round 2</th>
<th>Interquartile Deviation Round 2</th>
<th>Consensus Reached?</th>
<th>Grouped Median Round 3</th>
<th>Interquartile Deviation Round 3</th>
<th>Consensus Reached?</th>
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</thead>
<tbody>
<tr>
<td>k. require the use of multiple steps for a successful solution</td>
<td>3.28</td>
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<td>3.25</td>
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<td>l. may be purely contrived mathematical problems</td>
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<td>m. can be puzzles</td>
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<td>n. can be games of logic</td>
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<td>o. involve the consideration of mathematical constructs</td>
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<td>3.6</td>
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<tr>
<td>p. involve non-routine, open-ended, or unique situations</td>
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<td>0.5</td>
<td>NO</td>
<td>3.56</td>
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</tbody>
</table>

(Chamberlin, 18).