UTILITY OF IONOSPHERE AND TROPOSPHERE MODELS FOR EXTENDING THE RANGE OF HIGH-ACCURACY GPS

David William Dodd
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UTILITY OF IONOSPHERE AND TROPOSPHERE MODELS FOR
EXTENDING THE RANGE OF HIGH-ACCURACY GPS

by

David William Dodd

A Dissertation
Submitted to the Graduate Studies Office
of The University of Southern Mississippi
in Partial Fulfillment of the Requirements
for the Degree of Doctor of Philosophy

December 2007
UTILITY OF IONOSPHERE AND TROPOSPHERE MODELS FOR EXTENDING THE RANGE OF HIGH-ACCURACY GPS

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David William Dodd

Abstract of a Dissertation
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December 2007
ABSTRACT

UTILITY OF IONOSPHERE AND TROPOSPHERE MODELS FOR EXTENDING THE RANGE OF HIGH-ACCURACY GPS

by David William Dodd

December 2007

This dissertation studied the use of NOAA real-time ionosphere and troposphere products in extending the range of long-baseline, high-accuracy DGPS for real-time positioning. The question being addressed by this work is; can existing real-time ionosphere and troposphere models reduce the observation uncertainties to the level where they can be used to reliably resolve integer ambiguities, in real-time, over long baselines (>30km). In-house GPS processing software (USM_OTF) was developed to ingest the models and compute epoch-to-epoch, float and fixed ambiguity position solutions. Single baseline processing, ranging from 20 to 740 km, over several days in four separate sessions (July 2004, January 2005, August 2005 and July 2006) incorporating four regions of the U.S.A. (Michigan, California, Central and the South East), were evaluated. The first session looked at the NOAA real-time troposphere model and the second session looked at the NOAA real-time ionosphere model. The third and fourth sessions looked at the use of both the NOAA real-time ionosphere and troposphere models. Results showed that the NOAA troposphere model reduced the height bias uncertainty by up to 30 cm, under high activity conditions. They also showed that the troposphere model increased the uncertainty standard deviation under these high activity conditions. The results from the first tests of the real-time NOAA ionosphere model showed that, due to satellite coverage issues, it produced worse results than other real-time models. The NOAA model suffered from lack of satellite coverage corrections, especially in areas near the limits of the model, and at the beginning and end of a
satellite's flight path. A reduction in satellite numbers lead to weaker geometry and less reliable position solutions. These tests showed that it was better to provide less accurate ionosphere estimates than to leave the satellites out of the solution. These problems were addressed by NOAA prior to the final tests. The final ionosphere testing results showed that, overall, the ionosphere-free float solution with the NOAA troposphere model produced the best results. The float solution determined from the four observables (L1, L2, P1 and P2) using the real-time ionosphere model, when combined with the L1-L2 observation, produced results similar to, but slightly worse than, the ionosphere-free solution. For the short (~20 km) baselines, the four-observable, fixed solution produced the best results, but as the range increased the ability of the ambiguity algorithm to resolve the correct integers, reliably, was degraded due to un-modeled residual ionosphere and troposphere effects.

The real-time NOAA ionosphere and troposphere models, along with the methods developed for this research, greatly reduce the position uncertainty for both float and fixed solutions. However, residual effects hamper the processes ability to reliably fix ambiguities. The four-observation float solutions are comparable to the ionosphere-free solutions, but if the float solutions do not lead directly to fixed solutions, the computational and logistical overhead associated with using ionosphere models is not worth the effort, for real-time differential applications.
DEDICATION

This work is dedicated to my wife Christina van Driest and my children; Kirsten, Ryan and Avery. Without your sacrifice, this would not have been possible.
ACKNOWLEDGEMENTS

The author would like to acknowledge the collaboration of Sunil Bisnath in all of the studies conducted for this dissertation. The generous support of USM and the Department of Marine Science is greatly appreciated. The author would also like to recognize the financial support of the US Coast Guard, C2CEN for two of the studies presented here.

Portions of this dissertation were presented at the 2005 US Hydro Conference in San Diego, the 2006 ION NTM conference in Monterey, and the 2006 ION GNSS conference in Fort Worth. Some results were also published in the August 2006 issue of the International Hydrographic Review.
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<tbody>
<tr>
<td>$1\sigma$</td>
<td>One-sigma standard uncertainty (68% confidence level)</td>
</tr>
<tr>
<td>$2\sigma$</td>
<td>Two-sigma 95% uncertainty</td>
</tr>
<tr>
<td>95% OS</td>
<td>95% ordered statistic, or 95\textsuperscript{th} percentile</td>
</tr>
<tr>
<td>3D</td>
<td>Three-dimensional</td>
</tr>
<tr>
<td>4Obs</td>
<td>USM_OTF processing algorithm that uses four observables (L1, L2, P1 and P2)</td>
</tr>
<tr>
<td>AR</td>
<td>Ambiguity Resolution</td>
</tr>
<tr>
<td>C2CEN</td>
<td>U.S. Coast Guard Command &amp; Control Engineering Center</td>
</tr>
<tr>
<td>CA</td>
<td>Course Acquisition (L1 code)</td>
</tr>
<tr>
<td>CIRES</td>
<td>Cooperative Institute for Research in Environmental Sciences, NOAA and University of Colorado at Boulder.</td>
</tr>
<tr>
<td>CORS</td>
<td>Continuously Operating Reference Stations (GPS)</td>
</tr>
<tr>
<td>DF</td>
<td>Dual Frequency</td>
</tr>
<tr>
<td>DGPS</td>
<td>Differential GPS</td>
</tr>
<tr>
<td>DMS</td>
<td>USM Department of Marine Science</td>
</tr>
<tr>
<td>Doug I</td>
<td>MAGIC ionosphere model</td>
</tr>
<tr>
<td>Doug P</td>
<td>Ionosphere mitigation method developed by Doug Robertson</td>
</tr>
<tr>
<td>DOY</td>
<td>Day of Year</td>
</tr>
<tr>
<td>FAA</td>
<td>Federal Aviation Administration</td>
</tr>
<tr>
<td>Fixed</td>
<td>Ambiguity Fixed Solution</td>
</tr>
<tr>
<td>Float</td>
<td>Ambiguity Float Solution</td>
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<tr>
<td>GIM</td>
<td>JPL Global Ionosphere Map</td>
</tr>
<tr>
<td>GNSS</td>
<td>Global Navigation Satellite System</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
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<tr>
<td>HSRC</td>
<td>USM Hydrographic Science Research Center</td>
</tr>
<tr>
<td>IALA</td>
<td>International Association of Lighthouse Authorities</td>
</tr>
<tr>
<td>ION</td>
<td>Institute of Navigation</td>
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<td>Iono-free</td>
<td>Ionosphere-free processing algorithm</td>
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<td>Jet Propulsion Lab</td>
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<tr>
<td>L1</td>
<td>Primary GPS signal</td>
</tr>
<tr>
<td>L2</td>
<td>Secondary GPS signal</td>
</tr>
<tr>
<td>LAMBDA</td>
<td>Least-squares Ambiguity Decorrelation Adjustment</td>
</tr>
<tr>
<td>LBAS</td>
<td>Land Based Augmentation System</td>
</tr>
<tr>
<td>MAGIC</td>
<td>NOAA ionosphere model</td>
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<td>MDP</td>
<td>Modified Doug P</td>
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<td>NDGPS</td>
<td>Nationwide Differential GPS</td>
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<td>NDGPS-HP</td>
<td>NDGPS High Precision</td>
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<td>NOAA ICON</td>
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<td>NOAA/SEC</td>
<td>NOAA Space Environment Center</td>
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<td>No-iono</td>
<td>No Ionosphere mitigation</td>
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<tr>
<td>NWP</td>
<td>Numerical Weather Prediction</td>
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<tr>
<td>Acronym</td>
<td>Description</td>
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<td>The Ohio State University</td>
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<td>L1 Precise code</td>
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<td>L2 Precise code</td>
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<td>Precise Point Positioning</td>
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<td>Root Mean Square</td>
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<td>RTDGPS</td>
<td>Real-time Differential GPS</td>
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<tr>
<td>RTK</td>
<td>Real-time Kinematic</td>
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<td>SA</td>
<td>Selective Availability</td>
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<tr>
<td>SAAST</td>
<td>Closed form Saastamoinen Troposphere model</td>
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<tr>
<td>SBAS</td>
<td>Space Based Augmentation System</td>
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<td>SF</td>
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<td>USM</td>
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<td>USM_OTF</td>
<td>USM in-house, on-the-fly GPS processing software</td>
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<td>USTECT</td>
<td>US Total Electron Content</td>
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<td>USTEC S</td>
<td>USTEC Slant range TEC</td>
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<td>USTEC Z</td>
<td>USTEC Zenith TEC</td>
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<td>WAAS</td>
<td>Wide Area Augmentation System</td>
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<td>ZPD</td>
<td>Zenith Propagation Delay</td>
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CHAPTER I
INTRODUCTION

This dissertation details the research conducted at USM to evaluate the usefulness of NOAA generated ionosphere and troposphere models for long-range, high-accuracy GPS. It deals specifically with the use of these models for single baseline, real-time applications, such as those used in the maritime environment, where sub-decimeter level vertical positioning is essential if GPS is to be used for water level movement, whether it is used to determine vertical buoy movement or to monitor real-time under-keel-clearance for shipping.

There is no question that GPS has revolutionized positioning for hydrography. In the not so distant past, substantial time, effort and expense went into achieving positional accuracies in the 10s of meters. Now, a $~\$100$ stand-alone handheld GPS receiver can achieve the same precision, almost anywhere in the world. In the days of Selective Availability (SA), GPS receivers were capable of $\sim100$ m positional uncertainty, in real-time, stand-alone operations. Real-time Differential GPS (RTDGPS) reduced the horizontal positioning uncertainty to the meter level, making it a very valuable resource for offshore navigation and positioning applications. To assist in the safe navigation of coastal and inland water transport, many IALA (International Association of Marine Aids to Navigation and Lighthouse Authorities) member nations established networks of radio beacon base stations for the transmission of GPS corrections. This enabled anyone, with the proper radio equipment, to receive differential GPS corrections and subsequently achieve 1-3 meter level horizontal positioning uncertainty. [C2CEN, 2005; Hoppe, 2004; Strang & Borre, 1997]

Advancements in GPS hardware and software reduced real-time GPS uncertainty levels to the decimeter, and in some cases centimeter, level in all three-dimensions, when using carrier phase information. However, to achieve these uncertainty levels, it was necessary to be within $\sim20$ km of a base station and it was necessary to transmit all of the
GPS code and carrier information from the base station to the roving unit (or visa versa) [Hofmann-Wellenhof, Lichtenegger, and Collins, 2001].

Long-range RTDGPS uses processing techniques that greatly reduce uncertainties associated with satellite and receiver clocks as well as with satellite positions. However, atmospheric uncertainties remain; specifically, uncertainties associated with the ionosphere and the troposphere. DGPS greatly reduces the refractive effects of the atmosphere as long as the path between the base and a satellite, and the rover and the same satellite, are similar. As the range between base and remote increases, the likelihood of path similarity decreases. When there is a high variability in atmospheric conditions (e.g., a passing storm), path similarity decreases more rapidly.

An inter-agency team, including the US Coast Guard, Federal Aviation Administration, Federal Highway Administration, and others, was chartered by the US Department of Transport, to implement a modernized version of their highly successful DGPS base station network [Wolfe, Judy, Kritz, Chop and Parsons, 2004]. Rather than simply buying new GPS receivers, they looked to the future and how they could better serve the civilian community, both on land and at sea. One option for improvement was to transmit not only GPS code correctors, but also phase observation information, to enable the user to compute high-accuracy, three-dimensional positions.

There are many applications for long-range, high-accuracy offshore positioning. Coastal and harbor shipping could use high-accuracy vertical positioning for determining under keel clearance to allow for increased shipping capacity and improved navigation safety. The offshore construction industry would benefit greatly from high-accuracy three-dimensional positioning. Coast monitoring buoys could use high-accuracy vertical positioning for tide observations and wave height monitoring, and for tidal datum establishment. Tidal observations could be made in situ, to reduce uncertainties due to distance from shore tidal stations. All vertical coastal zone information (tides, vertical datums, water depths, land heights, reference stations, etc…) could be related to a GPS-defined reference ellipsoid. The relationship between all coastal zone entities would be
established to this common surface, and any interrelationship (e.g., depth relative to chart
datum) would be defined with respect to the ellipsoid (e.g., ellipsoid to chart datum and
ellipsoid to bottom).

The US Coast Guard planned to transmit enough GPS information to enable users
to compute high-accuracy three-dimensional positions. In order to extend the range of
the usefulness of this information, they also looked into transmitting atmospheric
information in the form of troposphere and ionosphere corrector maps. The
Hydrographic Science Research Center, of the University of Southern Mississippi,
conducted research into the use of corrector maps in long-range, high-accuracy GPS
position solutions. This research showed that the use of atmospheric correctors improved
the uncertainty of long baseline positioning to the point where ambiguity resolution
should be possible, well beyond the range of current methods (~20 km) [Hofmann-
Wellenhof et al., 2001]. In order to accomplish this testing, in-house software was
developed to compute float and fixed ambiguity GPS positions using sequential least-
squares algorithms and to allow for the applications of ionosphere and troposphere
models. Follow-up research was conducted to show the positioning improvements
obtained from using ionosphere and troposphere maps together in both static and highly
dynamic situations, using fixed and float solutions.

This dissertation is comprised of introduction and conclusion chapters, a chapter
with background information and four chapters containing specific tests and results.
Chapter two discusses the concepts and algorithms necessary for the reader to understand
the research and results. The next four chapters (three through six) contain the
description and results of four separate tests, which were conducted between 2004 and
2007, with data from four different times (July 2004, January 2005, August 2005 and
June 2006). The third and fourth chapters present the results of individual troposphere
and ionosphere model evaluations respectively. The fifth chapter presents the results of
tests using both the ionosphere and troposphere models together, with static and dynamic
GPS observations. The sixth chapter is comprised of an evaluation of combining both
ionosphere and troposphere models to help resolve integer ambiguities, over long static baselines. The final chapter contains overall conclusions and recommendations.

The third chapter deals with an evaluation of troposphere corrector models, with an emphasis on the National Oceans and Atmosphere Agency (NOAA) model. The evaluation was conducted using data collected in June 2004, from groups of stations in three regions around the US, with baselines between 140 and 740 kilometers in length. Only ionosphere-free, float solutions were used in the analysis. Results showed general improvement in the height component bias of centimeters to decimeters when using the NOAA troposphere model. The results of this study were delivered in a report to the US Coast Guard [Dodd & Bisnath, 2005], as well as in a paper for the 2005 US Hydro conference in San Diego, California [Dodd, 2005] and in the August 2006 volume of The International Hydrographic Review [Dodd & Bisnath, 2006d].

The fourth chapter deals with an evaluation of ionosphere corrector models, with an emphasis on the NOAA US Total Electron Content (USTEC) model. The evaluation was conducted using data collected in January 2005, from groups of stations in four regions around the US, with baselines between 140 and 740 kilometers in length. Only float solutions were used and, because this analysis was designed to evaluate ionosphere models, the NOAA troposphere model was not implemented. A significant solar event occurred during the time period of data collection, which proved to be very challenging for all of the ionosphere mitigation techniques. The USTEC model did not perform as well as many of the other models due to issues with satellite corrector coverage, which have since been addressed. The results of this study were delivered in a report to the US Coast Guard [Dodd & Bisnath 2006c], as well as in a paper for the 2006 Institute of Navigation (ION) National Technical Meeting in Monterey, California [Dodd & Bisnath, 2006b].

The fifth chapter deals with an evaluation of troposphere and ionosphere correctors together, with an emphasis on the NOAA models. Ambiguity-fixed (AR) solution algorithms were implemented and compared with the float solutions. The
evaluation was conducted using data collected prior to and during Hurricane Katrina in August 2005, from two base stations and one ocean-observing buoy located in south Mississippi. The three stations combined for one static baseline of 92 km and two dynamic baselines of 26 km and 99 km. The significant storm event proved to be very challenging for all of the GPS receivers as well as for the processing packages. The AR solutions using the NOAA USTEC model performed the best for the short 26 km baseline. The ionosphere-free solution performed the best over the 99 km dynamic baseline, with the USTEC fixed solution being very close. Use of the NOAA troposphere model did not improve the solutions. The results of this study were presented in a paper for the 2006 ION Global Navigation Satellite System (GNSS) conference in Fort Worth, Texas [Dodd & Bisnath, 2006a].

The sixth chapter deals with an evaluation of troposphere and ionosphere correctors together, with an emphasis on the NOAA models, for static baselines only. The evaluation was conducted using data collected in June 2006, from several stations located in south Louisiana. The baseline lengths ranged from 26 km to 300 km and there were no significant space or earth weather events. The NOAA troposphere maps as well as the NOAA USTEC maps had both been enhanced since the previous three studies. Ambiguity-fixed (AR) solution algorithms were compared with the float solutions. Significant improvement in the height bias was seen for certain baselines, when using the NOAA troposphere model.
CHAPTER II
BACKGROUND

The Global Position System (GPS) is used extensively in many applications around the world. Different applications require different accuracies and processing methods. One of the most challenging applications is high-accuracy, real-time positioning in the maritime environment, where users can expect high dynamics, variable weather patterns and long ranges to GPS reference stations and tidal stations. To understand the challenges in GPS positioning in these environments it is necessary to understand some of the concepts of GPS processing. This chapter begins with an overview of GPS, which is followed by a discussion of some of the uncertainties associated with GPS positioning. It looks into the position processing techniques used for the evaluations, such as sequential least-squares and fixed and float ambiguity solutions. This chapter also discusses some of the key previous research conducted in the area of long-range, high-accuracy GPS.

II.1 GPS Overview

There are several types of real-time GPS positioning modes, which require different equipment and result in different accuracies. Code point positioning, which is the simplest and least accurate mode, requires only one GPS receiver and antenna. Code Differential GPS (DGPS) positioning, which is more accurate than code point positioning, requires two receivers, antennas and a communication link for real-time applications. There are services that provide the differential corrections, in which case only a single rover station is required, along with a system to receive the correctors (usually a satellite or radio link). Precise Point Positioning (PPP), which can achieve high precision, requires a dual-frequency receiver and antenna, advanced processing algorithms and a subscription to a satellite-based correction service. Real-Time
Kinematic (RTK) positioning, which can achieve very high precision (sub-decimeter), over short ranges (< 20 km), requires two dual-frequency receivers and antennas as well as a communication link [Hofmann-Wellenhof et al., 2001]. This dissertation looks at the RTK positioning mode, but without the communication link. The USM_OTF processing methods have been developed to simulate a real-time environment, however all data were post-processed (Post-processed Kinematic or PPK).

The theory behind three-dimensional positioning using GPS is based on the fact that a unique solution can be derived from measuring the distances between an unknown location (roving receiver) and at least three known locations (satellites). The measured distance is computed from the time-of-travel of radio waves transmitted from each of the satellites to the roving receiver. A three-dimensional position (three unknowns x,y,z) can be computed from the three range observations. Because radio waves travel at the speed of light (299,792,458 m/s), extremely accurate time-of-travel estimates are necessary in order to compute a reasonable position (1 ms time uncertainty translates to a 300 km range uncertainty). To alleviate the need for precise clocks in receivers, the receiver clock uncertainty is included as a fourth unknown in the position computation algorithm, in which case a minimum of four ranges are required for a solution. Other uncertainty sources include:

- Satellite clock and position uncertainties
- Multipath uncertainties
- Atmospheric uncertainties
- Residual receiver clock uncertainties

For the code differential GPS technique, it is assumed that the satellite clock and position uncertainties, as well as the atmospheric uncertainties, are the same for two stations in close proximity. Differencing the observations between the two stations will effectively remove the satellite uncertainties, and greatly reduce atmospheric uncertainties. Computing a second difference between one “reference” satellite and the remaining satellites, removes the effect of the receiver clocks. This process is known as double differencing and allows for precise relative vector determination. If the position
of one of the receivers is known (base station), then the position of the other (roving receiver) can be determined through the double differencing process. As the distance between the base and rover increases, the path through the atmosphere diverges, resulting in a de-correlation of atmospheric effects, which in turn leads to a reduction in position uncertainty.

Code positioning refers to the use of the GPS signal code (CA, P1 and/or P2) for range computations. The carrier signal, which is used to transmit the code, has a much smaller wavelength (~20 cm) and GPS receivers can measure the received signal to a fraction of that wavelength. Once the receiver has locked onto a signal, it can keep track of the number of wavelengths that are received. If the initial integer count (ambiguity) can be determined, then the range to that satellite can be determined with a tracking uncertainty at the mm level. If the integer ambiguity can be determined, the solution is considered to be achieved through “integer estimation and fixing” (or simply fixed). If the ambiguities are estimated and the initial integers are not fixed, then the solution is considered to be achieved through “floating point estimation” (or float).

The GPS signal is transmitted at two frequencies, 1575.42 MHz (L1) and 1227.60 MHz (L2). On each frequency the observables are the carrier (L1 and L2 in number of cycles) and the code (CA or P1 and P2 in meters). When using a dual frequency receiver, all four observables are available to the processing software. Doppler shift and signal-to-noise observations can also be used in processing software, if available. The Doppler shift can be used to apply clock offsets and the signal-to-noise ratio can be used in solution weighting functions.

Two of the public services available for enhanced GPS positioning are the US Coast Guard’s Nationwide Differential GPS (NDGPS) and the Federal Aviation Administration’s Wide Area Augmentation System (WAAS). NDGPS is a land-based system where differential corrections are transmitted via medium-wave radio frequencies, from towers at reference station sites. WAAS is a satellite based augmentation system (SBAS) where corrections are delivered via communication satellites. The WAAS
service provides correctors for GPS satellite orbits and clocks as well as corrector grids for the ionosphere and troposphere. Currently, both the NDGPS and WAAS services are intended for single-frequency users and can provide horizontal accuracies at the meter level.

Several private sector services are also available that can provide a variety of accuracies. Some of the high precision options include wide area SBAS services that can provide near-RTK-type solutions with 10 cm vertical positioning (1σ) within a network of base stations. These systems provide ionosphere, troposphere, orbit and clock corrector estimates determined from the network of base stations. Another high precision SBAS option is a PPP-type solution that can achieve 20 cm vertical positioning (1σ). The SBAS provides precise orbit and clock correctors and the PPP algorithms use ionosphere-free carrier observations to smooth the ionosphere-free code observations. The troposphere, for the PPP solution, is generally dealt with by using a zenith propagation delay term as another unknown in the least-squares position solution. One drawback to these long-range systems is the long convergence time (i.e., tens of minutes) needed to achieve high precision result.

The processing methodologies used in the studies for this dissertation were developed based on the assumption that the information necessary for high-accuracy, long-range positioning would be transmitted from the NDGPS reference stations to users at sea. Users would expect to receive dual-frequency code and carrier observations for the base station as well as ionosphere and troposphere information. They would also expect to use the broadcast ephemeris for satellite clock and orbit computations, and rely on information from a single base station.

II.2 Atmosphere

The Earth's atmosphere is a layer of gas surrounding the Earth that is held in place by the Earth's gravitational force. For the most part, the gases become more…
rarified with increasing distance from the Earth, until they disappear altogether into outer space. The lower atmosphere makes up the bulk of atmosphere, with 99.9% of its mass beneath 50 km. The lower region is made up of nitrogen (N\textsubscript{2}), oxygen (O\textsubscript{2}), carbon dioxide (CO\textsubscript{2}), water (H\textsubscript{2}O) and trace gases and is relatively dense. With altitude the atmosphere becomes less dense and more stratified with heavier material at the lower altitudes and lighter material, such as hydrogen, in the outer regions [Hargreaves, 1992].

The atmosphere is divided into regions starting with the troposphere (to approximately 15 km), the stratosphere (to approximately 50 km), the mesosphere (to approximately 80 km), the ionosphere (to about 1500 km and includes the thermosphere and the exosphere) and the magnetosphere. The troposphere is the most variable of the regions and it is here that most of the terrestrial weather occurs. It is the densest region and contains approximately 75% of the atmosphere’s matter. It is generally characterized by a decrease in temperature and pressure with altitude. The stratosphere is more stable than the troposphere and is characterized by a gradual increase in temperature, with higher temperatures laying on top of lower a temperature, which leads to its stability. The increase in temperature with altitude is due to heating from the ultraviolet solar radiation. This layer contains the Ozone layer. The mesosphere layer lies between the stratosphere and the ionosphere and is characterized by a return to the decreasing of temperature with altitude. The ionosphere is of particular interest to GPS users and will be dealt with further in this discussion and in Appendix A. The magnetosphere plays an important role in the composition of the ionosphere; therefore; it too will be examined further.

The magnetic field of the Earth can be considered to be equivalent to a field produced by a dipole magnet located within the Earth, and oriented along the axis of rotation. The field extends through the troposphere out to several thousand kilometers (see Figure II-1). The magnetic field affects the motion of ionized particles. The greater the degrees of ionization, the more the particles are affected by the field. In general, ionization increases with altitude, as a result, the effects of the magnetic field also
increase with altitude. The magnetosphere is the region on the top of the ionosphere where the magnetic field is dominant. [Hargreaves, 1992; Budden, 1985].

The Earth's magnetic field is embedded in a conductor, which is the plasma of the ionosphere. A magnetic field embedded in a conductor cannot be easily changed; as a result, the plasma stream (solar winds), which contains an electric field, cannot penetrate the magnetosphere. The same is true of the magnetosphere penetrating the solar winds. As a result, the magnetosphere acts as a buffer, deflecting solar winds around the Earth. On the sun-side of the Earth, the magnetosphere is blunt (see Figure II-1) where it encounters the solar wind. On the leeside, the solar winds, magnetic field and magnetosphere extend out as a tail. The boundary between the solar winds and magnetosphere is known as the magnetopause, and it is through this boundary that energy is transferred from the solar winds into the magnetosphere. Some solar wind particles flow along the Earth's magnetic field and enter the atmosphere at the poles. Collisions between these charged particles and atmospheric molecules are responsible for the aurora (aurora borealis or northern lights in the northern hemisphere). The magnetosphere circulates in two regions; the lower side circulates with the Earth and the Earth's magnetic field. The outer region circulates with the solar winds.
II.2.1 Ionosphere

GPS signal refraction occurs as the waves travel through the ionosphere. The ionosphere is a region that covers from ~50 to ~1500 km above the Earth’s surface. It is an area that is characterized by electrically charged atoms known as ions. These charged particles interact with radio signals, such as GPS transmissions, as the signals pass through the region. The effect of this interaction on GPS signals is equal and in the opposite direction for the carrier and the code (advance of the carrier and delay of the code). The effect of the ionosphere is dispersive, meaning that it varies with the frequency of the signal.

The ionosphere affects the L1 and L2 GPS signals differently – as there exists a frequency-dependent relationship; as a result the L1 and L2 signals can be combined to create an “ionosphere-free” signal that compensates for the effects of the ionosphere. However, the resulting combined signal has a greatly increased noise level. This L1/L2
combination also removes the integer nature of the initial cycle count ambiguity. This, and the increased noise, makes ambiguity resolution difficult. As a result, the ionosphere-free combination is useful in an ambiguity float solution for long baseline distances (greater than ~20 km), and for initial ambiguity estimates.

L1 and L2 can be combined (L1-L2) to create a “wide lane” carrier observation with an 86.2 mm wavelength. This combination improves the chances of determining the ambiguity by increasing the size of the wavelength, and consequently increasing the size of the ambiguity search space, while maintaining the integer nature of the ambiguity. In the future, the GPS modernization scheme will include a third frequency, L5. This will enable a Three Frequency Ambiguity Resolution (TCAR) solution that will eliminate the ionosphere term. [Hofmann-Wellenhof et al., 2001]

A more detailed treatment of the ionosphere can be found in Appendix A.

II.2.2 Troposphere

The troposphere is the section of Earth’s atmosphere extending from the surface to approximately 10 to 20 km. It is generally characterized by a constant decrease in temperature with increasing height of approximately 6.5 °C/km, on average [Mendes, 1999]. Both the hydrostatic or “dry” and wet constituents of the troposphere refract (reduce speed and alter direction of) electromagnetic waves, such as those of GPS. Unlike the ionosphere, the troposphere is not a dispersive medium, therefore additional GPS signals of varying frequency cannot be used to estimate and eliminate tropospheric refraction.

The effect of the hydrostatic component on the GPS signals accounts for about 90% of the total troposphere delay (zenith delay of ~240 cm). It is a function of temperature and atmospheric pressure; however, it can be computed from atmospheric pressure observations at the receiving antenna [Leick, 2004]. This pressure value can be estimated from the height of the antenna above sea level; therefore, site-specific observations are not necessary. However, the wet delay component is more problematic.
It comprises about 10% of the total troposphere delay (zenith delay of up to ~40 cm), and is far more variable than the dry component [Leick, 2004]. The irregularity of water vapor content, while small in magnitude, represents the major obstacle to precise position estimation – i.e., cm-level [Dodd & Bisnath, 2005].

GPS signal delay resulting from the effects of the troposphere propagates almost directly into height position uncertainties. Effects on the horizontal position uncertainties are averaged out when the satellites are distributed relatively homogeneously around the sky. However, consistent troposphere delays will translate directly into the height because they are not offset by observations from below.

II.3 GPS Processing Techniques

The USM_OTF processing techniques used in this study were developed to simulate, as closely as possible, a real-time, dynamic marine environment. The software was developed to compute a position solution at every epoch, using broadcast ephemeris. Multiple position computation algorithms were developed to accommodate different processing scenarios, such as:

1. Stand alone, single frequency pseudo-range (code), point positioning
2. Double differencing, single frequency, pseudo-range positioning
3. Double differencing, dual frequency, code and carrier, ionosphere-free positioning with sequential-least-squares (SLS)
4. Double differencing, dual frequency, four observation (L1, L2, CA [or P1], and P2), with SLS (ambiguity float solution)
5. Double differencing, dual frequency, four observation (L1, L2, CA [or P1], and P2), with SLS (ambiguity fixed solution)
6. Versions of 3, 4 and 5 above were developed with a Zenith Propagation Delay (ZPD) term

The basic code and carrier double difference observation equations are as follows:

Code: $\nabla \Delta p_i = (p_i^c - p_i^0) + (\rho_h^c - \rho_h^0) - (\rho_h^b - \rho_h^0) + \nabla \Delta E_{\text{tropo}} + \nabla \Delta E_{\text{iono}} + \nabla \Delta E$

Carrier: $\nabla \Delta \phi_i = (\phi_i^c - \phi_i^0) + (\rho_h^c - \rho_h^0) - (\rho_h^b - \rho_h^0) - \lambda \left[ (N_i^c - N_i^0) - (N_h^c - N_h^0) \right]$

$+ \nabla \Delta E_{\text{tropo}} - \nabla \Delta E_{\text{iono}} + \nabla \Delta E$
Where:

\[ \rho \quad = \quad \text{observed pseudo-range} \]
\[ \rho \quad = \quad \text{actual range} \]
\[ \phi \quad = \quad \text{observed carrier, from start of lock} \]
\[ N \quad = \quad \text{ambiguity at start of lock} \]
\[ \lambda \quad = \quad \text{carrier wave length} \]
\[ \nabla \Delta E \quad = \quad \text{double differenced receiver noise and multipath and unmodeled atmospheric uncertainties} \]
\[ \nabla \Delta E_{\text{tropo}} \quad = \quad \text{double differenced modeled troposphere uncertainty} \]
\[ \nabla \Delta E_{\text{iono}} \quad = \quad \text{double differenced modeled ionosphere uncertainty} \]
\[ x, y, z \quad = \quad \text{satellite coordinates} \]
\[ X, Y, Z \quad = \quad \text{receiver coordinates} \]
\[ \text{Subscript } r \quad = \quad \text{remote receiver} \]
\[ \text{Subscript } b \quad = \quad \text{base receiver} \]
\[ \text{Superscript } i \quad = \quad \text{satellite} \]

A more detailed treatment of various observation equations and algorithms can be found in Appendix C.

The sequential-least-squares (SLS) processes were included for position and ambiguity resolution algorithms. Ambiguity estimates were included as unknowns in the least-squares process and, once established, did not change. Ambiguity estimates and uncertainties (aposteriori variance-covariance), estimated for one epoch, were used in the subsequent epoch’s solution. As the confidence in the ambiguity estimates increased, more weight was put on the values transferred from the previous epoch, to a point where they no longer changed. As part of the SLS process, positions and ZPD values (if applicable), along with their confidences, were also transferred from epoch to epoch. The SLS implementation was adapted from the precise point positioning application, presented by Kouba & Heroux (2001), to the double differencing application used in these studies.

Ionosphere-free and four observation (4Obs) float ambiguity solutions were used in all of the four studies presented in this dissertation. Initial double difference float ambiguity values were estimated from a position solution obtained from using code double differencing. These initial ambiguity values were sent to the next epoch and used
in an ambiguity float solution (ionosphere-free or 4Obs), where a new set of ambiguities were estimated. A new set of estimated unknowns (position, ZPD [if applicable], and ambiguities), along with their a posteriori variance-covariance matrix, were then passed along to the subsequent epoch. The a posteriori variance-covariance matrix was added to the a priori variance-covariance of unknowns, along with some system noise. As the confidence in the unknown values increased, the variances and covariances decreased. In order to simulate dynamics, the position system noise was set to 1.0 m/s² as an estimate of a vessel’s antenna movement. The assumption was made that, once they were considered to be known, the ambiguities would not change; therefore, their system noise was set to zero.

For ionosphere evaluations, the various ionosphere models were compared to the ionosphere-free solutions. For troposphere evaluations, the various solutions (including ionosphere-free) were processed with either the NOAA troposphere model or the standard Saastamoinen (SAAST) closed-form model, the algorithm for which was taken from Spilker et al. 1996. The standard Klobuchar ionosphere mitigation model and the standard Saastamoinen troposphere model were incorporated into the processing code. The Klobuchar algorithm was taken from Navtech, 1995. All of the other ionosphere and troposphere models were ingested as text files. MATLAB™ converters were developed to transform the ionosphere files from their source format to a standard text file format. Each line in the standard files contained a time and slant corrector values for each in-view satellite, in meters. The NOAA troposphere maps were converted to a standard file format where each line contained the time; zenith wet delay, zenith dry delay, and combined wet and dry delay, in mm. The Neill mapping function [Neill, 1996] was implemented in the USM_OTF processing code to map the zenith wet and dry delay values to slant correctors for each satellite.

In the last two studies (CHAPTER V and CHAPTER VI) fixed ambiguity solutions were included in the evaluation process. The LAMBDA method was used to determine the integer ambiguity values. LAMBDA (least-squares ambiguity
decorrelation adjustment method) is an integer least-squares estimator used to determine integer ambiguity values for carrier phase observations [Joosten, 2001]. The LAMBDA method was implemented in the USM_OTF processing software. It uses float ambiguities and the corresponding variance-covariance matrix (from the float solution) to establish a search area from which the optimum set of fixed ambiguity candidates are estimated. The resulting integer ambiguities were then used in the ambiguity fixed solution. A more detailed treatment of the LAMBDA method can be found in Appendix B.

The zenith propagation delay term (ZPD) was implemented in the least-squares position estimation algorithms. A ZPD term was included as another unknown along with the latitude, longitude and height in the position estimation process. The ZPD corrector term was translated from the zenith to a slant delay value using the Neill wet delay mapping function. This slant range corrector was then removed from the range observations. This term should absorb some of the residual troposphere delay biases.

All of the primary processing code was written in "C". Multiple MATLAB™ scripts were written to reformat and interpolate ionosphere files. MATLAB™ scripts were also written to drive the processing code and facilitate batch processing. MATLAB™ scripts were also used to compute statistics, create plots and compile results. Waypoint™ Navigation’s commercial GrafNav™ software was used to validate the in-house processing software and methods, and compare test results.

A mathematical treatment of many of the algorithms used in USM_OTF can be found in Appendix C.

II.4 Use of Statistics in Evaluations

Statistics were used throughout the dissertation to help in the analysis of the results. Bias (mean), standard deviation and root-mean-square (RMS) values were computed for each processing run from the uncertainty values determined by differencing the published CORS station coordinates and the coordinates computed by the algorithms.
Statistics were generated for the north, east and up uncertainties. The RMS indicated overall uncertainty, and included both systematic (bias) and random (standard deviation) uncertainties. In some cases, two-dimensional uncertainties were computed from the horizontal position differences, and in other cases three-dimensional uncertainties were determined from the differences in all three directions (north, east and up).

Uncertainties are expressed at a particular confidence level. One-dimensional, one-sigma (1σ) standard uncertainties are considered to be at the 68% confidence level. In hydrography, uncertainties are usually expressed at the 95% confidence level (2σ). If the uncertainties are randomly distributed, then the 95% confidence level, for one-dimensional observations, can be computed by multiplying by 1.96, in the two-dimensional case the multiplier is 2.45 and in the three-dimensional case the multiplier is 2.80 [Vanicek & Krakiwsky, 1996].

Another method for determining the 95% confidence level is to use the actual data. This can be done by sorting the absolute value of the difference between published and observed coordinates and determining under what value 95% of the differences lie. This value is known as the 95% ordered statistic (95% OS) or 95th percentile, and its determination does not rely on random distribution [Wells, 2007]. All systematic and random errors are included in this statistic. If the data are randomly distributed, then 95% OS and 2σ will be very similar.

For the most part, the statistics expressed in these studies are used to evaluate the changes (improvements) in results from the implementation of different methods, algorithms and models. They are not used to express the overall accuracy that can be expected from the different processes and applications. Therefore, for much of this work, expressing uncertainties at the 95% confidence level is not always applicable. However, whenever reference is made to accuracy expectations, the 95% RMS uncertainties are included. This is especially true for the final analysis described in 0. Histograms from baselines used in 0, which include the 2σ limits as well as the 95% ordered statistic limits are in Appendix D. These histograms show that, for the most part, the uncertainties
associated with these studies are randomly distributed, and that the 2σ and 95% OS are very similar.

II.5 Other Long-range, High-accuracy GPS Efforts

Several institutions have conducted research into extending the range of RTK positioning, including The University of New Brunswick (UNB), The University of Calgary (U of C) and The Ohio State University (OSU). This section looks at some of the research conducted by these, and other groups, in the area of high-accuracy, long-range GPS.

The University of New Brunswick ran a yearlong data collection program in Nova Scotia and New Brunswick to test the effects of the troposphere on long-range RTK in the marine environment. Dual-frequency GPS and meteorological data were collected at shore stations and on the Princess of Acadia ferry, which crossed the Bay of Fundy, between St. John and Digby, where the two shore stations were located. The shore station separation was 74 km. Several papers have been produced in relation to this project, looking at different troposphere mitigation techniques and how they affect high-accuracy, long-range positioning (Cove, Santos, Wells and Bisnath, 2004; Santos, Nieviniski, Cove, Kingdon and Wells, 2005; Nieviniski, Cove, Santos, Wells and Kingdon, 2005; Ahn, Kim, Dare and Langley, 2005; and Nieviniski, 2006). Cove et al. [2004] found that although the Numerical Prediction Models (NWP, produced from weather observations) improved the range observation, it translated into only a minimal improvement in the position uncertainty. Ray tracing through the NWP to determine delay values was investigated in Ahn et al. [2005] and Nieviniski [2006]. Both found some positional improvement using ray tracing through the NWP models, over the UNB3 closed form model, but at significant computational expense. Height root-mean-square (RMS) values of 5 and 6 cm (1σ) were achieved for the 74 km static baseline. Ahn et al. [2005] showed that ambiguity fixing, over the 74 km static baseline, using the ray traced NWP model, was much more reliable than when the UNB3 model was used. Ahn et al.
[2005] achieved height RMS values of 5 cm, with significant improvement in the bias, from the closed form solutions.

Alves et al. [2004], at The University of Calgary and in conjunction with the US coastguard, looked into using the NOAA troposphere model for long-range high-accuracy positioning. This research evaluated single baseline solutions, but concentrated mainly on the multiple baseline network approach. These tests saw single baseline height RMS values improve from 19 cm to 13 cm for a 170 km baseline and from 25 cm to 19 cm for a 230 km baseline.

Troposphere model studies were performed at The Ohio State University and the results were presented at ION NTM 2005 (Zhang & Bartone, 2005a). This study looked at the use of troposphere models in long-range, high-accuracy GPS for land based applications. The study evaluated the range uncertainties and did not address position uncertainties. A follow-on study by the same authors (Zhang & Bartone, 2005b) did evaluate horizontal position uncertainties. In this study they saw a 2D RMS of 12.8 cm and 22.5 cm 3D RMS over 295 km and 16.2 cm 2D RMS over 780 km, using NDGPS-High Performance (HDGPS-HP) which included the NOAA troposphere model.

Studies conducted to evaluate the use of ionosphere models in long-range, high-accuracy GPS positioning include Colombo, Hernandez-Pajares, Juan and Sanz [2002], and Grejner-Brzezinska et al., [2005]. Colombo et al., [2002], looked at using ionosphere tomography to help resolve double differenced ambiguities over long baselines (~400 km) in a network solution. A two-layer ionosphere model was created from TEC estimates derived from dual-frequency CORS station observations. Ionosphere uncertainty estimates were determined along the path between a satellite and receiver and removed from the range observations. The bulk of the troposphere uncertainty was removed through the differential process. The residual troposphere uncertainty was included as a nuisance parameter in the solution. Testing looked at the use of broadcast and precise ephemeris. Results showed a very high success rate for ambiguity resolution when using adjusted or precise orbits. Follow on tests (Colombo, Sutter and Evans,
2003) using broadcast ephemeris and baselines of 500 to 1000 km showed a height uncertainty that generally stayed within ±25 cm.

Grejner-Brzezinska et al., [2005] performed a study on the impact of external ionosphere models on the uncertainty of RTK position estimates for single baselines. In this study, comparisons between NOAA's MAGIC and ICON models were made. Two-hour segments of 30-second observations, from three CORS stations, making up two baselines of ~ 60 and ~100 km, were evaluated. Results showed ambiguity resolution within 18 epochs for the MAGIC solutions and 68 epochs for the ICON solutions. Once the ambiguities were resolved, the 60 km baseline showed a height uncertainty mean of -3.4 cm and standard deviation of 2.5 cm (~4.2 cm RMS), and the 100 km baseline showed a mean of ~ 4.8 cm and a standard deviation of 2.7 cm (~5.5 cm RMS).

Many tests have been conducted in the use of GPS buoys at sea for height determination. Two papers, Arroyo-Suarez, Hsiao and Mabey [2005] and Colombo, Evans, Vigo-Aguiar, Ferrandiz and Benjamin [2000], represent a small sampling of this research. These tests concentrated on the use of GPS buoys for long period (> 10 minutes) water level variations such as those caused by storms, seismic events and tides. As a result, the high frequency variations were filtered out. Arroyo-Suarez et al., [2005] looked at the use of the Precise Point Positioning (or ultra long-baseline) technique for positioning, whereas Colombo et al., [2000] looked at long-baseline (up to 1200 km) differential techniques. Both of the positioning methods used some form of precise ephemeris. Both studies showed averaged height determinations at the 10 cm level.

Previous studies tended to look at either ionosphere models or troposphere models. Some of the studies looked at only the effect of the models on the observed range, and not the resulting position. Very few of the previous studies attempted to look at ambiguity resolution over long ranges. Also, many of the previous studies relied on the use of precise ephemeris. The studies conducted for this dissertation looked at the use of troposphere and the ionosphere models separately, and then together. They also looked at the effect of troposphere and ionosphere models on the resulting position, with
an emphasis on instantaneous height variation (no averaging), as applicable to the marine environment, using only broadcast ephemeris.

The results obtained from the researched presented for this thesis are comparable to those achieved by others. A direct comparison is difficult because of the different methods used, different types of statistics presented, different weather conditions, data averaging and amount of data evaluated. In general, uncertainties of ~6 cm (1σ height RMS) for a 100 km baseline, and 20 cm for 200 km are typical. In this study, we saw 1σ height RMS values of ~10 cm for 100 km and 18 cm for 200 km.
CHAPTER III
ANALYSIS OF THE UTILITY OF NOAA-GENERATED TROPOSPHERIC REFRACTION CORRECTIONS FOR DGPS

There are several models and methods used to mitigate the effect of the troposphere on GPS signals. Closed form models, such as the Saastamoinen model, use standard meteorological observations, although local temperature and pressure observations can be used in modified versions. Troposphere wet and dry delay maps, created from meteorological observations, can be interpolated to establish corrections for individual locations. Differential processing can remove the bulk of the tropospheric effect, with the remaining uncertainty dependant on the variation in tropospheric conditions over the distance between stations. Another method for mitigating the effects of the troposphere is to add another parameter (zenith propagation delay term) to the least-squares estimation process. This method has the advantage of not requiring any outside information, but the disadvantage increasing the number of unknowns, which decreases the degrees-of-freedom of the process. There is also a possibility of absorbing antenna motion incorrectly into the troposphere uncertainty. The research presented here looks at the effectiveness of combining differential techniques with the NOAA troposphere maps to mitigate tropospheric uncertainties over medium to long baselines. The evaluation is based on the improvement of the results as compared to using the Saastamoinen closed-form model. The effects of the ionosphere were almost completely removed by combining the observations to create an ionosphere-free measurement, resulting in a float (real-valued) ambiguity solution.

The NOAA tropospheric delay model was developed by the Forecast Systems Lab at NOAA [Gutman, Fuller-Rowell and Robertson, 2003]. The model consists of a numerical weather prediction model in which GPS zenith delay data are assimilated. The GPS data are collected from a large subset of Continuously Operating Reference System (CORS) sites. One manner in which to view this technique is that it allows for the GPS data to constrain the integrated delay in the weather model, while the weather model
provides a physics-based method of interpolating and extrapolating delays in space and time. The inputs to the model are: latitude, longitude, ellipsoid height, and time, and the outputs are: zenith hydrostatic delay and zenith wet delay for the current time (last assimilation) and for a two-hour prediction. The estimation is currently realized in a suite of client software consisting of C, FORTRAN, and Perl programs, which access NOAA tropospheric grid files via FTP. These grids are produced hourly, with ~20 km grid spacing. Again, the grids contain nowcasts and two hour forecasts. Further information can be found at http://www.gpsmet.noaa.gov. [Dodd & Bisnath, 2005]

Much of this study was funded by the US Coast Guard as part of their National DGPS upgrade process [Wolf et al., 2004], and delivered in a 2005 report [Dodd & Bisnath, 2005]. The results were presented at the US Hydro conference in San Diego, in 2005 [Dodd, 2005], as well as in the August 2006 issue of The International Hydrographic Review [Dodd & Bisnath, 2006d]. The following sections discuss the methodology used to perform the tests, and present an analysis example for one baseline, followed by a summary of the results of the analysis using all of the baselines. The final section discusses the results.

III.1 Methodology

In order to test positional uncertainty improvements when using different troposphere models, in-house processing software (USM_OTF) was developed. This relative kinematic positioning software was designed to simulate Real-Time Kinematics (RTK) firmware in a GPS-based positioning / navigating scenario. The software processed ionosphere-free, double-difference observables to mathematically almost completely eliminate the effects of ionospheric refraction. This combination allowed for robust long baseline processing, and removed the ionospheric effect, which otherwise would have complicated the analysis, since both the ionosphere and troposphere affected GPS positioning in a similar fashion. Also, carrier phase ambiguities were not fixed to integers in the processing because an ionosphere-free solution was used, and, as such, the
observable did not contain integer ambiguities. More details on the USM-OTF algorithms can be found in Appendix C.

The USM_OTF software was configured to simulate a real-time environment with the roving receiver in motion. To this end, broadcast ephemerides were used and position dynamics for the sequential least-squares filter were set to 1 m/s². Position uncertainty statistics show solutions as good as 12 cm, 12 cm and 20 cm (north, east and up 1 σ RMS) for a 620 km baseline length.

The USM_OTF software was validated using the commercial GPS processing package GrafNav™ version 6.03. Several baselines were processed in GrafNav™ with one station designated as “kinematic” and using the float solution in the forward direction only. The GrafNav™ software was used to ensure that the USM software was producing reasonable results; therefore the options used were selected to correspond to the USM processing scenario, and not to produce the optimal GrafNav™ solution.

Baselines from three areas of CONUS were processed: Michigan, California, and the South East (see Figure III-1). Each baseline consisted of 18 to 24 hours of 30-second observations per day for three to six days. Each day was processed as a separate data set. The stations selected were Continuously Operating Reference Station (CORS) sites, given that their data were readily available via the Internet.

A comparison of positioning results using the Saastamoinen (USM SAAST) and NOAA (USM NOAA) tropospheric delay estimates was carried out. Due to its popularity, the Saastamoinen model was selected as the conventional solution. For comparison, the UST_OTF software was run using the Saastamoinen model, and then using the NOAA model. The only difference between the two software runs was the troposphere correction applied.

The Saastamoinen model is a closed-form model that does not require any external meteorological information. The troposphere slant delay for each satellite is computed from the satellite elevation angle, receiver height above mean sea level, and the receiver latitude [Saastamoinen, 1973]. The NOAA delay estimates are supplied in the
form of zenith grid maps (~13' x ~13') covering the continental US. The grid map values are computed from a combination of CORS station GPS observations and meteorological observations. It is left to the user to determine the slant range delay values, and for this study the zenith delay values are converted to slant values using the Neill mapping function [Neill, 1996]. A more detailed description of the Saastamoinen model and algorithm can be found in Appendix C.

Figure III-1: Map depicting the three regions used for position domain analysis, Michigan, California and the South East.

III.2 Analysis for Single Baseline

Processing results for the SUP3-FRTG baseline (430 km), from the Michigan region, are shown here as an example. Position uncertainties and related statistics were determined by comparing computed rover positions to the published CORS station coordinates. Table III-1 shows the mean, standard deviation (1 σ), and RMS from GrafNav™ (using Saastamoinen), USM SAAST (Saastamoinen troposphere model), and USM NOAA (using NOAA troposphere model). The statistics indicate that the USM SAAST results are comparable to the GrafNav™ solution, and that the USM NOAA solution is an improvement to both, in all directions. However, the improvement in the
up direction, for both bias (~10 cm) and standard deviation (~10 cm) is the most pronounced. Figure III-2, Figure III-3 and Figure III-4 show North, East, and Up uncertainty time series plots for each of the three processing runs.

<table>
<thead>
<tr>
<th>GrafNav&lt;sup&gt;TM&lt;/sup&gt;</th>
<th>Mean (m)</th>
<th>Stdev (m)</th>
<th>RMS (m)</th>
<th>95% OS (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>0.035</td>
<td>0.095</td>
<td>0.101</td>
<td>0.196</td>
</tr>
<tr>
<td>E</td>
<td>0.006</td>
<td>0.108</td>
<td>0.108</td>
<td>0.180</td>
</tr>
<tr>
<td>U</td>
<td>-0.207</td>
<td>0.238</td>
<td>0.315</td>
<td>0.556</td>
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</table>

<table>
<thead>
<tr>
<th>SAASTAMOINEN</th>
<th>Mean (m)</th>
<th>Stdev (m)</th>
<th>RMS (m)</th>
<th>95% OS (m)</th>
</tr>
</thead>
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<tr>
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<td>0.090</td>
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<td>E</td>
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<td>U</td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>NOAA</th>
<th>Mean (m)</th>
<th>Stdev (m)</th>
<th>RMS (m)</th>
<th>95% OS (m)</th>
</tr>
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<td>0.066</td>
<td>0.066</td>
<td>0.183</td>
</tr>
<tr>
<td>E</td>
<td>-0.041</td>
<td>0.081</td>
<td>0.091</td>
<td>0.162</td>
</tr>
<tr>
<td>U</td>
<td>0.016</td>
<td>0.103</td>
<td>0.104</td>
<td>0.166</td>
</tr>
</tbody>
</table>

Table III-1: Mean, Standard Deviation (1σ), RMS and 95% OS for Michigan baseline SUP3-FRTG (430 km).

Figure III-2: GrafNav<sup>TM</sup> position uncertainty plots for Michigan SUP3-FRTG.
Figure III-3: USM Saastamoinen position uncertainty plots for the Michigan SUP3-FRTG baseline.

Figure III-4: USM NOAA position uncertainty plots for the Michigan SUP3-FRTG baseline.
III.3 Analysis Summary for All Baselines

Six days of observations, July 11 (day of year [DOY] 193) through July 16 (DOY 198) 2004, were used for this analysis. Each day was processed separately. Due to gaps in the CORS GPS data, not all stations could be used on all days. In the cases where one or more stations were eliminated due to large data gaps, the entire day was eliminated for all baselines in that region.

The baseline analysis summary has been divided into the three regions of Michigan, California and the South East. In each region, plots showing the mean (bias), standard deviation (1σ) and RMS for each baseline were generated. In this summary, only the improvement in results between USM SAAST and USM NOAA are shown.

III.3.1 Michigan Region

Seven CORS sites, all equipped with Leica RS500 GPS receivers, were used for the Michigan region analysis (see Figure III-5). Various combinations of these seven sites produced ten baseline pairs with lengths varying from 190 km to 620 km (see Table III-2). Due to large data gaps in several stations, July 12 (DOY 194) was not used. Five days of data were used including July 11, and July 13 through 16, 2004 (DOY 193, DOY 195 through DOY 198).
Figure III-5: Baselines and CORS stations for Michigan region.

<table>
<thead>
<tr>
<th>Number</th>
<th>Baseline</th>
<th>Length (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>UNIV-FRTG</td>
<td>190</td>
</tr>
<tr>
<td>2</td>
<td>SUP2-NOR2</td>
<td>210</td>
</tr>
<tr>
<td>3</td>
<td>SOWR-FRTG</td>
<td>270</td>
</tr>
<tr>
<td>4</td>
<td>NOR2-FRTG</td>
<td>280</td>
</tr>
<tr>
<td>5</td>
<td>SUP1-NOR2</td>
<td>350</td>
</tr>
<tr>
<td>6</td>
<td>SUP2-SOWR</td>
<td>400</td>
</tr>
<tr>
<td>7</td>
<td>SUP3-FRTG</td>
<td>430</td>
</tr>
<tr>
<td>8</td>
<td>SUP2-FRTG</td>
<td>480</td>
</tr>
<tr>
<td>9</td>
<td>SOWR-SUP1</td>
<td>560</td>
</tr>
<tr>
<td>10</td>
<td>FRTG-SUP1</td>
<td>620</td>
</tr>
</tbody>
</table>

Table III-2: Baselines and corresponding lengths for Michigan region.

Visual inspection of the zenith wet delay for the time period shows fairly homogeneous conditions for this region (see Figure III-6). Even when conditions changed (see Figure III-7) with increasing wet delays, the change was consistent throughout the area. Over the six-day observation span the zenith wet delay ranged between about 10 cm and 25 cm. The troposphere delay images shown in this chapter were truncated at the shoreline for visualization only. The delay grids do cover the marine regions.
Figure III-6: Michigan zenith wet delay (in cm) for July 11, 2004. Image derived from the NOAA troposphere zenith delay maps. The red square delimits the test area.

Figure III-7: Michigan zenith wet delay (in cm) for July 13, 2004.
Figure III-8 shows the north, east and up 1σ RMS for all Michigan baselines, using NOAA troposphere model. The height RMS varies from a low of ~5 cm (10 cm at 2σ) to a high of 30 cm (60 cm at 2σ). Generally, the uncertainty increases with baseline length.

![Graph showing RMS variations](image)

Figure III-8: Michigan north, east and up 1σ RMS for all baselines, using NOAA troposphere model.

Figure III-9 shows the 1σ RMS (uncertainty) improvements in the north, east and up components for the processed data sets as a function of baseline length. For this data set, the RMS improvement did not appear to be a function of baseline length. The average improvement in the horizontal position was about 1 cm in both north and east. There was sizeable RMS improvement in the vertical component, ranging from a few centimeters to over a decimeter with the average being 5.5 cm. Closer analysis indicated that the improvement was due to both a reduction in the uncertainty bias, with an average improvement of 4.4 cm, and standard deviation (solution noise or dispersion), with an average improvement of 3.0 cm (see Figure III-10 and Figure III-11). For the most part,
position uncertainty improved with the use of the NOAA troposphere model; however, in a minority of cases the uncertainty did not change, or got slightly worse. Inspection of the bias and standard deviation plots showed that the NOAA-based position uncertainty improvement was due to a reduction in the uncertainty in both bias and standard deviation, compared to the SAAST-based solutions.

Figure III-9: Michigan RMS improvement in USM NOAA over USM SAAST.
Figure III-10: Michigan bias improvement in USM NOAA over USM SAAST.

Figure III-11: Michigan stdev improvement in USM NOAA over USM SAAST.
III.3.2 California Region

Six CORS sites, all equipped with Ashtech Z-XII3 GPS receivers, were used for the California region analysis (see Figure III-12). Various combinations of these six sites produced nine baseline pairs with lengths varying from 140 km to 740 km. There were also significant height differences between some baseline pairs with variations from 235 m to 1850 m (see Table III-3). All six days of data were used, including July 11 through 16, 2004 (DOY 193 through DOY 198).

![Figure III-12: Baselines and CORS stations for California region.](image)

<table>
<thead>
<tr>
<th>Number</th>
<th>Baseline</th>
<th>Length (km)</th>
<th>ΔH (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>BLYT-MONP</td>
<td>140</td>
<td>1756</td>
</tr>
<tr>
<td>2</td>
<td>TORP-MONP</td>
<td>200</td>
<td>1847</td>
</tr>
<tr>
<td>3</td>
<td>CHAB-CARH</td>
<td>260</td>
<td>250</td>
</tr>
<tr>
<td>4</td>
<td>TORP-CARH</td>
<td>310</td>
<td>485</td>
</tr>
<tr>
<td>5</td>
<td>COSO-BLYT</td>
<td>390</td>
<td>1541</td>
</tr>
<tr>
<td>6</td>
<td>COSO-CHAB</td>
<td>420</td>
<td>1225</td>
</tr>
<tr>
<td>7</td>
<td>MONP-CARH</td>
<td>500</td>
<td>1362</td>
</tr>
<tr>
<td>8</td>
<td>TORP-CHAB</td>
<td>560</td>
<td>235</td>
</tr>
<tr>
<td>9</td>
<td>MONP-CHAB</td>
<td>740</td>
<td>1612</td>
</tr>
</tbody>
</table>

Table III-3: Baselines and corresponding lengths for California region.
Visual inspection of the zenith wet delay for the time period showed a wide variety of conditions for this region (see Figure III-13). Over the six day observation span the zenith wet delay ranged between about 0 cm and 30 cm with, in many cases, both extremes occurring at the same time (see Figure III-14). The troposphere delay images shown in this chapter were truncated at the shoreline for visualization only. The delay grids do cover the marine regions.

Figure III-13: California zenith wet delay (in cm) for July 11, 2004.

Figure III-14: California zenith wet delay (in cm) for July 13, 2004.
Figure III-15 shows the 1σ RMS in the north, east and up components for the GrafNav™ (blue square) and USM SAAST (red circle) processing results for DOY 198 (July 16), as a function of baseline length. These plots indicate that the two processes produced very similar results.

Figure III-16 shows the north, east and up 1σ RMS for all California baselines, using NOAA troposphere model. The height RMS varies from a low of ~5 cm (10 cm at 2σ) to a high of almost 50 cm (100 cm at 2σ). Generally, the uncertainty increases with baseline length.
Figure III-16: California north, east and up 1σ RMS for all baselines, using NOAA troposphere model.

Figure III-17, Figure III-18, and Figure III-19 show the RMS, bias, and standard deviation (1σ) improvements, respectively, of the NOAA troposphere model over the Saastamoinen model. All of these plots show RMS versus baseline length. The RMS improvement does not appear to be a function of the baseline length. The RMS plots indicate general improvement in both horizontal and vertical position uncertainty. The average horizontal improvements were 2.9 cm in the north and 2.0 cm in the east. The average improvement in the height was 14.3 cm. In a few cases the USM NOAA solution was worse than the USM SAAST. For the most part, the USM NOAA results that were worse than USM SAAST (negative improvement) correspond to an increase in the USM NOAA bias uncertainty.
Figure III-17: California RMS improvement in USM NOAA over USM SAAST.

Figure III-18: California bias improvement in USM NOAA over USM SAAST.
The position improvements seen here are much larger than those seen in the Michigan data set. Much of this can be attributed to the greater height difference, as well as the greater range and variability in wet delay between baseline pairs. However, some improvement may be due to the incorporation of atmospheric pressure observations, used in the hydrostatic component of the NOAA model, rather than estimated pressure values from station height, which are used in the Saastamoinen solution. Figure III-20, Figure III-21 and Figure III-22 show the RMS, bias and standard deviation (1σ) improvements of the NOAA troposphere model over the Saastamoinen troposphere model versus the difference in height between baseline pairs. There does appear to be an increase in height RMS improvement with increasing station pair height difference, which would indicate that the height variation correctors used in the Saastamoinen model are not as sensitive to change as the NOAA based model.
Figure III-20: California RMS improvement using NOAA versus height difference.

Figure III-21: California bias improvement using NOAA versus height difference.

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III.3.3 South East Region

Eight CORS sites, all equipped with Ashtech Z-XII3 or UZ-12 GPS receivers, were used for the South East region analysis (see Figure III-23). Various combinations of these eight sites produced nine baseline pairs with lengths varying from 190 km to 690 km (see Table III-4). There were no significant height differences between baseline pairs. Only three days of data were used: July 11 (DOY 193), July 13 (DOY 195) and July 16 (DOY 198).

Visual inspection of the zenith wet delay for the observation time period showed some dry areas in the beginning with wet delay readings as low as 15 cm (see Figure III-24). However, this region of the US is usually hot and humid during the summer months and Figure III-25 is a more representative picture, with wet delay values consistently between 25 cm and 35 cm. Viewing all of the wet delay plots for the week,
in the form of a movie, showed that weather variations occurred very rapidly, as compared to the other two areas of Michigan and California. The troposphere delay images shown in this chapter were truncated at the shoreline for visualization only. The delay grids do cover the marine regions.

Figure III-23: Baselines and CORS stations for South East region.

<table>
<thead>
<tr>
<th>Number</th>
<th>Baseline</th>
<th>Length (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>OKCB-RMND</td>
<td>190</td>
</tr>
<tr>
<td>2</td>
<td>LSUA-LMCN</td>
<td>270</td>
</tr>
<tr>
<td>3</td>
<td>SIHS-LMCN</td>
<td>300</td>
</tr>
<tr>
<td>4</td>
<td>JXVL-OKCB</td>
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<td>6</td>
<td>PNCY-LMCN</td>
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</tr>
<tr>
<td>9</td>
<td>PRRY-LMCN</td>
<td>690</td>
</tr>
</tbody>
</table>

Table III-4: Baselines and corresponding lengths for South East region.
Figure III-24: South East zenith wet delay (in cm) for July 11, 2004.

Figure III-25: South East zenith wet delay (in cm) for July 13, 2004.

Figure III-26 shows the north, east and up \(1\sigma\) RMS for all South East baselines, using NOAA troposphere model. The height RMS varies from a low of \(\sim 5\) cm (10 cm at \(2\sigma\)) to a high of almost 50 cm (100 cm at \(2\sigma\)). Generally, the uncertainty increases with baseline length.
Figure III-26: South East north, east and up $1\sigma$ RMS all baselines using NOAA troposphere model.

Figure III-27, Figure III-28 and Figure III-29 show the RMS, bias, and standard deviation ($1\sigma$) improvements of the NOAA troposphere model over the Saastamoinen model. All of these plots show RMS versus baseline length. The RMS plots indicate general improvement in the vertical position uncertainty. The average horizontal improvement was 0.2 cm in the north and -0.4 cm in the east. The average height improvement was 4.3 cm. The RMS improvements do not appear to be a function of baseline length. In a few cases the USM NOAA solution is worse than the USM SAAST solution (negative improvement). For the most part, the USM NOAA results that are worse than USM SAAST results (negative improvement) correspond to an increase in the USM NOAA standard deviation. The average standard deviation improvement in the height was -8.0 cm. The overall bias differences show an improvement in all cases; however, in a few cases the standard deviation indicates a higher noise level for the

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NOAA solution, which leads to an overall negative RMS improvement. The rapid weather pattern movement may be partially responsible for this.

Figure III-27: South East RMS improvement in USM NOAA over USM SAAST.
Figure III-28: South East bias improvement in USM NOAA over USM SAAST.

Figure III-29: South East stdev improvement in USM NOAA over USM SAAST.
III.4 Conclusions

For all three study areas, the horizontal RMS improvements were negligible (few cm), whereas the vertical RMS improvements were consistently above one decimeter, and in some cases above three decimeters. As expected, the troposphere uncertainty has little effect on the horizontal position because the effect is averaged out when satellites are evenly distributed in the sky. The greatest effect is in the height because satellites are distributed only above the user. For the data used in this study, with baselines between 140 and 740 km, the RMS improvements in height from USM SAAST to USM NOAA did not appear to be a function of baseline length. Table III-5 shows the average position height uncertainty improvements (bias, standard deviation and RMS) for each study area. Note that these values refer to improvements (SAAST-NOAA) and consequently a negative improvement indicates that the NOAA troposphere model resulted in a worse solution. For the South East, the average standard deviation improvement was −8.0 cm; however, there was still an average RMS improvement of 4.3 cm.

<table>
<thead>
<tr>
<th></th>
<th>bias (cm)</th>
<th>sd (cm)</th>
<th>1σ RMS (cm)</th>
<th>2σ RMS (cm)</th>
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</thead>
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<tr>
<td>Michigan</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>SAAST</td>
<td>-0.9</td>
<td>15.9</td>
<td>19.5</td>
<td>39.0</td>
</tr>
<tr>
<td>NOAA</td>
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<td>12.8</td>
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<td>28.4</td>
</tr>
<tr>
<td>SAAST-NOAA</td>
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<td>3.0</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SAAST</td>
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<td>19.7</td>
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<td>71.2</td>
</tr>
<tr>
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<td>SAAST-NOAA</td>
<td>14.1</td>
<td>4.4</td>
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</tr>
<tr>
<td>South East</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SAAST</td>
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<td>SAAST-NOAA</td>
<td>6.5</td>
<td>-8.0</td>
<td>4.3</td>
<td>8.6</td>
</tr>
</tbody>
</table>

Table III-5: Average position height bias, standard deviation and RMS for each study area. Improvements are in bold.

For the data sets used in the positioning analysis it is quite apparent that the NOAA troposphere corrections improve long baseline, float solution positioning. All areas showed a general improvement in results, especially in the vertical. In some cases the vertical RMS improvement was as much as 50 cm. However, all areas also showed a
few situations where the application of the NOAA troposphere model actually made the
results worse. For the California area, increased RMS from the NOAA troposphere
model (less accurate result) resulted from an increase in the bias uncertainty. For the
South East area, increased RMS from the NOAA troposphere model appeared to
correspond to an increase in the standard deviation. For areas of high vertical variation,
such as California, application of the NOAA troposphere model had the greatest effect on
the vertical position bias (see Table III-5). Whereas in areas of rapid weather pattern
changes, such as the South East, application of the NOAA troposphere model resulted in
higher standard deviation. Due in part to the results obtained from this study, NOAA re­
evaluated and enhanced the modeling for the South East region [Gutman, 2006].

The results shown here are similar to those quoted by Alves et al. [2004], where
NOAA tropospheric data were incorporated into single and multi-baseline processing. A
significantly larger data set with longer baselines has been presented in the current work;
however, more processing is required to better quantify the improvements.

The purpose of this study was to evaluate the use of the NOAA troposphere
correctors in the Nationwide DGPS service. This chapter focused on the maritime user,
however; the stations used were all on land. The regions and CORS stations were
selected to simulate, as closely as possible, a maritime environment, and in most cases,
some of the satellites in use would have been over a body of water. Therefore, the results
shown here were indicative of what would be expected in a maritime environment. Any
further position domain study should include the analysis of dynamic data from the
marine environment (e.g. a data buoy).
CHAPTER IV
ANALYSIS AND EVALUATION OF VARIOUS IONOSPHERIC MODELS

There exist several models and methods for mitigating the effects of the ionosphere on GPS signals. Ionosphere uncertainties can be almost entirely removed by combining dual-frequency observations into an ionosphere-free measurement. The disadvantages of this method are that the integer nature of the ambiguity is removed, and the resulting observation noise is increased. For high precision, integer fixed solutions; the effects of the ionosphere must be removed by models, rather than by the combination of observations. More information on the observation equations and algorithms can be found in Appendix C.

The research presented here compares several ionosphere models, with the main emphasis being on NOAA’s USTEC (US Total Electron Content) model, for medium to long baselines. The closed-form troposphere model (Saastamoinen) was used in all testing and no attempt was made to fix ambiguities. The same regions, baselines and stations used in the previous troposphere study were also used here, with the addition of four stations from a central region.

This chapter presents the results of a study conducted to evaluate several different ionosphere uncertainty mitigation models and methods. It describes how satellite coverage, as a function of ionosphere corrector availability, plays a significant role in position uncertainty. This chapter begins with an overview of the ionosphere and how it interacts with the GPS signal. The following two sections discuss the models and data used in the study. Section four describes some of the coverage issues that arose during the course of this study and section five looks at the analysis methods used. Sections six and seven present the results of the range and position domain evaluation. The last section contains the conclusions and recommendations.
The results presented here were also delivered to the US Coastguard in a 2006 report [Dodd & Bisnath, 2006c]. They were also presented at the 2006 Institute of Navigation National Technical Meeting, in Monterey [Dodd & Bisnath, 2006b].

IV.1 Ionosphere Modeling in GPS

There are several ionosphere models available which may aid in facilitating the extension of baseline lengths for ambiguity-fixed position solutions. Some of these models, such as JPL GIM and WAAS models, use a grid of zenith delay estimates that can be mapped to a slant delay. These models assume an infinitesimal thin 2-dimensional ionosphere layer that is fixed at one height above mean sea level. The intersection of the GPS signal and the ionosphere layer, known as the “pierce point”, must be computed in order to interpolate the appropriate zenith delay estimate, and that value is then mapped to a slant delay estimate using the obliquity factor (See Figure IV-1). The obliquity factor is a function of the satellite elevation with respect to the ionosphere layer.

The units used to describe the ionospheric effect are Total Electron Count Units (TECU). These units can be converted to carrier cycles (L1 and L2) or meters through [e.g., Smith, 2004]:

![Figure IV-1: Ionosphere layer and Signal Pierce Points.](image-url)
Nominally, slant range correctors vary from 1 to 100 TECU (~0.16 to ~16 m) [Leick, 2004]; however, in this study, correctors went as high as 160 TECU (~26 m).

In order to achieve high precision ambiguity-fixed solutions, it is necessary to reduce the combined GPS uncertainties to about 1 wavelength (of the decimeter level). As a result, the uncertainty contribution from the ionosphere correctors should be no more than a few decimeters or ~1 TECU.

### IV.2 Description of Models Investigated

The USTEC model was the primary subject of the investigation. The NOAA-NGS ICON model was discontinued [Smith, 2005], but both zenith and slant delay values were available for the time period of this study. Several other models were added to portions of this study in order to validate the testing procedures and software. Not all models were used in all of the tests. The following subsections describe each model used, and their utility to this study. The following models or methods were assessed:

<table>
<thead>
<tr>
<th>Model/Method</th>
<th>Process</th>
<th>External Information for Processing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ionosphere-free</td>
<td>Real-time</td>
<td>None</td>
</tr>
<tr>
<td>No Ionosphere</td>
<td>Real-time</td>
<td>None</td>
</tr>
<tr>
<td>Klobuchar</td>
<td>Real-time</td>
<td>8 parameters in navigation message</td>
</tr>
<tr>
<td>NOAA ICON</td>
<td>Post-processed</td>
<td>Station corrector file</td>
</tr>
<tr>
<td>JPL GIM</td>
<td>Real-time</td>
<td>Zenith corrector grid map</td>
</tr>
<tr>
<td>WAAS</td>
<td>Real-time</td>
<td>Zenith corrector grid maps</td>
</tr>
<tr>
<td>USTEC Zenith Map</td>
<td>Real-time</td>
<td>Zenith corrector grid maps</td>
</tr>
<tr>
<td>USTEC Slant Map</td>
<td>Real-time</td>
<td>Slant corrector grid maps</td>
</tr>
<tr>
<td>MAGIC (Doug i)</td>
<td>Post-processed</td>
<td>Modified RINEX file</td>
</tr>
<tr>
<td>Doug p</td>
<td>Post-processed</td>
<td>Modified RINEX file</td>
</tr>
</tbody>
</table>

The intention of this study was to investigate ionospheric mitigation options for the US Coast Guard National Differential GPS service (NDGPS), consequently only the real-time products were an option. The other models and methods were incorporated into...
the study to help with process evaluation. The real-time options were: ionosphere-free, no ionosphere, Klobuchar, WAAS, JPL GIM, USTEC S, and USTEC Z.

IV.2.1 Ionosphere-Free

The ionosphere-free solution did not use any input variables other than the observation data. The processing observables were the combined L1 and L2 carrier and the combined L1 and L2 code. This solution was used as the "true" position for comparison to the results of the other model solutions.

IV.2.2 Klobuchar

The Klobuchar model was outlined in the GPS Interface Control Document [Navtech, 1995] as a standard method for dealing with the ionosphere for single frequency users. This model accepts eight ionosphere parameters (four alpha and four beta) transmitted with the satellite ephemeris information, and combines them with the receiver-to-satellite azimuth and elevation, and the receiver's horizontal position, to produce a slant-range corrector for each in-view satellite. A complete description of the Klobuchar algorithm can be found in Appendix C.

IV.2.3 JPL GIM

The Jet Propulsion Lab (JPL) produces 2.5° (latitude) x 5° (longitude) resolution Global Ionosphere Maps (GIM) of zenith correctors for real-time use. Correctors are determined from GPS observations at IGS stations around the world assuming a two-dimensional shell height of 450 km. These maps are updated every two hours. More information on JPL GIM can be found at "http://iono.jpl.nasa.gov/gim.html."
IV.2.4 WAAS

The FAA has developed the Wide Area Augmentation Service (WAAS) for single frequency GPS, standalone aircraft applications. Satellite clock and orbit uncertainties, as well as troposphere and ionosphere correctors are transmitted via communication satellites. The 5°x5° resolution ionosphere zenith grid maps cover the continental US with a maximum extent of 15°N to 60°N and 55°W to 155°W, and a shell height of 350 km. The corrector messages are updated every 5 minutes. The WAAS ionosphere corrector maps used in this study were retrieved from The University of New Brunswick (UNB), where the messages were decoded, separated and archived. More information on the WAAS system can be found at “http://iono.jpl.nasa.gov/waas.html.”

IV.2.5 MAGIC (Doug i)

The MAGIC model was developed by NOAA/NGS (National Geodetic Survey), NOAA/SEC (Space Environment Center) and the Cooperative Institute for Research in Environmental Sciences (Cires). MAGIC processes use dual frequency GPS observations from CORS stations to create a three-dimensional model of TEC. From this model, the TEC along a line between any two points (satellite and receiver) can be traced. The models are developed from any number of CORS stations (usually hundreds) and are produced days after the data has been collected [Spencer, Robertson & Mader, 2004]. As a result, they are used in post processing only.

For this study, the MAGIC model correctors were used for validation purposes. The USTEC model is based on the same algorithms as the MAGIC model. The primary difference is that the USTEC model is derived from fewer CORS stations (only uses data available in real-time); therefore, results obtained from the MAGIC correctors would be as good or better than those from the USTEC correctors. Data correctors were obtained as modified RINEX files from NOAA/SEC (produced by Doug Robertson specifically for this study). In these files, an ionosphere slant corrector observation was included for...
each satellite. Modified RINEX files were produced for each CORS station used in this study.

IV.2.6 USTEC

The US Total Electron Content (USTEC) model is a real-time version of NOAA’s MAGIC model (see subsection IV.2.5). It uses the same algorithms, just fewer CORS stations as only real-time CORS sites are used. Data files are produced every 15 minutes and include a single zenith propagation delay map (based on a shell height of 350 km), and multiple slant-range maps. One slant-range map is produced for each satellite that can be seen from inside the grid. Each grid (slant and zenith) has a $1^\circ \times 1^\circ$ resolution and covers the continental US from $20^\circ$N to $50^\circ$N, and $60^\circ$W to $130^\circ$W. More information on the USTEC model can be found in USTEC [2004] and USTEC [2006].

For this study, both slant (USTEC S) and zenith (USTEC Z) were evaluated as real-time options. Data were obtained from the USTEC website “http://www.sec.noaa.gov/ustec/”.

IV.2.7 Doug Robertson NGS/SEC, Boulder (Doug p)

During the process of creating the MAGIC correctors for the stations to be used in this project, Doug Robertson developed a method to extrapolate satellite slant correctors for epochs where they were missing, usually during a satellite’s rise or set. As a by product of this process, he created a new set of correctors from the L1-L2 observations and the MAGIC values. The correctors were derived by shifting the L1-L2 observation time series to overlay the MAGIC corrector time series. This removed some of the high frequency smoothing inherent with the MAGIC model (see Figure IV-2). The L1-L2 observations and MAGIC correctors were averaged over a satellite arc. The difference between the two averages (bias) was removed from the L1-L2 observations to derive Doug p correctors [Robertson, 2005]. Because this method requires the full satellite arc before correctors are produced, and it is based on the MAGIC correctors, it is only valid
as a post-processing solution, and was used to help validate the real-time options. Correctors were delivered as modified RINEX files with the ionosphere corrector as a new observable, similar to MAGIC.

![Figure IV-2: L2-L1 and MAGIC combination to create “Doug p” correctors.](image)

IV.2.8 NOAA ICON TKS

NOAA ICON was a TEC estimation method developed by NOAA for creating ionosphere corrector maps purely from dual-frequency GPS observations [Smith, 2004]. The process involved determining TEC values for satellite-receiver pierce-point tracks for all CORS stations. The pierce-point locations were determined assuming a shell of 300 km. Unambiguous TEC was determined through the evaluation of multiple track cross-over (or near cross-over) points. The final track values (ICON TKS) for the CORS stations were stored on the NOAA FTP site: ftp://ftp.ngs.noaa.gov/cors/ionosphere/icon/. These TEC values were determined after a delay of several days, so they were not evaluated as a real-time option. However, they were used for range comparisons and evaluation of the real-time options.
IV.3 Test Data

The following sections describe the CORS sites, baselines and time period used in the testing process. One concern with this type of testing is the incestuous nature of the information being tested. In some cases the GPS station used to evaluate a particular model was also used to produce that model. Using incestuous data can help to validate testing software and processes. It can also help to evaluate the level of smoothing that goes into the creation of a particular ionosphere corrector model.

Three stations in Michigan (KEW1, SAG1 and DET1), were selected to help in the validation of the software and testing processes. Tests included satellite availability within the models and model smoothing. These stations were used for algorithm validation only, and were not included in the ionosphere model studies conducted for this evaluation.

In the “Michigan”, “California” and “South East” regions none of the CORS stations used for the USTEC ionosphere corrector evaluation were used in the building of the USTEC corrector models. In the “Central” region three of the four stations used in this analysis were used in the development of the USTEC model. All of the stations used in this study were also used in the production of the MAGIC ionosphere correctors.

IV.3.1 Time series

A one-week period from January 16 through January 22, 2005 (DOY 016 through DOY 022) was selected for this study. This week contained several space weather events including a G2 (moderate) geomagnetic storm on January 18 (DOY 018) and a G3 (strong) geomagnetic storm on January 21 (DOY 021) (see Figure IV-3). Figure IV-4 depicts a relatively calm ionosphere activity map for January 20 (DOY 020), 2005 at 20:30 UTC (14:30 local time). Figure IV-5 depicts the same time only 24 hours later. The effects of the solar event are very evident in the latter figure, where values of up to 60 TECU are seen over the central USA.
Figure IV-3: Space Weather Alert Timeline for last two weeks of January 2005 (from NOAA/SEC [2005]).

Figure IV-4: TEC for January 20 (DOY 020), 2005 20:30 UTC (from NOAA/NGS [2005]).
IV.3.2 CORS Stations

The continental US was divided into four regions for this study: Michigan, California, Central and the South East. A total of 26 stations were combined to form 33 baseline pairs. One baseline was comprised of 24 hours of observations between two stations. Figure IV-6 is a map showing the locations of the four different regions.

Figure IV-6: Map depicting the four regions used for position domain analysis, Michigan, California, Central and the South East.
IV.3.2.1 Michigan Region

Seven CORS sites, all equipped with Leica RS500 GPS receivers, were used for the Michigan region analysis (see Figure IV-7). Various combinations of these seven sites produced ten baseline pairs with lengths varying from 190 km to 620 km (see Table IV-1). None of these stations were used in the USTEC model development. Local stations that were used in the USTEC model development are shown in Figure IV-7 (mauve colored).

![Figure IV-7: Baselines and CORS stations for Michigan region.](image)

<table>
<thead>
<tr>
<th>Number</th>
<th>Baseline</th>
<th>Length (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>UNIV-FRTG</td>
<td>190</td>
</tr>
<tr>
<td>2</td>
<td>SUP2-NOR2</td>
<td>210</td>
</tr>
<tr>
<td>3</td>
<td>SOWR-FRTG</td>
<td>270</td>
</tr>
<tr>
<td>4</td>
<td>NOR2-FRTG</td>
<td>280</td>
</tr>
<tr>
<td>5</td>
<td>SUP1-NOR2</td>
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</tr>
<tr>
<td>6</td>
<td>SUP2-SOWR</td>
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</tr>
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<td>7</td>
<td>SUP3-FRTG</td>
<td>430</td>
</tr>
<tr>
<td>8</td>
<td>SUP2-FRTG</td>
<td>480</td>
</tr>
<tr>
<td>9</td>
<td>SOWR-SUP1</td>
<td>560</td>
</tr>
<tr>
<td>10</td>
<td>FRTG-SUP1</td>
<td>620</td>
</tr>
</tbody>
</table>

Table IV-1: Baselines and corresponding lengths for Michigan region.

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IV.3.2.2 California Region

Six CORS sites, all equipped with Ashtech Z-XII3 GPS receivers, were used for the California region analysis (see Figure IV-8). Various combinations of these six sites produced nine baseline pairs with lengths varying from 140 km to 740 km. There were also significant height differences between some baseline pairs with variations from 235 m to 1850 m (see Table IV-2). None of these stations were used in the USTEC model development. Local stations that were used in the USTEC model development are shown in Figure IV-8 (mauve colored).

![Figure IV-8: Baselines and CORS stations for California region.](image)

<table>
<thead>
<tr>
<th>Number</th>
<th>Baseline</th>
<th>Length (km)</th>
<th>ΔH (m)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1756</td>
</tr>
<tr>
<td>2</td>
<td>TORP-MONP</td>
<td>200</td>
<td>1847</td>
</tr>
<tr>
<td>3</td>
<td>CHAB-CARH</td>
<td>260</td>
<td>250</td>
</tr>
<tr>
<td>4</td>
<td>TORP-CARH</td>
<td>310</td>
<td>485</td>
</tr>
<tr>
<td>5</td>
<td>COSO-BLYT</td>
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<td>1541</td>
</tr>
<tr>
<td>6</td>
<td>COSO-CHAB</td>
<td>420</td>
<td>1225</td>
</tr>
<tr>
<td>7</td>
<td>MONP-CARH</td>
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<td>1362</td>
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<tr>
<td>8</td>
<td>TORP-CHAB</td>
<td>560</td>
<td>235</td>
</tr>
<tr>
<td>9</td>
<td>MONP-CHAB</td>
<td>740</td>
<td>1612</td>
</tr>
</tbody>
</table>

Table IV-2: Baselines and corresponding lengths for California region.
IV.3.2.3 South East Region

Seven CORS sites, all equipped with Ashtech Z-XII3 or UZ-12 GPS receivers, were used for the South East region analysis (see Figure IV-9). Various combinations of these seven sites produced eight baseline pairs with lengths varying from 190 km to 690 km (see Table IV-3). There were no significant height differences between baseline stations. None of these stations were used in the USTEC model development. Local stations that were used in the USTEC model development are shown in Figure IV-9 (mauve colored).

![Figure IV-9: Baselines and CORS stations for South East region.](image)

<table>
<thead>
<tr>
<th>Number</th>
<th>Baseline</th>
<th>Length (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>OKCB-RMND</td>
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<tr>
<td>3</td>
<td>SIHS-LMCN</td>
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</tr>
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<td>4</td>
<td>JXVL-OKCB</td>
<td>360</td>
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<tr>
<td>5</td>
<td>PRRY-OKCB</td>
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<tr>
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<td>580</td>
</tr>
<tr>
<td>9</td>
<td>PRRY-LMCN</td>
<td>690</td>
</tr>
</tbody>
</table>

Table IV-3: Baselines and corresponding lengths for South East region.
IV.3.2.4 Central Region

A central region was added to the evaluation to help in the coverage analysis (see section IV.4). Four CORS sites, all equipped with Ashtech Z-XII3 GPS receivers, were used for the Central region analysis (see Figure IV-10). Various combinations of these four sites produced six baseline pairs with lengths varying from 352 km to 614 km (see Table IV-4). KAN2, SAL2 and MEM1 were used in the USTEC model development.

![Figure IV-10: Baselines and CORS stations for Central region.](image)

<table>
<thead>
<tr>
<th>Number</th>
<th>Baseline</th>
<th>Length (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>MEM2-STL3</td>
<td>352</td>
</tr>
<tr>
<td>2</td>
<td>SAL1-MEM2</td>
<td>418</td>
</tr>
<tr>
<td>3</td>
<td>KAN1-SAL1</td>
<td>421</td>
</tr>
<tr>
<td>4</td>
<td>KAN1-STL3</td>
<td>492</td>
</tr>
<tr>
<td>5</td>
<td>STL3-SAL1</td>
<td>576</td>
</tr>
<tr>
<td>6</td>
<td>MEM2-KAN1</td>
<td>614</td>
</tr>
</tbody>
</table>

Table IV-4: Baselines and corresponding lengths for Michigan region.
IV.4 USTEC Correction Coverage and Model Smoothing

Satellite availability and ionosphere model smoothing had significant effects on the positioning results. When a corrector was not available for a particular satellite, that satellite was removed from the solution. This reduction in coverage weakens the position solution geometry. All of the real-time ionosphere models supply corrector information via mapping grids. The user interpolates either the satellite/receiver pierce point position, or the receiver position (for USTEC S) to extract the appropriate correction. As a result, the model grids are a smoothed representation of reality. The larger the grid resolution, the more the correctors are smoothed. This chapter discusses the coverage and smoothing issues as they pertain to the USTEC model.

IV.4.1 USTEC Coverage

The ionosphere-free, no ionosphere and Klobuchar model solutions utilize all valid observations. The JPL-GIM solution is global and it also uses all valid observations. The Doug p solution uses an extrapolation technique to expand the coverage of the MAGIC solution, therefore, for the most part; it also uses all valid observations. The WAAS, USTEC slant grid, USTEC zenith grid and MAGIC solutions are all constrained by corresponding grid limits. This constraint causes some lower elevation satellites to be removed from the solution. Figure IV-11 shows the pierce point locations for three CORS stations in relation to the USTEC grid limits. For this study, only satellites with valid correctors are used in the solution.

There were two aspects to the USTEC slant map satellite coverage issue. The first was the existence of a satellite slant map in a particular USTEC corrector file, and the next was the existence of valid data for a user position within a satellites slant map. To illustrate the coverage issues, a timeline of events for satellite vehicle (SV) 6 on January 16, 2005 is shown in Figure IV-12 and Table IV-5.
Figure IV-11: Pierce Point locations for SUP1, CHAB, and RMND using 350 km height, for January 18, 2005.

<table>
<thead>
<tr>
<th>Time</th>
<th>SV 6 Event</th>
<th>PP Latitude</th>
<th>Use in Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>08h 37m 30s</td>
<td>SV 6 arrives</td>
<td>Pierce Point = 42° N</td>
<td>Used by all models except USTEC S grid</td>
</tr>
<tr>
<td>09h 45m 00s</td>
<td>USTEC slant map</td>
<td></td>
<td>Used by all models</td>
</tr>
<tr>
<td>10h 29m 30s</td>
<td>USTEC slant map no</td>
<td></td>
<td>Not used in USTEC S solution</td>
</tr>
<tr>
<td></td>
<td>longer includes grid for</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>SV6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10h 30m 30s</td>
<td>SV6 Outside USTEC</td>
<td>Pierce Point = 50° N</td>
<td>Not used in USTEC Z solution</td>
</tr>
<tr>
<td></td>
<td>grid</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10h 54m 30s</td>
<td>SV6 sets</td>
<td></td>
<td>No longer used in other solutions (elev &lt; 10°)</td>
</tr>
</tbody>
</table>

Table IV-5: SV 6 timeline for January 16, 2005 as seen from SAG1.
Figure IV-12: Pierce point locations for SVs as seen from SAG1 for four hours on January 16, 2005.

The Table IV-5 timeline indicated that there was no USTEC slant map for SV 6 until it had been visible (elevation > 10°) for more than one hour. After the satellite left the USTEC grid, it was still visible to the other solutions for another 25 minutes. To illustrate the effect of this coverage problem the DET1-SAG1 (160km) baseline was run with all in-view SVs for each model (see Figure IV-13). It was then run again using only the satellites seen by USTEC S (see Figure IV-14) for all models. The insets in Figure IV-13 and Figure IV-14 show a blowup of the height uncertainty for a ~1/2 hour section. The height position wander resulting from fewer satellites is very evident. These results show a similar response from JPL GIM, WAAS, USTEC Z, Doug i (MAGiC) and USTEC S when all are using the same satellites. The responses from Doug p and ICON TKS are not as severe, but still evident. This would indicate that the wander is caused unmodeled ionosphere effects, exacerbated by fewer satellites. In this case, the ICON TKS and Doug p models handle the ionosphere effect better than the other models.
Figure IV-13: DET1-SAG1, January 16 2005 all SVs in view.

Figure IV-14: DET1-SAG1, January 16 2005, only SVs seen by USTEC S.
Table IV-6 displays 3D RMS (at 1σ) for day 016, for models; JPL-GIM, WAAS, USTEC S, USTEC Z, MAGIC (Doug i) and Doug p when using all satellites (ALL SV) and only those seen by the USTEC S interpolation (USTEC SV). Also shown are the average number of SVs used for each epoch (#SV) and the difference between the two RMS values.

<table>
<thead>
<tr>
<th>Ionosphere Model</th>
<th># SV</th>
<th>3d RMS (cm) All SV</th>
<th>3d RMS (cm) USTEC SV</th>
<th>Delta (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>JPL GIM</td>
<td>7.92</td>
<td>37</td>
<td>50</td>
<td>13</td>
</tr>
<tr>
<td>WAAS</td>
<td>7.80</td>
<td>40</td>
<td>51</td>
<td>11</td>
</tr>
<tr>
<td>USTEC S</td>
<td>7.08</td>
<td>44</td>
<td>44</td>
<td>00</td>
</tr>
<tr>
<td>USTEC Z</td>
<td>7.80</td>
<td>37</td>
<td>50</td>
<td>13</td>
</tr>
<tr>
<td>Doug I</td>
<td>7.88</td>
<td>33</td>
<td>46</td>
<td>13</td>
</tr>
<tr>
<td>Doug p</td>
<td>7.88</td>
<td>13</td>
<td>18</td>
<td>05</td>
</tr>
</tbody>
</table>

Table IV-6: Coverage evaluation results (in cm).

In order to alleviate some of the coverage issues with the USTEC S model, some extrapolation was implemented. Extrapolation was only performed where a slant map existed in the USTEC S file and one or more of the surrounding grid values for a given location were valid. As a direct result of this evaluation, NOAA updated the USTEC model [Fuller-Rowell, 2006a].

IV.4.2 Model Smoothing

Mapping grids used to represent ionospheric correctors were finite in both spatial and temporal resolution; consequently, correctors were smoothed in order to best represent the region within a grid cell, for a particular time period. The amount of spatial smoothing depended on the size of the grid cell (resolution) and the amount of temporal smoothing depended on the map update rate. The validity of a grid map deteriorated as the time between updates increases.
Figure IV-15 shows two plots depicting the Doug p, MAGIC and USTEC S ionosphere correctors for satellite 21 at CORS stations FRTG and SAG1. FRTG was used in the development of the MAGIC model correctors, but not the USTEC model correctors. SAG1 was used in the development of both MAGIC and USTEC models. In both plots Doug p had high frequency variations that were not present in MAGIC or USTEC. USTEC and MAGIC were closely correlated at SAG1, but less so at FRTG. This was a result of model smoothing.

The MAGIC correctors were produced for each epoch; therefore, temporal smoothing was not an issue. USTEC was updated every 15 minutes and the prominent inflection points indicated update times (e.g., 2.25 and 2.75 hrs). From the SAG1 plot (see Figure IV-15) it was evident that there was some temporal smoothing in the USTEC model between 2.75 and 3.5 hrs, which created the only significant deviation between USTEC and MAGIC for this location and time period. Spatial smoothing in both MAGIC and USTEC was evident in both the high and low frequency deviations from Doug p.

Figure IV-15: SAG1 and FRTG SV 21 correctors for 2 hours on January 18, 2005.
IV.5 Analysis Methodology

The following discussion describes the testing process adopted for this study. The first evaluation was made of ionosphere correctors at each test station (range domain). Comparisons were made between zenith correctors interpolated at each test station. Because of the difficulty in determining the "true" ionosphere delay value, zenith comparisons were made between the models without trying to determine which was the most correct. Slant range corrector values were also compared at each station. Tests for this study have shown that the best double difference position uncertainty came from using the Doug p model; therefore, it was used as a reference for slant and double difference slant evaluation.

The second evaluation was made of positions determined from using the various models (position domain). Three position determination modes were used; code point positioning, sequential least-squares single-frequency double differenced positioning and dual-frequency double differenced point positioning. Code point positioning was included to evaluate the effect of the various models in a non-differencing environment, which would indicate the effectiveness of the actual corrector values, as opposed to the differenced correctors. The following subsections describe the processes in more detail.

IV.5.1 Zenith Range Domain Methodology

The zenith range evaluation compared the zenith delay values between various models. Only models that contained zenith grid values (ICON, USTEC Z, WAAS and JPL GIM) were used for the comparison. The Klobuchar model was included in the comparison by simply applying the algorithm for each 30-second epoch assuming an elevation angle of 90°. The results for one week for station FRTG in Michigan are shown in Figure IV-16 and the summary statistics for one day are shown in Table IV-7. For a comparison between models the ICON zenith delay correctors were selected as a reference. The ICON correctors were used because the evaluations
were conducted at stations where actual correctors were measured, not modeled. The results of this comparison for station FRTG in Michigan are shown in Figure IV-17 and summary statistics for the entire week are shown in Table IV-8.

![Graph showing ionosphere correction over time for stations FRTG, Klobuchar, JPL GIM, WAAS, ICON, and USTEC.]

Figure IV-16: Zenith delay comparison for FRTG for week of January 16, 2005

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>St Dev</th>
<th>$\sigma$</th>
<th>Max</th>
<th>Min</th>
<th>RMS $\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Klobuchar</td>
<td>11.65</td>
<td>6.71</td>
<td>21.72</td>
<td>4.95</td>
<td>13.44</td>
<td></td>
</tr>
<tr>
<td>JPL GIM</td>
<td>7.72</td>
<td>6.27</td>
<td>17.77</td>
<td>1.54</td>
<td>9.94</td>
<td></td>
</tr>
<tr>
<td>WAAS</td>
<td>5.80</td>
<td>5.59</td>
<td>15.04</td>
<td>0.00</td>
<td>8.06</td>
<td></td>
</tr>
<tr>
<td>ICON</td>
<td>7.72</td>
<td>6.14</td>
<td>18.87</td>
<td>1.65</td>
<td>9.87</td>
<td></td>
</tr>
</tbody>
</table>

Table IV-7: Zenith delay comparison statistics for FRTG for January 16, 2005 (in m).
Figure IV-17: Zenith delay comparison for FRTG for week of January 16, 2005, differenced from ICON.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>St Dev</th>
<th>Max</th>
<th>Min</th>
<th>RMS 1σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>FRTG Klobuchar</td>
<td>10.68</td>
<td>4.69</td>
<td>26.41</td>
<td>0.05</td>
<td>11.67</td>
</tr>
<tr>
<td>JPL GIM</td>
<td>5.66</td>
<td>1.99</td>
<td>18.84</td>
<td>-0.22</td>
<td>6.00</td>
</tr>
<tr>
<td>WAAS</td>
<td>1.52</td>
<td>1.57</td>
<td>13.43</td>
<td>-3.15</td>
<td>2.18</td>
</tr>
<tr>
<td>ICON</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>USTEC Z</td>
<td>2.08</td>
<td>1.96</td>
<td>13.23</td>
<td>-5.58</td>
<td>2.86</td>
</tr>
</tbody>
</table>

Table IV-8: Zenith delay comparison statistics (re ICON) for FRTG for week of January 16, 2005 (in m).

IV.5.2 Slant Range Domain Methodology

The slant range comparison evaluated the slant range TEC values, determined for each satellite, between the various models. All models were first converted into one standard format in TECU. If the models were comprised of zenith grid values, such as JPL-GIM, WAAS and USTEC Zenith map, the pierce points and obliquity values were
calculated for each visible satellite for each station. The zenith grid maps were then interpolated spatially and temporally and the resulting grid values were mapped to the slant via the obliquity factor. The USTEC slant grid model was interpolated relative to the station position, for each visible satellite, and the results exported to the standard format. Models that had correctors specific to a particular station, such as ICON TKS, MAGIC, and Doug p values, were simply converted into the standard format. Figure IV-18 shows the slant range corrections for KAN1, Figure IV-19 and Table IV-9 show the slant range corrections for KAN1 differenced from Doug p. Figure IV-20 and Table IV-10 show the slant range comparison for all satellites, using Doug p as a reference, and differenced between KAN1 and STL3, for January 16, 2005. Doug p was used as a reference because it produced the best double-difference results. The displayed correction difference indicates the difference the processing software would “see” between the various models.

Figure IV-18: Slant range correctors for KAN1 DOY 16 (January 16), 2005.
The plots shown in Figure IV-18 indicate that, for the most part, the magnitude of the correctors for all models is similar, showing relatively low values (<30 TECU) between 2 and 14 hours UTC, and relatively high values (up to 70 TECU) for 14 to 24 hours. The period of low corrector values correspond to local night, and the high values correspond to local daylight hours. This is consistent with low ionosphere activity during the night and high activity during the day. The Klobuchar correctors are the most regular because they are derived entirely from mathematical processes, with a constant minimum of ~15 TECU. The WAAS and JPL GIM models show more variability than the Klobuchar model, but the greatest variability comes from the USTEC, MAGIC (Doug i) and Doug p models. The greater the variability in the correctors indicates less smoothing from the model, and corresponds to correctors that are more representative of reality.

Figure IV-19: Slant range correctors for KAN1 DOY 16 (January 16), 2005 differenced from Doug p.
Table IV-9: Slant range corrector differences for KAN1 DOY 16 (January 16), 2005 (re Doug p) in TECU.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>St Dev 1σ</th>
<th>Max</th>
<th>Min</th>
<th>RMS 1σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>KAN1</td>
<td>Klobuchar</td>
<td>-10.31</td>
<td>5.17</td>
<td>4.27</td>
<td>-35.54</td>
</tr>
<tr>
<td>JPL GIM</td>
<td>-7.20</td>
<td>2.22</td>
<td>-1.38</td>
<td>-18.87</td>
<td>7.53</td>
</tr>
<tr>
<td>WAAS</td>
<td>-0.22</td>
<td>1.40</td>
<td>4.33</td>
<td>-5.91</td>
<td>1.42</td>
</tr>
<tr>
<td>USTEC S</td>
<td>-0.64</td>
<td>1.83</td>
<td>8.65</td>
<td>-9.02</td>
<td>1.94</td>
</tr>
<tr>
<td>USTEC Z</td>
<td>-0.60</td>
<td>2.00</td>
<td>9.06</td>
<td>-8.67</td>
<td>2.08</td>
</tr>
<tr>
<td>Doug i</td>
<td>0.00</td>
<td>0.98</td>
<td>5.01</td>
<td>-8.10</td>
<td>0.98</td>
</tr>
<tr>
<td>Doug p</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

The comparisons displayed in Figure IV-20 and Table IV-9, show how well the models compare to Doug p, assuming the Doug p slant ranges are closest to reality. The plots indicate that the Klobuchar model is the furthest from Doug p and the slant correctors are larger by an average of ~10 TECU. The JPL GIM correctors are also larger than Doug p and have an average offset of ~7 TECU. The remaining models are much closer to Doug p, with the two USTEC models averaging at ~2 TECU RMS, WAAS at 1.4 TECU RMS and MAGIC at ~1 TECU RMS. Doug P was derived from the MAGIC model; therefore it is not surprising that it is the closest. It is a little surprising that the two USTEC models are slightly further from Doug P than the WAAS model correctors, considering that they are derived from the same algorithms as MAGIC, and consequently, should be very similar. However, these results indicate that the WAAS, USTEC, and MAGIC model correctors are all very close to Doug p, within ~2 TECU 1σ RMS (~4 TECU 2σ RMS).

The final slant range comparison, displayed in Figure IV-20 and Table IV-10 show the slant range differences between correctors at KAN1 and STL3, with respect to Doug p. In this relative comparison, all are within ~1.5 TECU RMS (~ 3 TECU 2σ RMS). Again, as expected, Klobuchar is the furthest away from Doug P and MAGIC (Doug i) is the closest.
The results of the slant range evaluation indicate that, if the Doug p correctors are the closest to the truth, and produce the best position results, then the model ranking (from best to worst) should be as follows:

Doug p, MAGIC (Doug i), USTEC S, USTEC Z, WAAS, JPL GIM, Klobuchar
IV.5.3 Position Domain Methodology

For the spatial domain analysis, UST_OTF GPS processing software was used to compute positions for every epoch. The broadcast ephemeris was used for all computations and the troposphere was corrected for using the Saastamoinen model. One master MATLAB™ script file was then used to run the positioning analysis. Any number of station pairs, days, processing modes and ionosphere models could be run from this one application. Results, plots and statistics were generated and stored in separate directories and files for final evaluation.

The code point positioning results are presented here in order to show the effects of biases between the different models. The example results shown in Figure IV-21 and Table IV-11 are for the CORS station KAN1, from the central region. The point-positioning algorithm is basic and does not include the carrier (except for code smoothing), and does not attempt any epoch-to-epoch filtering.

The single-frequency position results are presented to evaluate effects of the ionosphere models on single-frequency users. This processing mode uses only P1 (or CA) and L1 observations and the sequential least-squares method for position determination. The example results shown in Figure IV-22 and Table IV-12 are for KAN1 to STL3, which is a 500 km baseline in the central region. No attempt was made to fix the ambiguities.

Sequential Least-squares (SLS) dual-frequency (SLS DF) results provide for the bulk of the evaluation. The example results shown in Figure IV-23 and Table IV-13 are from the KAN1 to STL3 500 km baseline in the central region: For the dual-frequency processing mode, USM_OTF used the combined L1/L2 code and carrier observations in the ionosphere-free solution, and used the four observables (L1 code and carrier, and L2 code and carrier) for all of the other solutions. In all cases the software used Sequential Least-squares (SLS) to produce the final ambiguity float solution. No attempt was made to fix the ambiguities.
Figure IV-21: Point Positioning, KAN1 DOY 16 (January 16), 2005.

Table IV-11: Point Positioning, KAN1 DOY 16 (January 16), 2005.

<table>
<thead>
<tr>
<th></th>
<th>N-RMS (m)</th>
<th>E-RMS (m)</th>
<th>U-RMS (m)</th>
<th>1σ 3d RMS (m)</th>
<th>2σ 3d RMS (m)</th>
<th>NSV</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.76</td>
<td>1.20</td>
<td>4.23</td>
<td>4.74</td>
<td>13.27</td>
<td>8.02</td>
</tr>
<tr>
<td>3</td>
<td>1.71</td>
<td>1.08</td>
<td>2.61</td>
<td>3.30</td>
<td>9.24</td>
<td>8.02</td>
</tr>
<tr>
<td>4</td>
<td>1.50</td>
<td>1.16</td>
<td>2.35</td>
<td>3.02</td>
<td>8.46</td>
<td>8.02</td>
</tr>
<tr>
<td>5</td>
<td>1.56</td>
<td>1.19</td>
<td>2.25</td>
<td>2.99</td>
<td>8.37</td>
<td>8.02</td>
</tr>
<tr>
<td>7</td>
<td>1.55</td>
<td>1.16</td>
<td>2.30</td>
<td>3.00</td>
<td>8.40</td>
<td>8.01</td>
</tr>
<tr>
<td>8</td>
<td>1.60</td>
<td>1.16</td>
<td>2.31</td>
<td>3.04</td>
<td>8.51</td>
<td>8.02</td>
</tr>
<tr>
<td>10</td>
<td>1.51</td>
<td>1.15</td>
<td>2.32</td>
<td>3.00</td>
<td>8.40</td>
<td>8.02</td>
</tr>
<tr>
<td>11</td>
<td>1.47</td>
<td>1.17</td>
<td>2.28</td>
<td>2.95</td>
<td>8.26</td>
<td>8.02</td>
</tr>
</tbody>
</table>

The code point positioning results shown in Figure IV-21 and Table IV-11 indicate that the Doug p solution produced the best results, with a 3D 2σ RMS of 8.26 m. MAGIC, JPL GIM, WAAS and the two USTEC solutions are within ~ 10 cm.
(3D 2σ RMS) of each other (8.4 to 8.5 m), so are very similar. Klobuchar came in at 9.24 m (3D 2σ RMS); and, as expected, the no-ionosphere corrector solution produced the worst results at 13.3 m (3D 2σ RMS).

![Figure IV-22: SLS single frequency, KAN1-STL3 DOY 16 (January 16), 2005.](image)

<table>
<thead>
<tr>
<th>KAN1-STL3 (492 km), SLS SF, from Jan 16 2005, 24 hrs</th>
<th>N-RMS (cm)</th>
<th>E-RMS (cm)</th>
<th>U-RMS (cm)</th>
<th>1σ 3d RMS (cm)</th>
<th>2σ 3d RMS (cm)</th>
<th>NSV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Iono-free</td>
<td>7</td>
<td>12</td>
<td>16</td>
<td>21</td>
<td>59</td>
<td>7.70</td>
</tr>
<tr>
<td>2 No ionosphere</td>
<td>17</td>
<td>29</td>
<td>35</td>
<td>48</td>
<td>134</td>
<td>7.70</td>
</tr>
<tr>
<td>3 Klobuchar</td>
<td>15</td>
<td>18</td>
<td>34</td>
<td>41</td>
<td>115</td>
<td>7.70</td>
</tr>
<tr>
<td>4 JPL GIM</td>
<td>15</td>
<td>15</td>
<td>29</td>
<td>36</td>
<td>101</td>
<td>7.70</td>
</tr>
<tr>
<td>5 WAAS</td>
<td>12</td>
<td>17</td>
<td>28</td>
<td>35</td>
<td>98</td>
<td>7.70</td>
</tr>
<tr>
<td>7 USTEC S</td>
<td>12</td>
<td>19</td>
<td>28</td>
<td>36</td>
<td>101</td>
<td>7.70</td>
</tr>
<tr>
<td>8 USTEC Z</td>
<td>12</td>
<td>16</td>
<td>29</td>
<td>35</td>
<td>98</td>
<td>7.70</td>
</tr>
<tr>
<td>10 Doug i</td>
<td>12</td>
<td>16</td>
<td>25</td>
<td>32</td>
<td>90</td>
<td>7.70</td>
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<tr>
<td>11 Doug p</td>
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<td>7.70</td>
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</tbody>
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Table IV-12: SLS single frequency, KAN1-STL3 DOY 16 (January 16), 2005.

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The single-frequency results shown in Figure IV-23 and Table IV-13 indicates that, of the options available to a single-frequency user, the Doug p solution gives the best results (76 cm 3D 2σ RMS) for the 500 km baseline. The ionosphere-free solution is included in the results, but is not an option for the single frequency user. The remaining models are ordered as expected with MAGIC, USTEC, WAAS and JPL GIM grouped together within ~10 cm (3D RMS 2σ) of each other (90 to 100 cm). Klobuchar faired the worst of the models at 115 cm. The no-ionosphere model produced the worst results at 134 cm.

The Dual-frequency results shown in Figure IV-23 and Table IV-13 show a similar sequence to the single-frequency, although they are slightly worse. The ionosphere-free solution is significantly better at 59 cm (3D 2σ RMS), than the others which range from 78 cm for Doug p to 168 cm for no-ionosphere.

![Figure IV-23: SLS Dual Frequency, KAN1-STL3 DOY 16 (January 16), 2005.](image)
Table IV-13: SLS Dual Frequency, KAN1-STL3 DOY 16 (January 16), 2005 (in cm).

<table>
<thead>
<tr>
<th></th>
<th>N-RMS (m)</th>
<th>E-RMS (m)</th>
<th>U-RMS (m)</th>
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<td>JPL GIM</td>
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IV.6 Range Domain Evaluation

The range domain analysis was divided into two sections; zenith delay and slant delay. For the zenith delay, the evaluation looked at one zenith corrector value per epoch, for each of the 26 stations in this study. For the slant delay, the evaluation looked at the average slant corrector of all in-view satellites, for each epoch, at each station.

IV.6.1 Zenith Range Domain Evaluation

For the zenith delay evaluation the average daily zenith delay was compared between stations. Presented here are the results from two days, one with low ionosphere activity (DOY 016, January 16) and one with high ionosphere activity (DOY 021, January 21). The following plots show the average TECU values for each station. Along the x-axis of each plot, stations are grouped by region and are ordered south to north within each region. The regions are also ordered south to north. For the zenith delay plots shown in Figure IV-24 and Figure IV-25, the average is computed from one day of 30-second epoch time series, at each station, for each model. Each average is computed from 2880 correctors.

Figure IV-24 depicts the daily (24 hour) average for DOY 016 (January 16) (low activity). There is a general south to north trend of decreasing TECU values, as would be
expected because of the higher ionosphere activity towards the equator. The zenith values range from 18 to 4, with the ICON values encompassing the entire range (14 TECU). Klobuchar shows the least amount of variation, only ranging from 16 to 14. JPL GIM, WAAS, and USTEC show the same basic form, with the same approximate range between high and low of about 6 TECU. However, JPL GIM shows an offset from WAAS and USTEC of about 5 TECU.

Figure IV-25 shows the same information as Figure IV-24 except that it displays the evaluation for DOY 021 (January 21). Overall, the average zenith correctors range from ~20 TECU in the south to ~6 TECU in the north. Klobuchar shows a fairly consistent slope from ~20 to ~17 TECU. The primary differences between the two days are the magnitude of the corrections (~2 TECU higher) and the “bump” in the trend over the Central region. This bump, recorded by all models except Klobuchar, is evident in DOY 016 (January 21), but to a lesser degree. One reason for this anomaly is that the Central and California regions are relatively close in latitude. Another reason was that the ionosphere storm was centered over the Central region (see Figure IV-5).
Figure IV-25: Daily average zenith ionosphere corrector for DOY 021 (January 21), 2005, all regions.

Figure IV-26 depicts the average zenith correctors for all 24 stations combined and plotted against the DOY. Each average is computed from 2880 epochs * 26 station = 74880 correctors. All models, except Klobuchar, show similar trends. Klobuchar does not vary as much as the other models, and consistently has the highest average correction. JPL GIM is greater than WAAS, ICON and USTEC by ~4 TECU. The high activity of DOY 021 (January 21) is evident in all of the models, but to a lesser degree in Klobuchar.
The zenith delay evaluation indicates that WAAS and USTEC are in close agreement and that JPL GIM has the same form, but is offset from the other two by a bias of ~5 TECU. This bias may be caused by different satellite/receiver hardware offset estimations. This bias will affect the point positioning results, but for DGPS positioning the double differencing process will remove it.

IV.6.2 Slant Range Domain Evaluation

For the slat delay evaluations, averages were computed from all in-view satellites, for each 30-second epoch over 24 hours. If we assume an average of 8 in-view satellites per epoch then there were 8 satellites* 2880 epochs = 23040 correctors per station (26), per 24 hour period, per model (8).

Figure IV-27 shows the average slant corrector for all satellites, at each station, for DOY 016 (January 16), which is a quiet ionosphere day. The stations are sorted by latitude (south to north). Note that there is a general trend (as expected) for all models: higher latitude, smaller corrector. The average values range from 30 to 8 TECU.
Figure IV-28 has the same properties as Figure IV-27 except that it is for a high activity day DOY 021 (January 21). The average range is now between 35 and 11 TECU. The general trend, high to low for south to north, seen on DOY 016 (January 16) is still there, except for the rise in the Central region. This corresponds to the ionosphere storm shown in Figure IV-5, which is centered over the central US.

Figure IV-27: Daily average slant ionosphere corrector for DOY 016 (January 16), 2005, all regions.

On DOY 016 (January 16) all models follow the same general trend, with the Klobuchar model having the highest values followed by the JPL GIM model. All of the others are grouped together. On DOY 021 (January 21) all of the models follow the same general trend, except for Klobuchar, which has less of a slope than the others.
An average slant value for each day is shown in Figure IV-29. Each average is computed from all of the slant correctors from all of the stations for each day; 8 satellites * 2880 epochs * 26 stations = 599040 correctors. Averaged values for each model are plotted against the DOY. Note the large spike on DOY 021 (January 21), which corresponds to the high activity day. Again most of the models follow the same basic trend, except Klobuchar, which does not have the means of responding to the high activity. The figure also shows a fairly consistent bias between WAAS and JPL GIM. Again, this bias may be a function of inconsistencies between the calculations of satellite/receiver offsets between the methods. For point positioning this offset should translate into a bias between position solutions. However, with double differencing, this offset should not translate into a bias between the SLS dual frequency results.

Figure IV-30 shows the 1σ RMS slant corrections averaged over all satellites as seen from all stations and differenced from Doug p. The results from all four regions are very similar, with the USTEC S correction grouped closely with WAAS, JPL GIM, and...
USTEC Z. The highest values (furthest from Doug P) come from Klobuchar, followed by JPL GIM. The smallest values (closest to Doug p) come from MAGIC.

Figure IV-29: Daily average slant ionosphere corrector from all stations.
Figure IV-30: Average RMS Slant correction for all regions (re Doug p).

Figure IV-31 shows the 1σ double differenced RMS slant correction difference (re Doug p) averaged over all station pairs. There appears to be more variation between the regions than in the previous figures. This variation is most pronounced on DOY 21 (January 21), which is when the G3 geomagnetic event occurred. These plots represent the difference the process would see between the different methods and Doug p. Klobuchar is still the largest (furthest from Doug p) and MAGIC is still the smallest (closest to Doug p), but now JPL GIM is grouped with WAAS and USTEC. The bias seen in Figure IV-30 was removed by double differencing.
Figure IV-31: Average RMS double differenced slant correction for all regions (With respect to Doug p).

If the Doug p corrector is considered to be the reference or "truth", the results shown in Figure IV-31 indicate the uncertainty that would be seen by the positioning processor when using one of the other models. In all regions and all days, except DOY 021 (January 21), the MAGIC correctors are less than 0.2 meters from Doug p, which is less than one L1 wavelength, and would indicate that ambiguity resolution is possible. USTEC S falls near or below 0.2 in all regions except California, again excluding DOY 021 (January 21), indicating that it may be helpful in ambiguity resolution, during times of normal ionosphere activity.
IV.7  Position Domain Evaluation

Twenty-six stations were combined to create thirty-three station pairs, over seven days, which were processed using four different modes (code point positioning [PP], code double differencing [DD], sequential least-squares code and carrier, single-frequency [SLS SF] and sequential least-squares code and carrier, dual-frequency [SLS DF]) and nine different methods to deal with the ionosphere (though only eight for point positioning). All of these analyses totaled to about 7700 runs through the process, which produced a corresponding number of plots and statistics files. The challenge became how to present results with enough information to evaluate the ionosphere models, without overwhelming the reader with plots and tables and still present the findings in a comprehensive manor. The ultimate goal of this analysis was to compare the various models over a range of spatial, temporal and processing scenarios.

Position domain result variations between regions are affected by differing: ionosphere correctors, troposphere delays, baseline lengths, satellite constellations and receiver types. Therefore, it is difficult to evaluate the results between regions. The relative RMS position uncertainties within each region are the best indicator of model/method performance.

IV.7.1  Number of Satellites

The relative number of satellites used by each process is an indication of coverage. Coverage varied for each of the models depending on the size of the grid used and available information within the models. Figure IV-32 shows the average number of satellites in each region, for each day. The ionosphere free, no ionosphere, Klobuchar, JPL GIM always show the same number of satellites. WAAS is the same as the previously mentioned in all areas, except California, where it showed slightly fewer satellites due to grid limits. In the Central region, where there should not be any coverage issues, all models have the same number of satellites for the first two days, and then USTEC S has fewer for the next 5 days. In all other regions, USTEC S, USTEC Z,
MAGIC and Doug p show fewer satellites than other model solutions. Reduced coverage was due to the lack of correctors for satellites near the limits of the model grid. Any satellites outside the grid did not have correctors available for them, and as a result, were not used in the solution.

The difference in average number of SV is very small (~0.3 satellites) but the values are averaged for an entire day, over many baselines, and therefore a small number is significant.

Figure IV-32: Average number of satellites in each region.
IV.7.2 Code Point-Positioning

The code point positioning results are presented here in order to show the effects of biases between the different models. The point-positioning algorithm is very basic and does not include the carrier (except for code smoothing), and does not attempt any between epoch filtering. Figure IV-33 shows three plots for DOY 016 (January 16) (low activity day) for the Central region. The plots represent:

- 3D RMS versus station
- Average number of SVs versus station
- Slant range differenced from Doug p versus station

![Graph showing code point positioning RMS statistics for Central region, DOY 016 (January 16), 2005.]

As expected, no-ionosphere produced the highest RMS at each station, and Klobuchar produced the next highest. Klobuchar also had the highest deviation from the Doug p slant range. Figure IV-34 displays another view of Figure IV-33 with an increased vertical exaggeration. Most of the models' average 3D RMS parallel each other and are ranked (lowest to highest RMS) Doug p, WAAS, USTEC S, USTEC Z and JPL GIM. For all but one station, MAGIC has the second lowest RMS, for STL3 the
RMS is higher than JPL GIM. This corresponds to a drop in the average number of satellites. For the most part, ranking of the 3D RMS position uncertainty corresponds to the ranking of the slant range deviation from Doug p. The exception is again with STL3 where WAAS is further from Doug p than USTEC S. Figure IV-35 and Figure IV-36 are similar to the above figures, except they correspond to DOY 021 (January 21). The 3D RMS ranking for DOY 021 (January 21) (from low to high) is: Doug P, WAAS, MAGIC, JPL GIM, USTEC Z and then USTEC S. USTEC shows fewer satellites than all of the other models. STL3 is an exception again. In this case Doug p, shows a much higher 3D RMS than USTEC S. The low ranking of USTEC models is due to reduced satellite coverage. This problem is especially acute on DOY 021 during the geomagnetic event.

Figure IV-34: Code PP RMS statistics for Central region, DOY 016 (January 16), 2005 – Zoom.
Figure IV-35: Code PP RMS statistics for Central region, DOY 021 (January 21), 2005.

Figure IV-36: Code PP RMS statistics for Central region, DOY 021 (January 21), 2005 – Zoom.
Figure IV-37 shows the 3D $1\sigma$ RMS position uncertainty averaged over all stations plotted against the region. As expected the no-ionosphere and Klobuchar solutions have the highest RMS values. The Klobuchar solution is fairly consistent at around 4.3 meters. The no-ionosphere solution is also consistent, except in the Michigan region where the results improve to less than the Klobuchar results. The remaining models are clustered together, with the closest grouping in the Central region where coverage is not as significant a factor.

Figure IV-38 is identical to Figure IV-37 except that the vertical scale is larger. From this figure, it can be seen that the JPL GIM results are consistently higher than the WAAS results, which have the lowest average RMS. This verifies the previous claim that the offset between JPL GIM and WAAS slant values would translate into a point position offset.

Figure IV-37: Code PP average 3D RMS for all days and stations.
IV.7.3 Sequential Least-squares Single Frequency

The sequential least-squares single frequency processing mode utilized the CA (or P1 if available) and the L1 carrier. In the following plots the ionosphere free solution, as described in section IV.7.4, is plotted with the other models for consistency. Figure IV-39 shows a series of plots similar to the ones used in point positioning, with the following features:

- 3D RMS versus baseline length
- Average number of SVs versus baseline length
- Slant range double difference, with respect to Doug p, versus baseline length

The series of plots in Figure IV-39 contains the results for DOY 016 (January 16) in the Central region. The 3D RMS plot indicates that, overall, "no ionosphere" produces the worst results, followed by Klobuchar, and ionosphere-free and Doug p produce the best results. Figure IV-40 shows a larger vertical exaggeration version of Figure IV-39. In this series of plots it can be seen that USTEC S performs the best of the real-time models (USTEC S & Z, WAAS, and JPL GIM) for 3D RMS position uncertainty. It is also the closest to the Doug p DD slant range. For the most part, all of the models use the same number of satellites so the only differing factor is the corrector. Of the real-time
models, the USTEC S correctors are the closest to Doug p. Of the real-time models, JPL GlM correctors are the furthest from Doug p, and they also produce the highest 3D RMS values.

Figure IV-39: SLS SF combined RMS statistics for Central region, DOY 016 (January 16), 2005.

Figure IV-40: SLS SF combined RMS statistics for Central region, DOY 016 (January 16), 2005 – Zoom.
Figure IV-41 and Figure IV-42 show the results from DOY 021 (January 21). Again no-ionosphere and Klobuchar produce the highest 3D position RMS and ionosphere-free and Doug p produce the lowest. The average number of satellites used in the USTEC S solution is consistently lower than all of the other models. However, for the most part, the 3D RMS ranking of the real-time models corresponds to the slant range double difference RMS ranking. For DOY 021 (January 21) the overall ranking (low to high RMS) is WAAS, JPL GIM, USTEC S and USTEC Z.

![Figure IV-41: SLS SF combined RMS statistics for Central region, DOY 021 (January 21), 2005.](image)

Figure IV-43 depicts the average 3D position RMS for all baselines from the Central region plotted against the DOY. As expected, no-ino and Klobuchar produce the highest RMS and Doug p and ionosphere-free produce the lowest. Of the real-time options USTEC S and Z show the best results (and they are very similar) followed by WAAS and JPL GIM. This pattern is reversed for the ionosphere storm day of DOY 021 (January 21).
Figure IV-42: SLS SF combined RMS statistics for Central region, DOY 021 (January 21), 2005 – Z.

Figure IV-43: SLS SF combined 1σ RMS statistics for Central region, averaged over all baselines.

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Figure IV-44 shows the overall average 3D position RMS for each region. Considering only the four real-time options (USTEC S and Z, WAAS and JPL GIM), Table IV-14 ranks the RMS (low RMS to high RMS) for each region.

<table>
<thead>
<tr>
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</table>

Table IV-14: Single frequency average ranking with respect to the 3D position RMS (low to high) for each region

Figure IV-44: Single frequency average RMS statistics for all regions.

IV.7.4 Sequential Least-squares Dual Frequency

The double difference sequential least square processing mode utilized the CA (or P1), P2, L1 and L2 observations. An example of compiling the results from all baselines.
in one region, for one day and plotting against the baseline length is shown in Figure IV-45. This figure shows three plots for DOY 016 (January 16) (low activity day) for the Central region:

- 3D RMS versus baseline length
- Average number of SVs versus baseline length
- Slant range double difference versus baseline length.

Figure IV-45: Dual frequency average 3D, N SV and SR DD for Central region, DOY 016 (January 16).

The slant range double difference (re: Doug p) represents the difference the processing software sees between using each model and the Doug p solution. For this evaluation it is assumed that Doug p creates the most correct solution. The 3D RMS plot indicates that Ionosphere-free and Doug p have the most accurate results, followed by MAGIC, WAAS, JPL GIM, USTEC S, and USTEC Z. In general, the closer the DD SR is to Doug p the more accurate the solution, however, the uncertainty is also influenced by the average number of satellites.

Figure IV-46 is the same as Figure IV-45 but for DOY 021 (January 21) (high activity day). The 3D RMS plot indicates that Ionosphere-free and Doug p have the most accurate results, followed by MAGIC, WAAS, JPL GIM, USTEC S, and USTEC Z.
There does appear to be a slight increase in the uncertainty and DD slant RMS over baseline length.

Figure IV-46: Dual frequency average 3D, N SV and SR DD for Central region, DOY 021 (January 21).

Figure IV-47 shows the average 3D RMS over all baselines plotted against the DOY for the central area. DOY 018 (January 18) and DOY 021 (January 21) show RMS spikes in all but ionosphere-free and Doug p solutions. The spike on DOY 021 (January 21) is due to known high ionosphere activity and DOY 018 (January 18) is due to high RMS uncertainties in the middle of DOY 18 (January 18) for all baselines, and may be attributed to a G2 event that occurred on that day. Ionosphere-free and Doug p give best results followed by MAGIC, JPL GIM, WAAS, and both USTEC solutions, which are similar. Klobuchar and No-Ionosphere again give the least accurate results.
Looking at the averaged RMS position uncertainty for all baselines and days, plotted against the region, gives an overall indication of relative position uncertainty, as see in Figure IV-48. When compared to the average number of satellites (see Figure IV-49) and the average double difference uncertainty (see Figure IV-50), it becomes evident that the position uncertainty is correlated with the number of satellites used and the relative uncertainty of the slant corrector (with respect to Doug p). The MAGIC solutions are the closest in uncertainty to the Doug p solution (disregarding ionosphere free). The double differenced slant correctors are also the closest to Doug p.
Figure IV-48: DF average RMS over all baselines and days, for each region.

As seen in the previous examples, ionosphere-free and Doug p give the best results, with MAGIC coming next. For the most part Klobuchar and no-ionosphere produce the worst results. The others are grouped in the middle, with USTEC S usually the worst of the middle cluster. However, the South East region is an exception. USTEC S is worse than Klobuchar. This is due to poor coverage, which is not seen in the average number of satellites (see Figure IV-49) because on many occasions the number of satellites available from USTEC S goes to four, at which time the processing software reverts to code differential, and uses all satellites seen by the base and remote receivers. This occurs for the USTEC S solution only.
Figure IV-49: Average Number of Satellites for all days and regions.

Figure IV-50: Slant double difference averaged over all baselines and days, for all Regions.
Table IV-15 and Table IV-17 list the north, east and up 1σ RMS values for DOY 016 and DOY 021 (January 16 and 21 respectively) for all baselines and regions combined (sorted from low to high). Table IV-16 and Table IV-18 contain the 2σ RMS for days 016 and 021 respectively. The results are consistent with other views of the data for DOY 016 (January 16), but USTEC S and Klobuchar switch on the high activity day of DOY 021 (January 21), again this can be attributed to coverage.

<table>
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Table IV-15: SLS DF 1σ RMS all Regions DOY 016 (January 16) (in cm).

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Table IV-16: SLS DF 2σ RMS all Regions DOY 016 (January 16) (in cm).

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Table IV-17: SLS DF 1σ RMS all Regions DOY 021 (January 21) (in cm).

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<td>U</td>
<td>68</td>
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<td>138</td>
<td>154</td>
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<td>160</td>
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</table>

Table IV-18: SLS DF 2σ RMS all Regions DOY 021 (January 21) (in cm).

The position domain analysis indicates that all of the models improve the positioning results. The questions as to whether or not any of the models are appropriate...
for long baseline ambiguity resolution remains unclear. The 3D RMS results from all models are at or above 0.3 meters, which is greater than one wavelength. The real-time options all have average 3D RMS values near 0.5 meters 1σ (1.4 m 2σ), which will make it difficult to use them in an ambiguity fixed solution, but using them will be better than using nothing. The poor results from the USTEC model can be attributed to coverage, and improvements in this area will make it more viable.

IV.8 Conclusions

Table IV-19 shows the north, east, and up mean RMS (1σ) (Table IV-20 shows 2σ), in meters, of the point positioning results for all stations, in all regions over all days, sorted from most to least accurate. Overall, WAAS performed the best, followed closely by Doug p. The least accurate solution was no ionosphere, followed closely by the Klobuchar results. It is interesting to note that, on occasion, the Klobuchar solution was worse than using no corrector. Grouped in the middle were (in order of uncertainty) USTEC Z, GIM, MAGIC and then USTEC S. The MAGIC solution is not suitable for real-time applications, but it gives an indication of what USTEC S can achieve. USTEC Z and JPL GIM both performed better than the USTEC S model; this was attributed to coverage issues, which were evident in all regions, but especially in the Michigan, California and South East regions. USTEC Z performed better than USTEC S, because the USTEC S suffered from fewer usable satellites due to missing slant maps.

<table>
<thead>
<tr>
<th></th>
<th>WAA</th>
<th>p</th>
<th>US Z</th>
<th>GIM</th>
<th>MAG</th>
<th>US S</th>
<th>Klob</th>
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<td>U</td>
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<td>2.90</td>
<td>2.92</td>
<td>4.05</td>
<td>4.32</td>
</tr>
</tbody>
</table>

Table IV-19: PP 1σ RMS all stations, all regions, all days (in meters).
Table IV-20: PP 2σ RMS all stations, all regions, all days (in meters).

Table IV-21 shows the north, east and up mean RMS (1σ) (Table IV-21 shows 2σ) of the sequential least-squares dual frequency position results for all baselines, in all regions over all days, sorted from most to least accurate. The Doug p solution performed the best (almost identical to the ionosphere-free results). However, the Doug p solution can only be used in post processing, and was used in this study for validation and comparison. The next best solution was MAGIC, but again it can only be used in post processing, but showed how well the USTEC solution should be able to do. Of the real-time options, JPL GIM produced the overall best results followed by WAAS, USTEC Z and the USTEC S. As with the point positioning results, using no correctors produced the worst results, with Klobuchar performing better. It is interesting to note that, with the exception of Doug p, the single frequency process performed better than the dual frequency. A possible reason for this is higher noise level associated with the P2 code observation. The Doug p results were identical for both methods.

Table IV-21: SLS DF 1σ RMS all stations, all regions, all days (in cm).

Table IV-22: SLS DF 2σ RMS all stations, all regions, all days (in cm).
The uncertainty estimation of the USTEC models was hampered greatly by lack of coverage. JPL GIM performed the best overall, and this was due to global coverage, and the solution was not constrained by loss of satellites due to missing correctors, whereas all of the other real-time options: (other than Klobuchar and no-ionosphere) USTEC S, USTEC Z and WAAS, were. The WAAS solution fared better than both USTEC solutions because of coverage, and USTEC Z fared better than USTEC S again because of coverage.

Of the four primary real-time solutions (JPL GIM, WAAS, USTEC Z and USTEC S), the best results were obtained from the solution with the best coverage (JPL GIM). However, in the Central region where coverage is less of an issue, the USTEC S and Z correctors performed well. As the number of reference stations used to produce the USTEC model increases, and its coverage comes closer to the MAGIC solution, it will be a more viable option.

This study incorporates land stations only. If we consider the maritime community, where vessels will be hundreds of kilometers from a base station, and much closer to grid boundaries, reduction in coverage, leading to reduced uncertainty, will become more pronounced than with the stations used in the present analysis. It has been shown here that it is better to produce a lower uncertainty estimate for an ionosphere corrector, and expand the grid, than to remove the satellite from the solution. The maritime community would be better served by NDGPS if ionosphere modelers developed a method to expand coverage, rather than having to deal with corrector-less satellites at the receiving end.

It should be noted that the data used for this study was collected during a week of relatively high ionosphere activity. Also, since 2005, more real-time stations have been added to the USTEC model. A more up-to-date analysis, with the latest model and a less challenging ionosphere was conducted for 0.
CHAPTER V
IMPLEMENTATION OF IONOSPHERE AND TROPOSPHERE MODELS FOR HIGH PRECISION GPS POSITIONING OF THE USM OCEANOGRAPHIC DATA BUOY DURING HURRICANE KATRINA

The studies in the previous two chapters looked at the use of NOAA's troposphere and ionosphere products to help mitigate the effects of the atmosphere on GPS signals. These studies looked at medium to long static baselines. This study looked at the effectiveness of combining the NOAA troposphere and ionosphere maps, for an integer ambiguity fixed solution, in a dynamic environment. The GPS data used were collected from an oceanographic observing buoy in the northern Gulf of Mexico, and its associated base stations, during hurricane Katrina. This study addresses the following questions:

1. Do the atmosphere models improve positioning uncertainty for 100 km baselines?
2. Is the improvement enough to enable ambiguity resolution?
3. How do positioning solutions for 100 km static baselines compare to dynamic baselines?
4. How do models and algorithms help positioning performance in high dynamics and extreme weather events?

This chapter summarizes the results from processing seven days of GPS data collected at three stations prior to and during Hurricane Katrina. Two of the stations were on land and the third was on an oceanographic buoy. The stations combined for two dynamic baselines (99 km and 22 km) and one static baseline (92 km). The weather during the week varied from extremely wet to dry. The vertical movement of the buoy ranged from very low (~0.1 m) to extremely high (~8m). The chapter is divided into six sections. The first section describes the oceanographic buoy and associated GPS base stations. The second section gives an overview of Hurricane Katrina with significant weather events, water levels and buoy movements. Sections three and four describe the data and methods used in the evaluation. Section five presents the results and section six gives the conclusions.
USM deployed an ocean-observing buoy in December 2004 in the Northern Gulf of Mexico as part of the Central Gulf of Mexico Ocean Observing System (CenGOOS, website http://www.cengoos.org/). It was located approximately 20 km south of the Barrier Islands at the 20 m isobath. The USM buoy was originally developed for research on extending the baselines for GPS RTK positioning in the marine environment, and subsequently has been developed for ocean observing as part of the Integrated Ocean Observing System (IOOS, website: http://www.ocean.us/what_is_ioos). In support of the GPS research; the buoy, with a seafloor package, carried a suite of instruments for characterizing the local meteorological parameters and the physical oceanography at the site (see Figure V-1). The measurements included atmospheric winds, temperature, barometric pressure and humidity, ocean temperature and conductivity at the surface and near the seafloor, vertical profiles of ocean currents, sea level height, and seafloor pressure. The effort involved collaboration with the Geochemical Research Group (GERG) at Texas A&M University. GERG has extensive experience in outfitting and maintaining ocean buoys, and in providing real-time data serving via the web through their Texas Automated Buoy System (TABS). The TABS program maintains buoys on the shelf of Texas and the western edge of Louisiana. The data from the USM mooring was served in near real-time on a USM web server at “www.cengoos.org” [Howden, Gihousen, Guinasso, Walpert and Sturgeon, 2006].
To support high-accuracy, long baseline research, base stations were established on Horn Island (HORN), in Gulfport (GBOB) and at the Stennis Space Center (STEN). Figure V-2 shows the baselines used for this experiment. All stations were equipped with NovAtel® OEM-4 dual frequency receivers and logged data to local hard drives at 1 Hz. STEN was located at USM’s main building on Stennis Space Center and data were stored locally. HORN was located on a concrete support for a pier on Horn Island, and stored data were retrieved directly from its computer every few months. Data from the buoy were retrieved through a wireless link every few months. Data from GBOB were downloaded automatically via the Internet on a weekly basis. GBOB was completely destroyed by the storm and all data for the study period were lost.
V.2 Hurricane Katrina

On August 29 (DOY 241) 2005, Hurricane Katrina made landfall along the Mississippi/Louisiana Gulf Coast. The eye of the storm followed the Pearl River (Mississippi-Louisiana border) and passed directly over the Stennis Space Center. It had sustained winds of 205 km/hr (125 mph) and the eye had a central pressure of 920 mbar. The surge accompanying the storm reached about 8 m. in some locations. Figure V-3 displays a MODIS (Moderate Resolution Imaging Spectroradiometer) image of cloud top temperatures of the hurricane when the eye passed over STEN.

Figure V-4 depicts an image of the zenith wet delay estimates derived from the NOAA troposphere model. Note that the direction of the baseline was parallel to the wet delay contours, and that for the two long baselines, the zenith wet delay values appeared to be very similar. Even though there was significant wet delay (up to 50 cm), these delays were similar for both stations and most of the delay should have been removed by double differencing. A visual inspection of all the hourly plots indicated that this situation occurred for the duration of the storm. The effect of the NOAA model would have been more pronounced for a baseline running perpendicular to the contours.
Figure V-3: MODIS image displaying cloud top temperatures for Hurricane Katrina at 1545 UTC on August 29 (DOY 241), 2005 (from CISSM [2006]).

Figure V-4: Zenith wet delay estimates from the NOAA troposphere model, at 1700 UTC on August 29 (DOY 241), 2005. Contour interval is 1 cm.
V.3 Data

Data from the storm (DOY 241) along with the six preceding days (235 through 240, or August 23 through 29) were used in this study. Dual frequency data were recorded at all sites (STEN, GBOB, HORN and BUOY) at 1 Hz. Due to the storm, all of the shore stations stopped recording before the end of August 29 (DOY 241). GBOB was completely destroyed by the storm; therefore no data were recovered for this study. HORN stopped recording at 0727 UTC when the batteries were washed away. The receiver, computer, antenna, and solar panels however, all survived. STEN stopped logging at 1720 UTC on DOY 241 (August 29) when the backup power supply for its logging computer failed. Stennis Space Center lost power at about 1200 UTC and did not regain it for 7 days. 100% of the power infrastructure for Hancock County, in which Stennis Space Center is located, was destroyed by the storm.

The buoy survived the storm and continued to record data until it was recovered two weeks later. The buoy, along with its four-ton anchor, moved 2.5 km to the northwest and then ~14 km to the southeast. Horizontal buoy movement started at ~1000 UTC and ended at about ~2200 UTC.

Figure V-5 shows the vertical movement of the buoy, relative to the WGS 84 ellipse, which was computed with GrafNav using the USM base station at Stennis. The height (in meters) is plotted against the date (in DOY). The plot shows that there is very little vertical movement during the first three days of the week. Buoy dynamics started to increase on DOY 238 (August 26), three days before the storm arrived, and reached a maximum vertical oscillation of about 8m (with a ~13 second period) at about 1400 UTC on DOY 241 (August 29). The buoy reached its maximum surge of about 3 m at 1400 UTC. Vertical buoy spectra was compared to accelerometer spectra to verify that the vertical buoy movement determined from GPS was true movement and not contaminated by processing or data collection [Bender, 2006].

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For some of the analysis presented in this chapter, DOY 236 (August 25) was used as example of a calm day, and DOY 241 (August 29) was used as an example of a rough, or high dynamics, day.

![Time series of vertical buoy positions.](image)

**Figure V-5:** Time series of vertical buoy positions.

### V.4 Evaluation Methodology

In kinematic positioning, it is difficult to distinguish between antenna movement and atmospheric effects [Leick, 2004]. This problem becomes much more acute as the effects of the atmosphere increase with increasing baseline length. To mitigate these effects, the baselines must be kept short (low satellite/receiver path decorrelation), or the effects of the atmosphere must be minimized. The USM GPS base station locations were selected to enable evaluation of short and long kinematic baselines and long static baselines. The short baseline removed the effects of the atmosphere, and the static baseline removed the effect of movement. Three baselines were created from the three
stations (see Figure V-2):

- HORN to BUOY (22 km kinematic)
- STEN to HORN (92 km static)
- STEN to BUOY (99 km kinematic)

The HORN to BUOY baseline was used as a reference for the long baseline results. It was also used to evaluate the effects of receiver dynamics with minimal atmospheric effects. The STEN to HORN baseline was used to evaluate the effects of the atmosphere and the use of the NOAA models, without the interference of antenna movement. Because STEN to HORN and STEN to BUOY baselines were geographically similar, atmospheric effects would also be similar. The STEN to BUOY baseline was used to evaluate the use of the NOAA models for a kinematic long baseline.

The USM_OTF GPS processing software developed for previous tests was used for this analysis. As in the previous studies, all processing was designed to simulate a real-time environment. Positions were computed for each epoch, using broadcast ephemeris, with a sequential least-squares algorithm for position and ambiguity smoothing. Position filter noise was set to a high level to ensure that any real antenna movement was not filtered out. Both ambiguity float and ambiguity resolved (AR), or fixed, solutions were computed. The AR algorithm incorporated the LAMBDA method [de Jonge & Tiberius, 1996]. See APPENDIX B for a more detailed treatment of LAMBDA.

The following processing modes were used in this study:

- Ionosphere-free float solution
- No-ionosphere float solution
- USTEC ionosphere model float solution
- No-ionosphere fixed solution
- USTEC ionosphere model fixed solution

The no-ionosphere solutions do not use any ionosphere mitigation except double differencing. They are included to show where ionosphere mitigation may or may not be necessary.
The USTEC methods included a modified real-time L2-L1 algorithm to introduce a high frequency ionosphere component into the USTEC corrector. This method combined cumulative averages of the USTEC slant correctors and the L2-L1 observations with the instantaneous L2-L1 observation to produce the modified USTEC value. As a result, at the beginning of a processing run, and whenever a new satellite arrives, the corrector was essentially the USTEC value. This method, known here as Doug p, is discussed in more detail in 0.

All methods were processed using both the Saastamoinen closed form troposphere model and the NOAA troposphere model. The NOAA model was interpolated using NOAA supplied software and with in-house interpolators. Two in-house interpolators were developed. The first simply interpolated wet and dry (pressure) values for each station. This method produced unsatisfactory results in the solutions, where in many cases, using the NOAA model produced worst results than the standard Saastamoinen model; consequently, a second interpolation method was developed. This second method interpolated zenith delays for each station, as well as for a troposphere pierce-point for each satellite. The pierce-point coordinates were computed assuming a troposphere height of 15 km. In the USM processing software, the troposphere corrector applied was a combination of the station zenith delay and the pierce point zenith delay. This pierce-point method produced the best results, and was used for all NOAA troposphere processing.

The commercial GPS processing package GrafNav™ version 7.5 was used to provide industry standard baseline positioning solutions. Final positions were derived from a combination of the forward and reverse processes. For the longer baselines (92 km and 99 km) GrafNav™ used an ionosphere-free float solution. All USM software runs were processed in the forward direction only.

Each of the seven days of data used in this study was processed individually. Although all 24 hours (8 for, 241 [August 29]) were processed, only the first 8 hours were used for illustrations and computation of performance statistics. The first hour of
each day was left out of the statistics calculations to very conservatively account for filter convergence.

V.5 Results

The following subsections discuss the results of the static baseline, short and long kinematic baselines on a calm day (DOY 236, August 25) and short and long kinematic baselines on a rough day (DOY 241, August 29).

V.5.1 Static Baseline

The static 92 km baseline between STEN and HORN was used to evaluate atmospheric effects, similar to those seen by the long kinematic baseline STEN to BUOY, without the influence of antenna motion. Figure V-6 and Figure V-7 show the north, east and up uncertainties for the first eight hours of DOY 236 (August 25) and DOY 241 (August 29) respectively, using the Saastamoinen troposphere model. Table V-1 and Table V-2 show the corresponding up bias and RMS statistics.
These tables and figures indicate that using no ionosphere correctors (“no ionosphere float” and “no ionosphere fixed”) for the 92 km baseline produced worse results than solutions using ionosphere mitigation (as expected). For both DOY 236 and DOY 241 (August 25 and 29 respectively) the “no ionosphere float” solution produced better results than the “no-ionosphere fixed” solution. This would indicate that the “no ionosphere float” ambiguities were not close enough to the correct ambiguities to enable reliable AR. For DOY 236 (August 25) the “USTEC fix” solution was comparable to the
"ionosphere free" solution, which would indicate that the float ambiguities derived with the modified USTEC correctors were good enough for AR. On DOY 241 (August 29) all of the solutions, other than the ionosphere free solutions, had problems at start up. This would indicate that there were residual ionosphere effects that were not adequately compensated for by the modified USTEC model. In the sequential least-squares algorithm, estimated float ambiguities became essentially fixed with time and if erroneous values were initially estimated, the filter required a significant time to recover. In the case of DOY 241 (August 29) (Figure V-7), it would appear that the initial USTEC values were not close enough to the actual ionosphere uncertainties, resulting in poor initial ambiguity estimates that caused the solution to drift.

Figure V-7: North, East and Up uncertainties for static baseline, DOY 241 (August 29).
Table V-2: Up bias and RMS for 92 km static baseline, DOY 241 (August 29).

<table>
<thead>
<tr>
<th></th>
<th>Up Bias (cm)</th>
<th>1σ Up RMS (cm)</th>
<th>2σ Up RMS (cm)</th>
</tr>
</thead>
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<td>Iono free</td>
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Figure V-8 and Figure V-9 show the up bias, up standard deviation and the up RMS, for the “ionosphere-free”, “USTEC float” and “USTEC fixed” solutions, plotted against the DOY for the 92 km static baseline. Figure V-8 shows the results from using the Saastamoinen model and Figure V-9 shows the results from using the NOAA troposphere model. The two figures are very similar, and only deviate from each other by a few centimeters, as see in Figure V-10, which shows the difference between the two figures (Saastamoinen minus NOAA). If the NOAA model improved the position solution, the resulting difference should produce a positive value. There does appear to be some small (few cm) bias improvement on days 236, 237 and 238. It was expected that the largest improvement would take place on the storm days of 240 and 241, but this does not appear to be the case. This result would indicate that the sensed troposphere delay was essentially the same for the satellites as seen from HORN and STEN. Also, there was no appreciable increase in the uncertainties between non-storm days and storm days. This would indicate that the NOAA troposphere model does not improve positioning capabilities over the 92 km baseline. It is known that there are problems with the troposphere model in the Southeast region of the US, and that NOAA is attempting improvements [Gutman, 2006].

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Figure V-8: Bias, standard deviation and RMS for 92 km static baseline, Saastamoinen Troposphere model.

Figure V-9: Bias, standard deviation and RMS for 92 km static baseline, NOAA Troposphere model.
V.5.2 Calm Day (DOY 236, August 25)

Using the buoy vertical motion as a guide (see Figure V-5), two days were selected for close scrutiny; DOY 236 (August 25) as an example of a calm day and DOY 241 (August 29) as an example of a rough day. Figure V-11 and Table V-3 show the results of the 22 km kinematic baseline between HORN and BUOY on DOY 236 (August 25). These results are with respect to the GrafNav™ smoothed solution. The table indicates that the “no ionosphere fixed” and the “USTEC fixed” solutions are comparable to the GrafNav™ solution (within 10 cm 2σ RMS) and that all solutions are within 20 cm (2σ RMS) of the GrafNav™ solution. This would indicate that, over a 22 km baseline, with relatively low dynamics, the USM filtered solutions and the GrafNav™ smoothed solution are comparable.
Figure V-11: North, East and Up uncertainties for 22 km kinematic baseline, DOY 236 (August 25) (reference GrafNav™).

<table>
<thead>
<tr>
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<th>Up Bias (cm)</th>
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<th>2σ Up RMS (cm)</th>
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</tr>
<tr>
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<td>10</td>
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</tbody>
</table>

Table V-3: Up bias and RMS for 22 km kinematic baseline, DOY 236 (August 25) (reference GrafNav™).

Figure V-12 and Table V-4 show the results of the 99 km dynamic baseline between STEN and BUOY on DOY 236 (August 25). These results are with respect to the GrafNav™ smoothed HORN to BUOY (22 km) baseline. The results in the figure and table are very similar to those of the 92 km static baseline, shown in Figure V-6 and Table V-1, for the same time period. The height deviation at about hour 6 is evident in both figures and the “ionosphere-free” and “USTEC fixed” 1σ RMS values are the same.
for both baselines. This would indicate that the "ionosphere-free" and "USTEC fixed" solutions are following the buoy dynamics correctly. As seen in the static baseline, the solutions that do not account for the ionosphere are less accurate than those that do.

<table>
<thead>
<tr>
<th></th>
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<th>1σ Up RMS (cm)</th>
<th>2σ Up RMS (cm)</th>
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<td>GrafNav</td>
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<td>26</td>
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</tbody>
</table>

Table V-4: Up bias and RMS for 99 km kinematic baseline, DOY 236 (August 25) (reference GrafNav™ HORN to BUOY).

Figure V-12: North, East and Up uncertainties for 99 km kinematic baseline, DOY 236 (August 25) (reference GrafNav™ HORN to BUOY).
V.5.3 Rough Day (DOY 241, August 29)

DOY 241 (August 29) provided for a very challenging GPS positioning environment. The zenith wet delay was high (up to 50 cm) and the pressure was low in the area of the storm (down to 920 mbar in the eye). The buoy moved vertically between 5 and 8 meters every 13 seconds, which created a very demanding signal reception and lock environment. Figure V-13 and Table V-5 show the results of the 22 km dynamic baseline between HORN and BUOY on DOY 241 (August 29). These results were with respect to the USM ionosphere-free solution. The RMS statistic for the “no ionosphere float” and the “USTEC float” solutions are the same for the rough day and the calm day. However, all of the fixed solutions (“no iono fix”, “USTEC fix and GrafNav™) are higher. The greatest difference is with the “no iono fix” solution, which has a 2σ RMS of 42 cm on the rough day as opposed to 8 cm (2σ) on the calm day. This would indicate that the residual ionosphere, combined with the buoy dynamics, is affecting the solution’s ability to resolve the correct ambiguities.

![Figure V-13: North, East and Up uncertainties for 22 km kinematic baseline, DOY 241 (August 29) (reference USM ionosphere-free).](image-url)
Table V-5: Up bias and RMS for 22 km kinematic baseline, DOY 241 (August 29) (reference USM ionosphere-free).

<table>
<thead>
<tr>
<th></th>
<th>Up Bias (cm)</th>
<th>1σ Up RMS (cm)</th>
<th>2σ Up RMS (cm)</th>
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</tbody>
</table>

Figure V-14 and Table V-6 show the results of the 99 km dynamic baseline between STEN and BUOY on DOY 241 (August 29). These results are with respect to the USM ionosphere-free solution between HORN and BUOY (which represents the short baseline). The ionosphere-free solution performed very well with a 2σ RMS of 20 cm, considering the sea state, buoy motion and troposphere conditions. The other solutions did not fair as well. This may be due in part to the same ionosphere startup effects seen on the 92 km static baseline for the same time period, described earlier (see Figure V-7). Solution oscillations in the vertical, seen in the static baseline, are similar in form to the ones shown in the 99 km dynamic baseline. A combination of high-dynamics and a rapidly changing troposphere with varying pressure and moisture content, lead to float solutions with large uncertainties. With the float solutions so unstable, it was not possible for the ambiguities to be resolved correctly.

Table V-6: Up bias and RMS for 99 km kinematic baseline, DOY 241 (August 29) (reference USM ionosphere-free HORN to BUOY).

<table>
<thead>
<tr>
<th></th>
<th>Up Bias (cm)</th>
<th>1σ Up RMS (cm)</th>
<th>2σ Up RMS (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iono free</td>
<td>-3</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>no iono float</td>
<td>-2</td>
<td>29</td>
<td>58</td>
</tr>
<tr>
<td>USTEC float</td>
<td>0</td>
<td>18</td>
<td>36</td>
</tr>
<tr>
<td>no iono fix</td>
<td>-10</td>
<td>35</td>
<td>70</td>
</tr>
<tr>
<td>USTEC fix</td>
<td>1</td>
<td>22</td>
<td>44</td>
</tr>
<tr>
<td>GrafNav</td>
<td>-11</td>
<td>42</td>
<td>84</td>
</tr>
</tbody>
</table>
Figure V-14: North, East and Up uncertainties for 99 km kinematic baseline, DOY 241 (August 29) (reference USM ionosphere-free HORN to BUOY).

V.5.4 Summary Plots

Figure V-15 shows the up bias, up standard deviation and the up RMS, for all model solutions, plotted against the DOY for the 22 km dynamic baseline, HORN to BUOY. Table V-7 shows the average, without DOY 241 (August 29), for the up bias, standard deviation and RMS. The “no ionosphere fix” solution showed the lowest bias uncertainty for all but the first and last days, where it performed the worst. The “USTEC fix” solution was comparable on most days, and better on some. Overall, for the short kinematic baseline, the “USTEC fix” and “no ionosphere fix” solutions performed the best, with the lowest overall average 2σ RMS of 10 cm; although, all solution RMS statistics were within 4 cm of each other; therefore, their differences are not statistically significant.
Figure V-15: Bias, standard deviation and RMS for 22 km kinematic baseline, Saastamoinen (reference GrafNav™ HORN to BUOY).

<table>
<thead>
<tr>
<th></th>
<th>Up Bias</th>
<th>Up StDev</th>
<th>1σ Up RMS</th>
<th>2σ Up RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iono free</td>
<td>3</td>
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<td>7</td>
<td>14</td>
</tr>
<tr>
<td>no iono float</td>
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<td>14</td>
</tr>
<tr>
<td>USTEC float</td>
<td>3</td>
<td>4</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>no iono fix</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>USTEC fix</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

Table V-7: Average up bias, standard deviation and RMS (1σ and 2σ), in cm, for 22 km dynamic baseline (reference GrafNav™ HORN to BUOY).

Figure V-16 shows the up bias, up standard deviation and the up RMS, for all model solutions, plotted against the DOY for the 99 km dynamic baseline, STEN to BUOY. Table V-8 shows the average, without DOY 241 (August 29), for the up bias, standard deviation and RMS. As expected, the bias and standard deviation are larger for the long dynamic baseline than for the short one. The solutions that did not use any ionosphere mitigation performed the worst with average 2σ RMS values of over 40 cm.

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The ionosphere-free and USTEC fix solutions show the lowest average biases of 6 cm and lowest 2σ RMS values of 26 and 28 cm, respectively. If not for the standard deviation spike on DOY 238 (August 27), the RMS values would be much lower. This summary indicates that buoy dynamics combined with un-modeled atmospheric uncertainties, negatively affected the buoy positioning uncertainty. Even so, the filtered solutions using NOAA model data were slightly better than the commercial smoothed solutions.

Figure V-16: Bias, standard deviation and RMS differences for 99 km kinematic baseline, Saastamoinen (reference GrafNav™ HORN to BUOY).

<table>
<thead>
<tr>
<th></th>
<th>Up Bias (cm)</th>
<th>Up StDev (cm)</th>
<th>1σ Up RMS (cm)</th>
<th>2σ Up RMS (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iono free</td>
<td>6</td>
<td>10</td>
<td>13</td>
<td>26</td>
</tr>
<tr>
<td>no iono float</td>
<td>9</td>
<td>19</td>
<td>21</td>
<td>42</td>
</tr>
<tr>
<td>USTEC float</td>
<td>9</td>
<td>11</td>
<td>15</td>
<td>30</td>
</tr>
<tr>
<td>no iono fix</td>
<td>8</td>
<td>19</td>
<td>21</td>
<td>42</td>
</tr>
<tr>
<td>USTEC fix</td>
<td>6</td>
<td>12</td>
<td>14</td>
<td>28</td>
</tr>
<tr>
<td>GrafNav</td>
<td>9</td>
<td>11</td>
<td>15</td>
<td>30</td>
</tr>
</tbody>
</table>

Table V-8: Average up bias, standard deviation and RMS, in cm, for 99 km kinematic baseline (reference GrafNav™ HORN to BUOY).
V.6 Conclusions

The data and baselines used in this study were selected because of the buoy dynamics and troposphere conditions caused by Hurricane Katrina. Unfortunately, the NOAA troposphere model did not improve the solution over using the closed-form Saastamoinen model. Overall, the ionosphere-free solution performed the best with an overall average 1σ RMS of 13 cm (26 cm 2σ) for the 99 km dynamic baseline. This should improve with better mitigation of the troposphere.

The USTEC fixed solution came in a close second with an overall RMS of 14 cm (28 cm 2σ). In the ionosphere study described in CHAPTER IV, the modified ionosphere correctors using L1-L2 performed as well as the ionosphere-free solution. In those tests the L1-L2 modification could only be performed in post processing because the algorithm required the entire satellite flight time series of ionosphere correctors and L1-L2 observations. The application of the L1-L2 corrector used in this study employed a coarse cumulative average technique that was at its least accurate during process start-up when the initial ambiguity estimates were being formed. If this process could be improved to the point where the float solutions (USTEC float) are as good as the ionosphere-free solutions, then the USTEC fixed solution should be even better.

The no ionosphere solutions (both float and fixed) performed very well for the short 22 km baseline. However, they were the least accurate over the longer (92 and 99 km) static and dynamic baselines; as expected, since the ionosphere is highly correlated and hence double-differenced away at short distances (22 km), and less so at longer distances.

The buoy dynamics did have an effect on the solutions, especially on the very rough DOY 241 (August 29), and over the longer baseline. The effects were diminished for the solutions that used ionosphere mitigation, and were the least for the ionosphere-free solution.
CHAPTER VI
IMPLEMENTATION OF MODIFIED IONOSPHERE AND TROPOSPHERE MODELS FOR HIGH PRECISION LONG-RANGE GPS POSITIONING

The final evaluation component of this dissertation looks at ionosphere and troposphere mitigation using updated models and processing techniques. The previous tests looked at data from 2003, 2004 and 2005. Since 2005, NOAA has updated both the troposphere model and the USTEC ionosphere model (Gutman, 2006; Fuller-Rowell, 2006a). The USM DGPS data processing software has also been updated since the previous tests to include a Zenith Propagation Delay (ZPD) term and an algorithm to introduce high frequency ionosphere perturbations into the ionosphere models (known here as the Modified Doug p [MDP] algorithm). The ZPD term is described in more detail in Appendix C. The tests conducted for this evaluation were performed in a similar manner to the tests presented in previous chapters. Data from southern Louisiana were processed in ambiguity float and fixed modes, employing ionosphere mitigation techniques (ionosphere-free and USTEC model), using either the NOAA troposphere maps or the Saastamoinen models, with the ZPD and MPD options on and off. Southern Louisiana was chosen because of the availability of data and challenging troposphere and ionosphere weather patterns. The tests were performed to answer the following questions:

- Does the NOAA troposphere model improve the ionosphere-free and USTEC four-observable solutions?
- Can the USTEC four-observable float solution achieve the accuracies of the ionosphere-free solution?
- Do the ZPD term and the MDP algorithm help the USTEC float solution achieve the accuracies of the ionosphere-free solution?
- Can integer ambiguities be reliably resolved, over long baselines, using the USTEC float solution?

The first section of this chapter describes the data used, including the CORS stations, baselines, days and weather conditions. The second section discusses the MDP...
algorithm and its application. The third section describes the processing and evaluation methodology. The fourth section presents the results and the final section the conclusions.

VI.1 Data

Five Continuously Operating Reference Station (CORS) sites from southern Louisiana were selected for this study. Pairs of sites were combined to create six baselines ranging from 26 to 302 km (see Figure VI-1 and Table VI-1).

Figure VI-1: CORS sites.
Four days with varying troposphere activity were used, including June 29, July 5, 6 and 7, 2006 (DOY 180, and 186 through 188). Figure VI-2 and Figure VI-4 show typical wet delay maps for June 29 (DOY 180) and July 07 (DOY 188), respectively. Both of these days had high variability in the troposphere delay, whereas July 05 (DOY 186) (Figure VI-3) had a more homogeneous pattern. All of the processing algorithms used for this study employed differential GPS processing techniques; therefore, it was the relative difference in wet delay between stations that had the greatest effect on the results, rather than the absolute delay values. The troposphere map shown in Figure VI-2 depicts the wet delay for 0900 hrs on June 29 (DOY 180). This map clearly indicates a high gradient between the land and water, with very dry air on land (low zenith wet delay of < 10 cm) and very wet air over the water (high zenith wet delay of > 30 cm). As a result, stations inland experienced less signal refraction effect from the troposphere than stations nearer the coast. Baselines where both stations were near the coast experienced similar troposphere delays, as did stations where both were further inshore (SIHS-HAMM). However, baselines with one station inshore and one near the coast (LMCN-HAMM and LMCN-SIHS), experienced vastly different troposphere effects. Behavior of the ionosphere for this time period was relatively benign, with the usual daytime highs and nighttime lows.
Figure VI-2: Troposphere wet delay for June 29 (DOY 180), 2006.

Figure VI-3: Troposphere wet delay for July 05 (DOY 186), 2006.
VI.2 Modified Doug p Algorithm

The concept of the Doug p ionosphere correction enhancement was introduced in CHAPTER IV. The original post-processed method was developed to introduce high frequency ionosphere variations into the ionosphere uncertainty estimated from the NOAA MAGIC corrector. The process required that the average L1-L2 observation be differenced from the average model corrector, for a complete satellite pass. As such, this method was only valid as a post processing technique. The tests performed in CHAPTER IV showed that the results of using this method, with the four observables (L1, and L2 carrier and CA and P2 code), was comparable to the ionosphere free solution. Only float solutions were determined for these initial tests, but it was speculated that the method could greatly enhance the ability to perform reliable ambiguity fixing. For the Hurricane Katrina tests of CHAPTER V, a real-time version of the Doug p method was developed and utilized, but not thoroughly examined. It was
used simply because the results of both the fixed and float solutions were much better when using the method, due to increased fidelity of the corrections. Evaluation of the MDP algorithm was included as part of the study for this chapter and a description of its implementation follow here.

The MDP algorithm can be applied to any estimate of the ionosphere uncertainty. The algorithm computes a cumulative average of the difference between L1-L2 and the ionosphere corrector and removes it from the L1-L2 observation.

\[
I_i = \frac{(L_1 - L_2 - X_i)}{0.646944}
\]

\[
X_i = \frac{X_{i-1} * (i - 1) + [(L_1 - L_2) - IM_i * 0.646944]}{i}
\]

Where:

- \(I_i\) is the modified ionosphere uncertainty in meters (MDP)
- \(L_1\) is the L1 carrier observation in meters
- \(L_2\) is the L2 carrier observation in meters
- 0.646944 accounts for the difference between the L1 delay and the L1-L2 delay
- \(IM_i\) is the slant range ionosphere model corrector
- \(X_i\) is the cumulative average of the bias between L1-L2 and the modeled uncertainty
- \(i = 1 \ldots n\), where \(n\) is the number of epochs the satellite is in view

Note: This algorithm has been adapted from "phion.f" FORTRAN code developed by Doug Robertson, December, 2004.

The cumulative average bias \((X_i)\) is determined at each epoch from the previous average and the instantaneous bias for the current epoch. When a satellite rises, the MDP corrector closely resembles the model value. As the satellite sets, the MDP corrector closely resembles the Doug P corrector, and is exactly the same for the last epoch, when the cumulative average MDP bias is the same as in the post-processed Doug P process.

The advantage of the Doug P (and MDP) method is that it uses a locally observed ionosphere estimate. It has the disadvantage of using the dual frequency carrier
observations to determine the uncertainty, which introduces noise into the process similar to the ionosphere-free solution. It also increases the correlation between the L1 and L2 observation, which is not accounted for in the position estimation process. Including this correlation into the position estimation solution will be a topic of future research.

Figure VI-5 shows a plot of the Doug P, MAGIC and MDP (determined from MAGIC) slant ionosphere correctors for the duration of a single satellite arc. Figure VI-6 depicts the difference between MAGIC and Doug P, as well as the difference between MDP and Doug P. The MDP correctors follow the MAGIC correctors to begin with, then they tend towards the Doug P correctors near the middle of the flight and stay relatively close (± 2 cm) for the remainder of the transit. This would indicate that the MDP algorithm performs well in the last half of the flight, and is similar to the MAGIC model in the first half. The MDP corrector is the same as the Doug P corrector at the end of the flight, where the cumulative average bias is the same as the overall bias computed in the Doug P process.

![Figure VI-5: Doug P, MAGIC and MDP ionosphere correctors for SV 1.](image-url)
VI.3 Processing and Evaluation Methodology

One baseline was considered to be 24 hours in duration. The same six baselines were selected for each of the 24-hour periods (DOYs) used in this study. A single processing session included three runs through the software for one baseline from one day. The three runs were:

- Ionosphere-free (Iono Free).
- USTEC float (USTEC S), which kept L1 and L2 as separate observations and applied the USTEC ionosphere model in a floating ambiguity solution.
- USTEC fixed (USTEC SF), which used the USTEC float solution to attempt to resolve the integer ambiguities, which were then used in an ambiguity fixed solution.

The processing sessions for each baseline and day were repeated eight times with the following options:

- Troposphere option: Saastamoinen closed form (SAAST) or NOAA troposphere model
- Modified Doug p (MDP); on or off.
- Zenith Propagation Delay (ZPD); on or off
VI.4 Results

As part of the evaluation process it was assumed that the “optimum solutions” came from using the “optimum methods”, which were obtained when the NOAA troposphere model, the ZPD term and the MDP algorithm were utilized. For comparison with standard practices, the “conventional method” was considered to be solutions resulting from using the Saastamoinen troposphere model and the ZPD term, but with the MPD algorithm off. The “conventional solution” was obtained from using the ionosphere-free algorithm with the “conventional method”.

The “Results” section is divided into multiple subsections. The first and second subsections look at the conventional and optimum methods, respectively. These subsections show the results from using the conventional and optimum methods on three sample baselines from DOY 180 (June 29). The third subsection compares the height uncertainties from the conventional and optimum solutions, from all baselines for DOY 180 (June 29). The fourth subsection evaluates the effect of using the different troposphere models, the ZPD term and the MDP algorithm for DOY 180 (June 29), looking at the height uncertainty only. The fifth subsection evaluates the effect of using the troposphere models, the ZPD term and the MDP algorithm, by looking at the position uncertainties in all three dimensions, for all baselines, averaged over all of the days. The final subsection looks at the effectiveness of the ambiguity fixing process using the optimum method.

VI.4.1 Conventional Method

One method for determining real-time, high-accuracy positions over long ranges is to use the ionosphere-free solution with a closed-form troposphere model, such as Saastamoinen. This is referred to here as the “conventional solution”, and is used as a benchmark for comparison purposes. The “conventional method”, which includes the use of the Saastamoinen troposphere model and the ZPD delay term, has been applied to the USTEC four-observation algorithms as well as the ionosphere free algorithms in
order to show the relative improvement in the results. Also included in the following evaluation are results from using GrafNav™ 7.60 ionosphere-free float solution. The following plots and tables are the results obtained from conventional methods for three baselines (26 km, 62 km and 141 km) from DOY 180 (June 29). The convectional method and conventional solution are used in the evaluation to show the results that can be expected from using traditional processing techniques, as opposed to the results that can be achieved when using the models, terms and algorithms presented here.

The north, east and up uncertainty results for the short baseline (26 km) are depicted in Figure VI-7 and the statistics are shown in Table VI-2. These results indicate that the ionosphere-free, USTEC fixed and GrafNav™ solutions are all very close (within 1 cm RMS), and that the USTEC float solution is the worst, especially in the height (3 to 4 cm RMS). In all cases the bulk of the uncertainty RMS is due to the standard deviation.

![Figure VI-7: Conventional short baseline (26 km) north, east and up uncertainties.](image-url)
Table VI-2: Conventional short baseline (26 km) north, east, up (height) uncertainty bias, standard deviation (1σ), RMS (1σ) and 95% OS, for DOY 180 (June 29).

<table>
<thead>
<tr>
<th></th>
<th>Bias (cm)</th>
<th>StDev (cm)</th>
<th>1σ RMS (cm)</th>
<th>95% OS (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N  E  U</td>
<td>N  E  U</td>
<td>N  E  U</td>
<td>N  E  U</td>
</tr>
<tr>
<td>Iono-Free</td>
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<td>2  3  7</td>
<td>2  3  7</td>
<td>5  11  17</td>
</tr>
<tr>
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<td>3  5  9</td>
<td>7  10  18</td>
</tr>
<tr>
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<td>2  3  6</td>
<td>5  5  12</td>
</tr>
<tr>
<td>GrafNav</td>
<td>1  -1  2</td>
<td>2  3  7</td>
<td>2  3  7</td>
<td>4  6  14</td>
</tr>
</tbody>
</table>

The north, east and up uncertainty results for the medium baseline are depicted in Figure VI-8 and the statistics are shown in Table VI-3. These results indicate that the ionosphere-free solution gives the best results in all three dimensions. The USTEC float and fixed and the GrafNav solutions are within 2 cm RMS of each other in all dimensions. The uncertainty bias has increased from the shorter baseline; this is especially true with the ionosphere-free solution, where the bias is greater than the standard deviation. This may be an indication that the effects of the variable troposphere are having an influence.
Figure VI-8: Conventional medium baseline (62 km) north, east and up uncertainties.

Table VI-3: Conventional medium baseline (62 km) north, east and up (height) uncertainty bias, standard deviation (1 σ) and RMS (1 σ) in cm.

<table>
<thead>
<tr>
<th></th>
<th>Bias (cm)</th>
<th>StDev (cm)</th>
<th>1σ RMS (cm)</th>
<th>95% OS (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>N  E  U</td>
<td>N  E  U</td>
<td>N  E  U</td>
</tr>
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<td>4 3 10</td>
<td>11 6 18</td>
</tr>
<tr>
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<td>6 9 12</td>
<td>15 21 28</td>
</tr>
<tr>
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<td>7  7 11</td>
<td>7 8 12</td>
<td>22 20 29</td>
</tr>
<tr>
<td>GrafNav</td>
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<td>5  7 12</td>
<td>5 7 13</td>
<td>10 14 26</td>
</tr>
</tbody>
</table>

The north, east and up uncertainty results for the long baseline are depicted in Figure VI-9 and the statistics are shown in Table VI-4. These results indicate that the ionosphere-free solution gives the best results in the height, and GrafNav gives the best results in the north and east, but the USM ionosphere-free solution and GrafNav solution are within 2 cm of each other in all directions. The USTEC float and fixed height RMS
uncertainties are slightly higher than the GrafNav solution. The height bias increased considerably, which accounts for the majority of the uncertainty RMS. The most striking difference between the ionosphere-free solutions (USM and GrafNav) and the USTEC float and fixed solutions is the standard deviation. Residual ionosphere effects, not present in the ionosphere-free results, appear to be affecting the USTEC fixed and float solutions, which would indicate that the USTEC models do not remove all of the ionosphere uncertainties.

![Graph showing uncertainties comparison](image)

**Figure VI-9: Conventional long baseline (141 km) north, east and up uncertainties.**

<table>
<thead>
<tr>
<th></th>
<th>Bias (cm)</th>
<th>StDev (cm)</th>
<th>1σ RMS (cm)</th>
<th>95% OS (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N  E  U</td>
<td>N  E  U</td>
<td>N  E  U</td>
<td>N  E  U</td>
</tr>
<tr>
<td><strong>Iono-Free</strong></td>
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<td>5  3  6</td>
<td>5  4  28</td>
<td>10  8  46</td>
</tr>
<tr>
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<td>8  9  31</td>
<td>16  18  62</td>
</tr>
<tr>
<td><strong>USTEC Fix</strong></td>
<td>2   -9 -26</td>
<td>9  12 18</td>
<td>9  15 32</td>
<td>18  30 64</td>
</tr>
<tr>
<td><strong>GrafNav</strong></td>
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<td>3  3  7</td>
<td>3  3  30</td>
<td>6  6  60</td>
</tr>
</tbody>
</table>

**Table VI-4: Conventional long baseline (141 km) north east and up uncertainty bias, standard deviation (1σ) and RMS (1σ) in cm.**

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These results indicate that the USM conventional solution (ionosphere-free with Saastamoinen troposphere model) is comparable with the GrafNav solution and, as a result, can be used as a benchmark to indicate how much the solution can be improved by using the models and techniques presented here.

VI.4.2 Optimum Method

The basic assumption made here is that the optimum method for long-range, high-accuracy positioning is with the use of the NOAA troposphere model, with a ZPD term and using the MDP algorithm. It is also assumed that the optimum solution (ionosphere-free, USTEC float or USTEC fixed) will come from using the optimum method. The following subsection evaluates the same baselines used in the previous conventional method subsection.

The short (26 km) baseline results shown in Figure VI-10 and Table VI-5 indicate that there is very little uncertainty bias in any of the solutions, and the RMS uncertainty is almost entirely made of the standard deviation. The USTEC fixed solution gives the best overall results; however, all three solutions are within 2 cm of each other in all dimensions.
Figure VI-10: Optimum short baseline (26 km) north, east and up uncertainties.

Table VI-5: Optimum short baseline (26 km) north, east, up (height) uncertainty bias, standard deviation (1σ) and RMS (1σ) in cm.

<table>
<thead>
<tr>
<th></th>
<th>Bias (cm)</th>
<th>StDev (cm)</th>
<th>1σ RMS (cm)</th>
<th>95% OS (cm)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>N  E  U</td>
<td>N  E  U</td>
<td>N  E  U</td>
<td>N  E  U</td>
</tr>
<tr>
<td>Iono-Free</td>
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<td>2  3  7</td>
<td>2  3  7</td>
<td>4  7  15</td>
</tr>
<tr>
<td>USTEC Float</td>
<td>1  -2  -1</td>
<td>3  2  6</td>
<td>3  3  6</td>
<td>5  5  11</td>
</tr>
<tr>
<td>USTEC Fix</td>
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<td>2  2  5</td>
<td>2  2  5</td>
<td>4  3  9</td>
</tr>
</tbody>
</table>

The medium baseline (62 km) results, shown in Figure VI-11 and Table VI-6, indicate that the height bias is still very low (at the 2 cm level) and the east bias is starting to increase (to the 3 cm level). However, the RMS values are still under 10 cm in all dimensions. The ionosphere-free solution gives the best results, but the USTEC float solution is only very slightly worse. The USTEC fixed solution gives similar results in the horizontal, but the height RMS is 3 cm larger due to a higher standard deviation.
Figure VI-11: Optimum medium baseline (62 km) north, east and up uncertainties.

<table>
<thead>
<tr>
<th></th>
<th>Bias (cm)</th>
<th>StDev (cm)</th>
<th>1σ RMS (cm)</th>
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<tr>
<td></td>
<td>N  E  U</td>
<td>N  E  U</td>
<td>N  E  U</td>
<td>N  E  U</td>
</tr>
<tr>
<td>Iono-Free</td>
<td>0  -3 -2</td>
<td>2  3  4</td>
<td>2  4  5</td>
<td>3  8  9</td>
</tr>
<tr>
<td>USTEC Float</td>
<td>1  -3 -2</td>
<td>3  2  6</td>
<td>3  4  6</td>
<td>6  6  14</td>
</tr>
<tr>
<td>USTEC Fix</td>
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<td>2  3  8</td>
<td>2  3  8</td>
<td>4  8  15</td>
</tr>
</tbody>
</table>

Table VI-6: Optimum medium baseline (62 km) north, east and up (height) uncertainty bias, standard deviation (1σ) and RMS in cm.

The long baseline (141 km) results, shown in Figure VI-12 and Table VI-7, indicate that uncertainty biases, in all directions are very small (2 cm level). The bulk of the RMS uncertainty comes from the standard deviation. The ionosphere-free solution gives the best horizontal results. The ionosphere-free and USTEC float solutions have the same height RMS. The USTEC fixed height uncertainty has increased significantly due to a high standard deviation.
These results indicated that the methods and models used in this study can achieved high precision positioning (sub decimeter RMS at 1 \( \sigma \)) over baselines up to 140 km from the ionosphere-free and USTEC float solutions. These evaluations also showed that the USTEC float and ionosphere-free solutions were compatible. The uncertainty bias remained relatively low for all baselines and processing methods and the increase in uncertainty with baseline length was due to an increase in standard deviation. This would
indicate that the NOAA troposphere model was performing well for this data set. The
noise level from the USTEC fixed solution increased significantly with baseline length,
this was especially true with the height uncertainty. Visual inspection of Figure VI-12
indicated that the ambiguities were incorrect for much of time series.

VI.4.3 Conventional Versus Optimum

In the previous two subsections results from using the conventional method and
the optimum methods from three sample baselines were evaluated separately. In this
subsection, the conventional and optimum results are compared to each other. The
resulting evaluations are separated into two parts. The first looks at a direct comparison
between the baselines from the previous subsections and the second looks at a
comparison between the height components of all baselines from the same day (DOY
180, June 29).

For the first comparison, the uncertainty statistics resulting from using the
optimum methods were subtracted from those using the conventional method, which
produced an indication of "improvement". Each of the processing algorithms
(ionosphere-free, USTEC float or USTEC fixed) was differenced from itself to produce
an indication of relative improvement. A negative improvement indicated that the
optimum solution produced worse results.

Table VI-8 shows the results from the short (26 km) baseline. These results
indicated little change in the horizontal (± 1 cm) bias and an improvement of 1 to 3 cm in
the height bias. The ionosphere-free solution showed no change in the standard
deviation, in any dimension, and no overall (RMS) improvement in the horizontal. There
was a very modest 1 cm overall improvement in the height, which was due to bias
reduction. The USTEC float solution experienced the largest overall improvement with 3
cm in the east RMS and 4 cm in the height. USTEC fixed solution showed a modest 2
cm improvement in the east RMS and 1 cm in the height RMS.
Table VI-8: Conventional-optimum (improvement) short baseline (26 km) north, east, up (height) uncertainty bias, standard deviation (1σ) and RMS (1σ) in cm.

<table>
<thead>
<tr>
<th>Improvement</th>
<th>Bias</th>
<th>Standard Deviation</th>
<th>Root Mean Square</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>North</td>
<td>East</td>
<td>Up</td>
</tr>
<tr>
<td>Iono-Free</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>USTEC Float</td>
<td>0</td>
<td>-1</td>
<td>3</td>
</tr>
<tr>
<td>USTEC Fix</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table VI-9 shows the improvement in the medium (62 km) baseline. All processing methods showed an overall uncertainty improvement in all dimensions. Improvement in the north uncertainty ranged from 2 to 4 cm, in the east from 0 to 6 cm and in the height from 4 to 6 cm. With the ionosphere-free solution, the majority of the height improvement was in the uncertainty bias; for the USTEC float, the majority of the height improvement was in the uncertainty standard deviation; and in the USTEC fixed, the improvement came in both bias and standard deviation.

<table>
<thead>
<tr>
<th>Improvement</th>
<th>Bias</th>
<th>Standard Deviation</th>
<th>Root Mean Square</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>North</td>
<td>East</td>
<td>Up</td>
</tr>
<tr>
<td>Iono-Free</td>
<td>0</td>
<td>-1</td>
<td>6</td>
</tr>
<tr>
<td>USTEC Float</td>
<td>0</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>USTEC Fix</td>
<td>1</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

Table VI-10 shows the improvement in the long (141 km) baseline. All processing methods showed a significant improvement in the height bias of 25 to 26 cm. This was attributed to the strong wet delay gradient along the coastline on DOY 180 (June 29) (shown in Figure VI-2), which was accounted for in the NOAA troposphere model. The stations used for the baseline were LMCN, which was on the coast, and HAMM which was inland.
<table>
<thead>
<tr>
<th>Improvement</th>
<th>Bias</th>
<th>Standard Deviation</th>
<th>Root Mean Square</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>North</td>
<td>East</td>
<td>Up</td>
</tr>
<tr>
<td>Iono-Free</td>
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<td>0</td>
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</tr>
<tr>
<td>USTEC Float</td>
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</tr>
<tr>
<td>USTEC Fix</td>
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<td>25</td>
</tr>
</tbody>
</table>

Table VI-10: Conventional-optimum (improvement) long baseline (141 km) north east and up uncertainty bias, standard deviation (1 σ) and RMS in cm.

The second part of the conventional/optimum evaluation is the comparison of the height uncertainty, over all baselines, for DOY 180 (June 29). The height results from all baselines, for 20 hours of 30-second observations, are shown in Figure VI-13 and Figure VI-14. Figure VI-13 and Table VI-11 shows the results from using the Saastamoinen closed form troposphere model with MDP and ZPD turned off (conventional). Figure VI-14 and Table VI-12 shows the results from using the NOAA troposphere model, with MDP and ZPD turned on (optimum).
Figure VI-13: Conventional method height uncertainty for all baselines, in meters, using Saastamoinen troposphere model and ZPD and MDP off.

The ionosphere-free solutions are consistently the best for most baselines, in both figures, especially in the longer baselines. However, when using the ZPD and MDP (Figure VI-14) the USTEC float and fixed solutions come much closer. The MDP algorithm is not used in the Ionosphere-free solution.
### Table VI-11: Conventional method North, East and up uncertainty bias, standard deviation and RMS for all baselines, in cm, using Saastamoinen troposphere model and ZPD and MDP off.

It is evident from the two figures that the solution improves considerably when using the NOAA troposphere model and with ZPD and MDP turned on. This is especially true as the baseline length increases. Bias improvements are very evident in the LMCN-SIHS and LMCN-HAMM baselines, which have both coastal and inland stations.
Figure VI-14: Optimum method height uncertainty for all baselines, in meters, using NOAA troposphere model and ZPD and MDP on.
<table>
<thead>
<tr>
<th></th>
<th>Dist (km)</th>
<th>North</th>
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<th>Up</th>
<th>Standard Deviation</th>
<th>Root Mean Square</th>
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</thead>
<tbody>
<tr>
<td>D-N</td>
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<tr>
<td>D-H</td>
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<td>-2</td>
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<td>2</td>
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<tr>
<td>L-N</td>
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<td>2</td>
<td>2</td>
</tr>
<tr>
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<td>-2</td>
<td>2</td>
<td>2</td>
</tr>
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</tr>
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<th>East</th>
<th>Up</th>
<th>Standard Deviation</th>
<th>Root Mean Square</th>
</tr>
</thead>
<tbody>
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<td>D-N</td>
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<td>-2</td>
<td>-1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>D-H</td>
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<td>-3</td>
<td>-2</td>
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<tr>
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<td>1</td>
<td>-2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>L-H</td>
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<td>-1</td>
<td>2</td>
<td>-2</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>S-H</td>
<td>186</td>
<td>-4</td>
<td>1</td>
<td>4</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>L-H</td>
<td>302</td>
<td>4</td>
<td>-1</td>
<td>-7</td>
<td>7</td>
<td>9</td>
</tr>
</tbody>
</table>

<table>
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<th>Dist (km)</th>
<th>North</th>
<th>East</th>
<th>Up</th>
<th>Standard Deviation</th>
<th>Root Mean Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>D-N</td>
<td>26</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>D-H</td>
<td>62</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>2</td>
<td>2</td>
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<tr>
<td>L-N</td>
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<td>1</td>
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<td>L-H</td>
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<tr>
<td>S-H</td>
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<td>L-H</td>
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<td>-6</td>
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<td>9</td>
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</table>

Table VI-12: Optimum method North, East and up uncertainty bias, standard deviation and RMS for all baselines, in cm, using NOAA troposphere model and ZPD and MDP on.

VI.4.4 Troposphere, ZPD and MDP Evaluation for DOY 180 (June 29)

In the previous subsections the optimum solutions were assumed to come from the use of the optimum methods. In this subsection the validity of these assumptions are examined. An in-depth evaluation of the effects of the different troposphere mitigation methods, the addition of a ZPD unknown in the least-squares solution and the usefulness of the MDP algorithm, was performed for DOY 180 (June 29). Plots showing the height uncertainty statistics (RMS, bias and standard deviation) for every combination were generated (see Figure VI-15, Figure VI-16 and Figure VI-17). Each plot displayed graphs depicting the height uncertainty statistic (Y-axis) in meters, versus the baseline.
length (X-axis) in kilometers (see Table VI-1 to correlate baseline length to stations). The ionosphere-free solution with the Saastamoinen troposphere model, and using a ZPD delay term was considered to be the "conventional" solution and was included in all plots as a reference (IF SAAST ZPD). The results shown were from processing runs using the conventional solution, the ionosphere-free solution, the USTEC float and the USTEC fixed solutions. The eight plots, resulting from all on/off combinations of MDP and ZPD using the NOAA and Saastamoinen troposphere models were combined into a single image.

In each image, plots “A” through “D” correspond to the Saastamoinen troposphere with plot “A” corresponding to MDP off, ZPD off; plot “B” corresponding to MDP off, ZPD on; plot “C” corresponding to MDP on, ZPD off; and plot “D” corresponding to MDP on, ZPD on. Also, in each image, plots “E” through “G” correspond to the NOAA troposphere model, with plot “E” corresponding to MDP off, ZPD off; plot “F” corresponding to MDP off, ZPD on; plot “G” corresponding to MDP on, ZPD off; and plot “H” corresponding to MDP on, ZPD on (see Table VI-13). On the plots themselves, a “1” indicates that the option is on and a “0” indicates that it is off. One image was created for each of the three statistics. See Figure VI-15 for the RMS results, Figure VI-16 for the bias results and Figure VI-17 for the standard deviation results. The vertical (Y-axis) scale for the RMS images (Figure VI-15) was set to 1 meter, while for the bias and standard deviation images the Y-axis was set to 0.5 meters.

<table>
<thead>
<tr>
<th>Pane</th>
<th>Troposphere</th>
<th>MDP</th>
<th>ZPD</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>SAAST</td>
<td>off</td>
<td>off</td>
</tr>
<tr>
<td>B</td>
<td>SAAST</td>
<td>off</td>
<td>on</td>
</tr>
<tr>
<td>C</td>
<td>SAAST</td>
<td>on</td>
<td>off</td>
</tr>
<tr>
<td>D</td>
<td>SAAST</td>
<td>on</td>
<td>on</td>
</tr>
<tr>
<td>E</td>
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<td>off</td>
</tr>
<tr>
<td>F</td>
<td>NOAA</td>
<td>off</td>
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</tr>
<tr>
<td>G</td>
<td>NOAA</td>
<td>on</td>
<td>off</td>
</tr>
<tr>
<td>H</td>
<td>NOAA</td>
<td>on</td>
<td>on</td>
</tr>
</tbody>
</table>

Table VI-13: Plot panels and test reference.
Figure VI-15: Height RMS (m) comparison for all baselines, using Saastamoinen (A through D) and NOAA Troposphere Model (E through H).
The most striking improvement came from using the NOAA troposphere model. This was evident from the large RMS improvements in most of the baselines, over using the Saastamoinen model, (Figure VI-15). The greatest improvement occurred for the longer baselines that had both coastal and inland stations (140 and 300 km). The improvement would be almost linear with baseline length if not for the 186 km baseline, for which both stations (SIHS-HAMM) were inland and; consequently, had similar tropospheric effects. As shown in the image, there is very little improvement from using the ZPD term, this is especially apparent from the ionosphere-free solution of Figure VI-15 A, where the uncertainty, when using ZPD (IF SAAST ZPD) is only slightly less than when it is not used (Iono Free). Both the USTEC float and fixed solutions showed a little improvement when using ZPD, but showed much greater improvement when using the MDP algorithm (see Figure VI-15 F and H). The USTEC float solution improved to the point where it was comparable to the ionosphere-free solution, as seen in Figure V-13 H.

The Saastamoinen bias plots located in Figure VI-16 (A through D) showed very similar results for all processing runs and all combinations of MDP and ZDP. This was in stark contrast to the NOAA troposphere results (E through H), where the bias was significantly smaller than the conventional solution for all of the processing runs. There was a maximum improvement of about 40 cm in the vertical for the longest baseline. The USTEC float and fixed solution biases showed improvement when the MDP was used (Figure VI-16 G and H). Only a very slight improvement was seen when using the ZPD (Figure VI-16 E and G). The optimal solution, considering the height bias, came from using the NOAA troposphere delay with ZPD and MDP turned on (Figure VI-16 G). The height uncertainty bias was less than 5 cm for baselines under 150 km, and less than 8 cm for baselines under 300 km. As seen in the RMS results, the 186 km baseline uncertainty improvement was much less than the other baselines of 100 km or more because of the location of the stations, both of which were in-land, where the variation in the troposphere wet delay was much lower.
Figure VI-16: Height bias (m) comparison for all baselines, using Saastamoinen (A through D) and NOAA Troposphere Model (E through H).
The variations in the standard deviation indicated variations in the uncertainty noise level. The height standard deviations (depicted in Figure VI-17) showed very little change when using the ZPD with the Saastamoinen troposphere (A through D) for the ionosphere-free solution; however, the USTEC float solutions showed a very slight improvement. When the NOAA troposphere model was used (depicted in Figure VI-17 E through H), the evaluation of the ZPD term was similar to that seen with the Saastamoinen model. The NOAA ionosphere-free solution standard deviations were actually a little worst than the Saastamoinen solution. This would indicate that the NOAA troposphere model injected some noise into the solution. As mentioned earlier, the double differencing process removed the majority of the troposphere uncertainties, leaving variations in the atmospheric effects between stations. As seen in the bias evaluation, the NOAA troposphere corrections improved the height uncertainty considerably, but the results here indicated that noise levels were slightly increased. The USTEC float solutions displayed similar effects. The greatest reduction in standard deviation came from the improvement in the USTEC (fixed and float) solutions when the MDP was used, as shown in Figure VI-17 C, D, G and H. Although the USTEC fixed and float solution improved relative to themselves, their standard deviations were always greater than those from the conventional ionosphere-free solution.
Figure VI-17: Height StDev (m) comparison for all baselines, using Saastamoinen (A through D) and NOAA Troposphere Model (E through H).
From the evaluations conducted to this point, looking only at height uncertainties for DOY 180 (June 29), it was established that:

- The USM ionosphere-free solution with the Saastamoinen troposphere model was comparable to conventional methods.
- For all baselines over 26 km, the USM ionosphere-free solution produced the best results. For the 26 km baseline, the USM ionosphere-free, USTEC float and USTEC Fix solutions produced comparable results.
- The optimal results for the four-observable solutions (USTEC) came from using the MDP algorithm.
- The four-observable float solution using the USTEC ionosphere model with MDP came very close to the ionosphere-free solution.
- The ZPD term marginally improved the solution in most instances.
- The NOAA troposphere model improved the solution in most instances, especially when the two stations experienced significantly different wet delay environments.
- The NOAA troposphere model improved the height uncertainty bias.
- The MDP improved the USTEC standard deviations.

VI.4.5 Troposphere, ZPD and MDP Evaluations Averaged Over All Days

The following subsection compiles the results from all four days of testing (DOY 180, 186, 187, and 188; June 29, July 5, July 6, and July 7), looking at the evaluation of the troposphere models, ZPD and MPD for the ionosphere-free, USTEC float and USTEC fixed solutions. In contrast to the previous subsection, all three dimensions (north, east and up [height]) are presented in the plots and tables. Each plot displays line graphs depicting the average north, east and up uncertainty statistics (Y-axis) in meters, versus the baseline length (X-axis) in kilometers (see Table VI-1 to correlate baseline length to stations). The ionosphere-free solution with the Saastamoinen troposphere model, and using a ZPD delay term was considered to be the "conventional" solution and was included in all plots as a reference (IF SAAST ZPD). The results shown were from processing runs using the conventional solution, the ionosphere-free solution, the USTEC float and the USTEC fixed solutions. The four plots resulting from all on/off combinations of MDP and ZPD using the Saastamoinen troposphere model, were
combined into a single image and the four plots using the NOAA troposphere model were combined into a second image. In each image plot “A” corresponds to MDP off, ZPD off; plot “B” corresponds to MDP off, ZPD on; plot “C” corresponds to MDP on, ZPD off; and plot “D” corresponds to MDP on, ZPD on. One pair of images was created for each of the three statistics. See Figure VI-18 and Figure VI-19 for the RMS results, Figure VI-20 and Figure VI-21 for the bias results and Figure VI-22 and Figure VI-23 for the standard deviation results. The vertical (Y-axis) scale was set to 0.3 meters for the horizontal position uncertainty components and 0.60 meters for the up uncertainty. The following discussion looks at the “improvement” of position uncertainty relative to each of the processing options, and with respect to the conventional solution.

The average height uncertainty (up) RMS for the Saastamoinen troposphere model (see Figure VI-18) had a similar pattern to those seen for DOY 180 (June 29) (see Figure VI-15), except that they were less pronounced. This was due to the fact that DOY 180 (June 29) had higher troposphere wet delay gradients than the other days, which resulted in larger height uncertainties for the Saastamoinen solutions. The ZPD term had a minimal effect on the height uncertainty, as seen in the DOY 180 (June 29) evaluation; however, it had a larger effect on the longitude (east) RMS uncertainty (comparing A to B and C to D).

The ionosphere-free east RMS (shown in Figure VI-18 A), was larger than the reference conventional solution, which used ZPD. The separation between the reference (IF SAAST ZPD) and the ionosphere-free solution increased with baseline distance to a maximum of approximately 4 cm. The east RMS for the USTEC float solution also showed improvement for the longer baselines. The USTEC float RMS uncertainty for all three dimensions showed some improvement when the MDP algorithm was used (comparing A to C and B to D), with the improvement increasing with baseline length.
Figure VI-18: North, east and up RMS (m) comparison for all baselines, averaged over all days, using Saastamoinen troposphere model.

The RMS results from using the NOAA troposphere model (shown in Figure VI-19) indicated similar trends to the Saastamoinen results. As expected, the height uncertainties were all lower than the Saastamoinen results. The ZPD term improved the east uncertainties slightly (comparing A to B and C to D), but not to the same extent as in the Saastamoinen case. Using the MDP option (comparing B to D) improved the USTEC
float solution in the north and height components to where they were comparable to the ionosphere-free solution. This was also the case in the east uncertainty for all but the longest baseline, where the uncertainty actually got slightly worst. This increase in the average east RMS value was due to an east uncertainty spike on DOY 186 (July 5).

Figure VI-19: North, east and up RMS (m) comparison for all baselines, averaged over all days, using NOAA Troposphere model.
The average north, east and up uncertainty biases resulting from the Saastamoinen troposphere model are shown in Figure VI-20 and the NOAA troposphere model results are shown in Figure VI-21. The Saastamoinen results indicated that there was a slight improvement in the east bias when using the ZPD term, but no significant change in the north or up components (comparing Figure VI-20 A to B and C to D). The use of the MDP algorithm, without ZPD (comparing A to C) showed slight improvement in the east bias, but no change in the north or up components. The NOAA troposphere bias results (Figure VI-21) showed almost identical north and east components to the Saastamoinen results. However, the height bias was significantly reduced, as was seen in the DOY 180 (June 29) evaluation. This would indicate that there was little effect on the uncertainty bias by using the ZPD term or the MDP algorithm, and that the most significant improvement came in the up bias from using the NOAA troposphere model. Also, in terms of the uncertainty bias, the ionosphere-free, USTEC float and USTEC fixed solutions were comparable, in all three dimensions.
Figure VI-20: North, east and up bias (m) comparison for all baselines, averaged over all days, using Saastamoinen Troposphere model.
Figure VI-21: North, east and up bias (m) comparison for all baselines, averaged over all days, using NOAA Troposphere model.

The average north, east and up uncertainty standard deviations resulting from the Saastamoinen troposphere model are shown in Figure VI-22 and the NOAA troposphere model results are shown in Figure VI-23. The Saastamoinen results indicated that the ZPD term improved the uncertainty standard deviations for all of the processing methods (ionosphere-free, USTEC float and USTEC fixed) in all directions, with the biggest
improvement occurring in the east uncertainty. The MDP algorithm also improved the uncertainty standard deviation for all of the processing methods, in all three dimensions.

Figure VI-22: North, east and up StDev (m) comparison for all baselines, averaged over all days, using Saastamoinen Troposphere model.

The NOAA troposphere results showed very little difference in uncertainty standard deviation from the Saastamoinen results. The north uncertainty standard
deviation showed the most improvement over Saastamoinen, and of the three processing methods the ionosphere-free solution had the largest improvement.

Figure VI-23: North, east and up StDev (m) comparison for all baselines, averaged over all days, using NOAA Troposphere model.
From these evaluations the following conclusions can be drawn:

- The NOAA troposphere model improves the up uncertainty bias.
- The NOAA troposphere model has little effect on the north and east uncertainty bias.
- The ZPD term slightly improves the east uncertainty bias, but has little to no effect on the north and up bias.
- The MDP algorithm has very little effect on the uncertainty bias.
- Ionosphere-free, USTEC float and USTEC fixed all have very similar uncertainty biases in all three dimensions.
- The NOAA troposphere model had little effect on the uncertainty standard deviation.
- The ZPD term slightly improved the standard deviation in all dimensions, with the greatest improvement in the east component.
- The MDP algorithm improved the standard deviation for all methods in all directions (except of course in the case of the ionosphere-free solution where it is not used).
- The ZPD term had a greater effect on the Saastamoinen solutions than on the NOAA troposphere solutions.

From the above evaluation, it is apparent that the NOAA troposphere model, the ZPD term and the MDP algorithm improve the uncertainty of the position estimation.

The following tables summarize the uncertainty RMS, bias and standard deviation, averaged over all days, for the conventional solution and for the optimum solution. All results are in cm.

The conventional solution results are in Table VI-14 (RMS), Table VI-15 (bias) and Table VI-16 (standard deviation). These results were derived from using the Saastamoinen troposphere mode, with the ZPD term and without the MDP algorithm.

<table>
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Table VI-14: Conventional solution average north, east and up RMS (1σ) in cm.
Table VI-15: Conventional solution average north, east and up bias in cm.

<table>
<thead>
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Table VI-16: Conventional solution average north, east and up standard deviation (1σ) in cm.

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The optimum solution results, using the NOAA troposphere model with the ZPD term and the MDP algorithm, are compiled in Table VI-17 (RMS), Table VI-18 (bias) and Table VI-19 (standard deviation).

Table VI-17: Optimal solution average north, east and up RMS (1σ) in cm.

<table>
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<tr>
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Table VI-18: Optimal solution average north, east and up bias in cm.

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</table>

Table VI-19: Optimal solution average north, east and up standard deviation (1σ) in cm.

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In the previous visual evaluation, the improvement of each processing method was only compared to itself. For example, the USTEC float solution using the MDP algorithm had a lower standard deviation than when it was not used. In the final evaluation of the troposphere models, ZPD and MDP presented here, the “improvement” was computed by differencing each of the optimum solution uncertainties from the conventional solution for the ionosphere-free method. The results are shown in Table VI-20, Table VI-21 and Table VI-22. Note: A negative improvement indicates that the uncertainty was greater for the optimal solution than it was for the conventional ionosphere-free solution.

The RMS improvements from the optimum methods over the ionosphere-free conventional solution, shown in Table VI-20, indicated that there was a decrease in the height RMS for most of the baselines and methods, and that improvement increased with baseline length. The exceptions to this were the ionosphere-free and USTEC float for the
26 km baseline, which showed an increase in RMS of 1 cm, and the USTEC fixed solution for the 186 km baseline (both stations inland), which showed an increase of 2 cm. The optimum ionosphere-free solution showed marginal RMS improvement, or no effect, in the north and east. The USTEC float solution showed no improvement in the north, except for the longest baseline, and showed an increase in east RMS in all but the shortest baseline. The USTEC fixed solution experienced either no effect or a slight increase in the north RMS, except for the longest baseline, and an increase in all east RMS except for the shortest baseline.

The ionosphere and troposphere models are used in this study to help mitigate atmospheric uncertainties as the signal path between base and remote start to decorrelate with increasing baseline length. For the shorter baselines, the signal path is highly correlated, and the differential processes effectively remove atmospheric uncertainties. Uses of the ionosphere and troposphere models for these short baselines will have little effect, and may even add uncertainty, worsening the results.

<table>
<thead>
<tr>
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</thead>
<tbody>
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</table>

Table VI-20: Optimal solution improvement over conventional, RMS (1σ) in cm.

The bias results, shown in Table VI-21, indicated improvement in the height for all processing methods over all baselines except the shortest. Improvement tended to increase with baseline length, except for the 186 km baseline. The north and east uncertainty biases from all baselines and methods showed very little difference from the conventional solution.
Table VI-21: Optimal solution improvement over conventional, bias in cm.

<table>
<thead>
<tr>
<th>BIAS: re conventional</th>
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<th>East</th>
<th>Up</th>
</tr>
</thead>
<tbody>
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<td>U FL</td>
<td>U FX</td>
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<tr>
<td>LMCN-SIHS 302</td>
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Table VI-22: Optimal solution improvement over conventional, standard deviation (1σ) in cm.

<table>
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<th>Up</th>
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<tr>
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<td>-1</td>
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<tr>
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</table>

These results indicate that the processing methods using the optimum options are an overall improvement over the ionosphere-free conventional solutions and that the greatest improvement is in the height bias. This indicates that the NOAA troposphere...
model improves the solution, and that improvement is mainly in the bias. They also indicate that, compared to the ionosphere-free conventional solution, the standard deviations are larger in the height for the USTEC float and fixed solutions. This indicates that the ionosphere models add noise to the process. The USTEC float solution improves as much as the ionosphere-free solution in the height bias, but some of that improvement is lost in the overall RMS due to a higher standard deviation.

Appendix D contains the 95% RMS for the north, east and up uncertainties for the conventional and optimal methods. Both the 2σ and 95% ordered statistic (95th percentile) are presented. Histograms displaying the height uncertainties for all solutions, with the conventional and optimal methods are also presented. A sample for the LMCN to HAMM (141 km) baseline is shown in Figure VI-24. The 2σ (blue line) and 95% OS (red line) are also plotted. This plot clearly indicates a shift towards zero (reduced bias) and a reduction in width (lower standard deviation) with the optimal method. There is also an alignment of the 2σ and 95% OS limits, which would indicate that the results are more normally distributed.

Figure VI-24: Up uncertainty histograms for LMCN to HAMM
VI.4.6 Ambiguity Fixing

All of the processing runs for these evaluations included the USTEC ambiguity float (USTEC S) and the USTEC ambiguity fixed (USTEC SF) solutions. Integer ambiguity values were estimated at every epoch using the ambiguity float estimates and associated variance co-variance matrix from the float solution. Once the ambiguity estimates converged (after several minutes), fixed position solutions were computed at every epoch. The test results from the previous subsections showed that of the three processing methods, the USTEC fixed solution consistently produced the least accurate results, except for the short (26 km) baseline, where it produced the best results. The problem with the fixed solution for the longer baselines was fixing on the correct integer ambiguity set. The plots shown in Figure VI-25 depict a short period of time, for each baseline, when the ambiguities are fixed. These plots are used to compare the fixed solutions to the ionosphere-free and four-observation float solutions, and help to determine when and why the integer ambiguities are correct, or not correct. Figure VI-25 displays an image with six plots, one for each baseline length, with three graphs per plot (north, east and up), of the uncertainty plotted against time for DOY 180 (June 29). Table VI-23 contains the north, east and up uncertainty statistics associated with the plots of Figure VI-25. All processing methods (ionosphere-free, USTEC float and USTEC fixed) used the optimum options (NOAA troposphere, ZPD term, MDP algorithm). The vertical (Y-axis) for each plot ranged from -0.2 to 0.2 meters, and the horizontal (X-axis) covered four hours (at 30 seconds). The plots were arranged in order of baseline length from short (A) to long (F). The four-hour time segments were selected to show sections of the data where the USTEC fixed solution converged on the correct ambiguity set.

The first four baselines (Figure VI-25 A through D) covered the same four-hour time period. The first two plots (26 km and 62 km) showed very close alignment between all processing methods, in all three dimensions.

In the 92 km baseline (see Figure VI-25 C) the ambiguity fixed north and east uncertainties were generally linear, but offset from the ionosphere-free and USTEC float
solutions. There were apparent ambiguity shifts as evidenced by solution steps. The height uncertainty showed signs of solution wander, resulting from incorrect ambiguity values.

Figure VI-25: Four hours of epoch-to-epoch north, east and up uncertainties for all baselines on DOY 180 (June 29).

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<table>
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<tr>
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<td>3</td>
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<tr>
<td>L-H</td>
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<td>-3</td>
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Table VI-23: North, east and up bias, standard deviation (1σ) and RMS (1σ) from four hours of epoch-to-epoch uncertainties for all baselines on DOY 180 (June 29), all in cm.

In the 141 km baseline (see Figure VI-25 D) there was ambiguity realignment at around 1100 hrs, where all three dimensions showed a solution step. Another shift occurred at shortly before 1300 hrs where the solution locked onto the correct values.

The two longer baselines (186 km and 302 km) covered different time periods from the shorter baselines, and from each other. The 186 km baseline (see Figure VI-25 E) showed very good results for the north and east USTEC fixed uncertainties, producing better results than both of the other methods. However, the height uncertainty showed a couple of steps, as well as some wander that followed the trend of the ionosphere-free.
and USTEC float solutions. The 302 km baseline (see Figure VI-25 F) showed steps in all three dimensions; however, the trend did follow the other two solutions.

More insight into the ambiguity fixed solution uncertainties can be gained by looking at the range residuals and ambiguities. Residuals are the difference between each observed double differenced range and the corresponding double differenced range determined from the computed position. Figure VI-26, Figure VI-27 and Figure VI-28 display residuals and ambiguities for the 26 km (DSTR-NOLA), 141 km (LMCN to HAMM) and 302 km (LMCN to SIHS) baselines, respectively, from the ionosphere-free, 4Obs float and 4Obs fixed solutions. Each figure contains five plots, A through E. The first plot (A) displays a height position uncertainty time series, in meters, to show relative position uncertainties. The next three plots display time series of the residuals for all in-view satellites from the ionosphere-free (B), float (C) and fixed (D) solutions, in meters. The final plot (E) displays the difference between the L1 float ambiguities and the L1 fixed ambiguities (float – fixed) in L1 cycles.

The up position uncertainty from the fixed solution of the 26 km baseline (Figure VI-26 A) shows an ~8 cm jump at ~10.3 hrs, which brings the uncertainty closer to zero, and a ~ + 8 cm jump at ~ 13.6 hrs that takes it further away from zero. Both jumps correspond to a one cycle ambiguity difference (float – fixed) shift in a satellite, as seen in Figure VI-26 E. The residual plots from the ionosphere-free solution (Figure VI-26 B) and the float solution (Figure VI-26 C) are very similar, except for the occasional spike in the float residuals. The spikes correspond to the first epoch of new satellites, where the initial ambiguity estimation is relatively coarse due, in part, to the start-up of the MDP bias estimator. The residuals from the fixed solution (Figure VI-26 D) are similar to the float solution, where the position uncertainties are similar. In the beginning and at the end, where the fixed solution height uncertainty increases, the residuals also increase.
Figure VI-26: DSTR to NOLA (26 km), up position uncertainties (A). Ionosphere free (B) float (C) and fixed (D) double difference residuals (m) and δ ambiguity (float – fixed in L1 cycles, E) for all in-view satellites.
The results from the 141 km baseline are shown in Figure VI-27. The height uncertainty, shown in Figure VI-27 A, indicates that the fixed solution is using the incorrect set of ambiguities until ~1100 hrs where a ~20 cm jump occurs, to what appears to be the correct set of ambiguities. The fixed position uncertainty is close to zero in all three dimensions (see Figure VI-25 D) until another ~20 cm jump occurs at ~11.35 hrs. This short period of correct ambiguities corresponds to an increase in several of the fixed residuals (see Figure VI-27 D), and a shift in the entire set of delta ambiguity values (see Figure VI-27 E). The initial jump to the correct ambiguity set occurs at a change in reference satellite. The subsequent shift to an incorrect ambiguity set, at 11.35 hrs, corresponds to a one cycle shift in two of the ambiguities, as seen in Figure VI-27 E. Throughout this short period of fixed position variation, the ionosphere-free and float solution height uncertainties and residuals (Figure VI-27 B and C) remain relatively consistent.

The results from the 302 km baseline are shown in Figure VI-28. The vertical scale for the delta ambiguity plot (Figure VI-28 E) is set to ±2 L1 cycles, as opposed to ±1 L1 cycles used in the previous plots; all other plot dimensions remain the same. All three height uncertainties shown in Figure VI-28 A follow the same basic trend. The fixed solution has several height uncertainty jumps, some of which are seen in the other two solutions. Three of the jumps (approximately 13.5 hrs, 13.75 hrs and 15.35 hrs) are not reflected in the other two solutions. These correspond to single, one cycle changes in δ ambiguity, as seen in Figure VI-28 E, which correspond to a revised integer ambiguity. Several other single, one cycle changes occur, but do not translate into height uncertainty shifts. The fixed residuals (Figure VI-28 D) show jumps at every instance of a δ ambiguity jump. Determining if the correct ambiguity set have been established with this baseline is difficult. Residual ionosphere and troposphere uncertainties are likely creeping into the solution, making the fixed solution less reliable than the floating solutions.
Figure VI-27: LMCN to HAMM (141 km), up position uncertainties (A). Ionosphere free (B) float (C) and fixed (D) double difference residuals (m) and $\delta$ ambiguity (float – fixed in L1 cycles, E) for all in-view satellites.
Figure VI-28: LMCN to SIHS (302 km), up position uncertainties (A). Ionosphere free (B) float (C) and fixed (D) double difference residuals (m) and $\delta$ ambiguity (float – fixed in L1 cycles, E) for all in-view satellites.
VI.5 Conclusions

The NOAA troposphere model greatly improves the position uncertainty, especially when dealing with highly variable weather conditions between the base and roving stations, which will be the case with base stations on the shoreline and remote stations at sea. The weather pattern depicted in Figure VI-2 shows a distinct wet delay gradient along the land sea interface, and this is more often than not the case. The ionosphere-free solution, which does not use any ionosphere model, overall produces the best results. However, the float solution using USTEC (USTEC S) and MDP comes very close, and it is the only method, of those considered for this study, that can lead to a reliable ambiguity fixed solution (USTEC SF).

Using ZPD does help the solutions, but only marginally. Using MPD with the USTEC ionosphere model greatly improves the four observable float solutions, which improves the software's ability to create a reliable fixed solution. The fixed solutions (USTEC SF) perform well for the short baseline, but the uncertainty starts to increase with baseline length because of the fixing on incorrect ambiguity sets. The following lists the main conclusions that can be drawn from this evaluation:

- The USM ionosphere-free solution with the Saastamoinen troposphere model was comparable to GrafNav conventional solution.
- For all baselines over 26 km, the USM ionosphere-free solution produced the best results. For the 26 km baseline, the USM ionosphere-free, USTEC float and USTEC Fix solutions produced comparable results.
- The optimal results for the four-observable solutions (USTEC) came from using the MDP algorithm.
- The four-observable float solution using the USTEC ionosphere model with MDP came very close to the ionosphere-free solution, in terms of position uncertainty.
- The USTEC fixed solution did not produce reliable integer ambiguities for the longer baselines, even with the improved float solution.
- The ZPD term marginally improved the solution in most instances.
- The NOAA troposphere model improved the solution in most instances, especially when the two stations experienced significantly different wet delay environments.
- The NOAA troposphere model improved the uncertainty bias.
- ZPD term improved east uncertainty.
• The ZPD term had a greater effect on the Saastamoinen solutions than on the NOAA troposphere solutions.
• The MDP algorithm improved the USTEC standard deviations.
• When using the NOAA troposphere model, ZPD term and MDP algorithm, the bias improves, but standard deviation gets worse; however, the overall RMS improves.
• Not only does the optimal method improve the RMS over the conventional method, it also leads to more normally distributed uncertainties. This indicates that the remaining uncertainties are random, meaning that the systematic effects from ionosphere and troposphere have been greatly reduced.

These results indicate that the processing methods using the optimum options are an overall improvement over the ionosphere-free conventional solutions and that the greatest improvement is in the height bias. They also indicate that, compared to the ionosphere-free conventional solution, the standard deviations are worse in the height for the USTEC float and fixed solutions. The USTEC float solution improves as much as the ionosphere-free solution in the height bias, but some of that improvement is lost in the overall RMS due to a higher standard deviation. Therefore, the overarching results summary is: ambiguity fixing is not reliable, even with the lower uncertainty of the USTEC float solution. However, they are much more reliable than if the NOAA troposphere model, ZPD term and MDP algorithm were not used.
CHAPTER VII
CONCLUSION AND RECOMMENDATIONS

Ionosphere and troposphere uncertainty mitigation is essential for long-range, high-accuracy, real-time DGPS positioning. Atmosphere mitigation is essential for all real-time applications, but it is even more critical in the highly dynamic marine environment. NOAA has developed real-time ionosphere (USTEC) and troposphere models that can be used to help reduce atmospheric effects. The following chapter recaps the results from the previous chapters, then puts forth of conclusions and finally adds some recommendations for future studies.

VII.1 Results Summary

The research conducted for this dissertation encompassed four separate studies. The first study looked exclusively at the troposphere where the real-time NOAA Numerical Weather Prediction (NWP) model was compared to the Saastamoinen troposphere model. This study looked at data for CORS stations in Michigan, California and the South East from the summer of 2004, with baselines ranging from 140 to 740 km. The second study investigated several ionosphere mitigation models and techniques, with an emphasis on the NOAA real-time USTEC model, during a period of high ionosphere activity. This study looked at data from Michigan, California, the South East and a central region in January 2005, with baselines ranging from 140 to 740 km. The third study examined both the NOAA real-time ionosphere and troposphere models in a dynamic marine environment during a period when the troposphere was very active (Hurricane Katrina). This study looked at stations in Mississippi during the week of August 28, 2005, with baselines from 22 to 100 km, using one second data. All of the other studies used thirty second data. The final study looked at the NOAA real-time models over a variety of static baselines, with a variety of tropospheric conditions. This last study was conducted from data collected in the summer of 2006, after modifications
had been made to both the ionosphere and troposphere models. Data were from stations in South Louisiana with baselines from 26 to 302 km.

The majority of the tests were conducted using differential techniques. As a result, the bulk of the uncertainties were removed during the differencing process, and the remaining uncertainties resulted from the local high-frequency atmospheric effects that were not accounted for by the models.

The ultimate goal of the studies conducted for this research was to determine if the NOAA real-time atmospheric models reduced the GPS signal uncertainties to the point where L1 and L2 double differenced integer ambiguities could be determined reliably, and used in an ambiguity fixed solution. As the studies progressed, so did the USM processing software. For the first NOAA troposphere model tests, an ionosphere-free double-differenced, sequential least-squares (SLS), floating ambiguity algorithm was developed. For the first ionosphere model test, a four observation (L1, L2, P1 and P2) double-differenced, SLS float solution was developed. For the last two studies, which combined the ionosphere and troposphere models, the integer ambiguity estimation routine (LAMBDA) was incorporated and a fixed solution algorithm was developed.

VII.2 Conclusions

During the first ionosphere model testing, Doug Robertson of NOAA/CiRES supplied NOAA MAGIC correctors as well as a set of correctors he produced from the L1/L2 observations combined with the MAGIC correctors (Doug P). The solutions derived from using the Doug P correctors produced the best results from an ionosphere model, and they were comparable to the ionosphere-free solution. The Doug P correctors were only available in post processing; therefore, for the last two tests a Modified Doug P (MDP) algorithm was developed to apply the same principles in real-time.

An important aspect of the testing process was the validation of the USM processing software. All tests showed that the USM software achieved results as good as,
if not better than, commercial software, using similar methods. This was true for the short baseline, ambiguity fixed solutions as well as the long baseline, ionosphere-free (ambiguity float) solutions.

For the most part, the use of the NOAA troposphere model improved the solution. This improvement was almost entirely in the height bias. In some cases, the use of the NOAA troposphere model increased the solution standard deviation, but not to the point where that increase lead to an overall increase in RMS. This increase in standard deviation was very evident in the south east region, and was attributed to a highly dynamic troposphere common in the area. For the most part, the solution RMS improved with the use of the NOAA troposphere model. Improvements were the most evident when there was a significant difference in troposphere activity between the two stations. In cases where the troposphere was relatively homogenous, there was little or no improvement. For the 2005 study of the data leading up to Hurricane Katrina, the NOAA troposphere models did not help, and in some cases made the solutions worse. During this study, even though the troposphere conditions were changing rapidly, those changes were similar for the stations at both ends of the baseline. For the last study, using static CORS data from the summer of 2006, the troposphere model improved the solution significantly where one end of the baseline was near the coast and the other end was inland. The prevailing weather pattern showed a significant wet delay gradient along the coastline. This caused the receivers at either end of the baseline to experience significantly different troposphere delays, which were effectively dealt with by the model.

The initial study of the ionosphere models showed that, for the most part, the ionosphere-free solution produced the best results, with the solutions derived from using the Doug P correctors coming very close. Of the real-time options, the global map (GIM) and the WAAS correctors produced very similar results, and were better than the others. The USTEC slant correctors faired very poorly, but this was attributed to poor satellite coverage within the model. This issue was addressed by NOAA/SEC subsequent to the
initial study and prior to final study. The results from the final study showed that the coverage issue had been resolved. For the baselines at ~20 km, the 4Obs fixed solution, using USTEC with MDP, produced the best results. For all other baselines, the ionosphere-free solution, using the NOAA troposphere model, produced the best results. The 4Obs float solution, using USTEC, MDP and the NOAA troposphere model came very close to the ionosphere free solution. The fixed solution showed jumps when the ambiguity estimator fixed on the incorrect set of integers. As the baselines increased, accompanied by an increase in differential atmosphere uncertainties, the frequency and magnitude of the uncertainties associated with the use of incorrect ambiguities increased. Even though the 4Obs fixed and float solutions were not as good as the ionosphere-free results, their solutions were far better when the USTEC model, with MDP and the NOAA troposphere model were used.

Not only does the optimal method improve the RMS over the conventional method, it also leads to more normally distributed uncertainties. This indicates that the remaining uncertainties are random, meaning that the systematic effects from the ionosphere and troposphere have been greatly reduced.

Results from using the MDP algorithm were not as good as the post-processed Doug P method. The main reason for this was the need to use a cumulative average of the bias between the L1-L2 and the model corrector, rather than the true average. The MDP algorithm estimated bias was the least accurate at the beginning of a satellites flight and the most accurate at the end, when it was the same as the one computed from the post-processed technique. At the beginning of a processing run, when the initial ambiguities were being estimated, the L1-L2/corrector biases for all satellites were also being estimated.

The increase in incorrect ambiguity estimates with baseline length may be related to the increase in uncertainty standard deviation from using the troposphere model. The integer ambiguity estimate process relies on the real-valued estimates as well as their
variance-covariance matrix. An increase in standard deviation due to the troposphere model estimates will degrade the algorithms ability to resolve the integers correctly.

VII.3 Recommendations for Future Study

Any future attempt at long range ambiguity resolution must include the real-time ionosphere and troposphere models, in a dynamic maritime environment. Future studies will incorporate the use of 1 Hz dynamic data from USM’s oceanographic buoy and associated base stations. The base station configuration will include the Stennis Space Center (STEN) and Horn Island (HORN) sites. In addition, a site at Diamondhead will be established to create a 20 km baseline to STEN and a site near Hattiesburg will be established to create a >200 km baseline to the buoy.

Improvements to the MDP algorithm will be investigated, with the goal of reliably fixing integer ambiguities. Modifications to the algorithm that will help computation of the L1-L2/model corrector bias merge to the “correct” value faster than the current method will be pursued. Further work will include a study of the effect of improper L1-L2/corrector biases at system startup, and how those start-up biases affect the initial ambiguity (float and fixed) estimates. Also, a process for mitigating the effect of incorrect start-up biases on newly acquired satellites will be developed. A post processed technique; similar to the one developed by Douglas Robinson, will be created to help evaluate the enhanced versions of the MDP algorithm.

Future studies must include ambiguity resolution techniques. The variance-covariance matrix passed to the LAMDBA method will be investigated further, as well as how constellation changes are handled. Effectively adapting to a change in reference satellite will be a primary consideration. Alternate ambiguity estimation techniques will also be considered.

The ZPD term did not have a significant effect on results for the studies conducted here. Future work will include a re-evaluation of the algorithm used. The
possibility of adding another term to account for residual ionosphere delay components will also be addressed. Fully compensating for the effects of the ionosphere and troposphere is necessary for the initial ambiguities to be resolved as integers.

All of the studies conducted for this dissertation ignored the effect of the orbit errors. The USM_OTF code will be updated to ingest precise ephemeris to evaluate its effect on ambiguity resolution. Comparisons between precise and broadcast ephemeris will be made.

Reduction in the uncertainty standard deviation will improve the chances of resolving the correct integer ambiguities; therefore, an improvement in the method of interpolating zenith propagation delay values may help. Currently, the troposphere model interpolator looks at only the zenith delay at the receiver. Taking into account the path of the signal as it travels through the troposphere will improve the delay estimate. Rather than trying to perform 3D ray tracing, the current two-dimensional grid map will be used to establish a horizontal corrector path between the user and the horizontal location of a satellite's troposphere pierce point. This will allow for the incorporation of troposphere delay variations in the vicinity of the user and base station.

The real-time ionosphere and troposphere models should have a significant effect on precise point positioning (PPP). A PPP algorithm should be added to the USM_OTF suite and used to evaluate the use of the NOAA models in this positioning technique. After all, PPP is the ultimate long-baseline positioning technique.
APPENDIX A

IONOSPHERE

GPS signal refraction occurs as the waves travel through the ionosphere. It is an area that is characterized by electrically charged atoms known as ions. These charged particles interact with radio signals, such as GPS transmissions, as the signals pass through the region. The effect of this interaction on GPS signals is equal and in the opposite direction for the carrier and the code (advance of the carrier and delay of the code). The effect of the ionosphere is dispersive, meaning that it varies with the frequency of the signal.

The ionosphere affects the L1 and L2 GPS signals differently – there exists a frequency-dependent relationship; as a result the L1 and L2 signals can be combined to create an “ionosphere-free” signal that compensates for the effects of the ionosphere. However, the resulting combined signal has a greatly increased noise level. The L1/L2 combination also removes the integer nature of the ambiguity. This, and the increased noise, makes ambiguity resolution difficult. As a result, the ionosphere-free combination is useful in an ambiguity float solution for long baseline distances (greater than ~20 km), and for initial ambiguity estimates.

Carriers L1 and L2 can be combined (L1-L2) to create a “wide lane” carrier observation with an 86.2 mm wavelength. This combination improves the chances of determining the ambiguity by increasing the size of the wavelength, and consequently increasing the size of the ambiguity search box, while maintaining the integer nature of the ambiguity. In the future, the GPS modernization scheme will include a third frequency, L5. This will enable a Three Frequency Ambiguity Resolution (TCAR) solution that will eliminate the ionosphere term. [Hofmann-Wellenhof, 2001]

The following discussion delves deeper into the makeup of the ionosphere and how it interacts with radio wave propagation. The ionosphere is a region of the atmosphere extending from approximately 80 to 1000 km. The major species found in
the ionosphere are oxygen, nitrogen, helium and hydrogen. Trace elements of ozone, nitric oxide, carbon dioxide, argon and water are also found.

High-energy photos from solar radiation collide with molecules in the atmosphere causing ionization and creating plasma. The amount of ionization depends mostly on the sun and its activity, with lesser contributions from cosmic rays. Variations in ionizing energy result from:

- Daily rising and setting of the sun
- Seasonal, with less radiation in the winter
- Solar Activity: sunspots (11 year cycle), solar flares and CMEs.
- Solar winds

Free-electron concentration varies vertically and horizontally. There are concentration spikes at the top of the ionosphere, where the ionosphere and magnetosphere meet. Interactions between electrons and protons traveling in the magnetosphere cause further ionization when they make contact with the atmosphere, causing ionospheric convection (horizontal variations as a result of vertical location). Horizontally, the highest concentrations are found near the equator because this is the region where solar radiation is most intense. There are also higher concentrations at the poles where ionizing particles contained in the solar winds travel along the Earth’s magnetic field. Vertical variations in concentration are not consistent from the lower ionosphere to the upper ionosphere. The ionosphere is divided into regions labeled D, E and F. Concentrations within each region vary, and the transition from region to region is continuous. The variation between regions is caused by; different types of gasses ionized, different degree of radiation absorption and different paths generated by the electromagnetic field [Leick, 2004].

The degree of ionization within this region is subject to the availability and energy of the radiation, and the density of the gas molecules. At high altitudes, high-energy ionizing solar radiation is abundant and the gas density is low, making recombination difficult. However, because of the low density, there are fewer molecules
to ionize, consequently ionization is small. As the altitude decreases, the number of available molecules increases, resulting in an increase in the ionization process. As the gas density increases further with lower altitudes, the molecules recombine more quickly leading to a decrease in ionization. At even lower altitudes the heavier molecules require more energy for ionization and once ionized, they tend to recombine very quickly due to their close proximity, also the energy available for ionization decreases because of absorption in the higher altitudes. This process leads to several highs and lows in the ionization process and combined with variations in atmospheric composition, leads to the different ionosphere layers (D, E and F) [HAARP, 2006].

The F layer is usually divided into two layers, F1 and F2, in the presence of solar radiation. F2 is the outermost layer of the ionosphere and it covers from approximately 180 km to the magnetosphere. It contains the highest concentration of free electrons, which is at a maximum between 200 km and 400 km. This layer is characterized by a decreasing (with altitude) electron production rate and a slow recombination rate, which leads to an electron density maximum. This layer continues through the night, but at a reduced intensity, due to the presence of ionizing protons from the magnetosphere. A seasonal anomaly affecting the northern hemisphere is known as the winter anomaly. The ionization rate is higher in the summer due to greater amounts of radiation; however, due to seasonal changes in molecular content, the recombination rate is even higher. This leads to a lower summer electron concentration [Hargreaves, 1992].

The F1 layer extends from approximately 120 km to 180 km and peaks at approximately 160 km. Radiation with wavelengths between 20 nm and 90 nm are absorbed during the ionization process. Ionization is higher than in F2, but so is recombination, leading to a lower electron density overall. The F1 layer merges with F2 at night [Hargreaves, 1992].

The E layer extends from approximately 90 km to 120 km and has its maximum at around 100 km. This layer almost disappears at night because the energy source is
removed. The maximum rises in altitude at night because recombination is faster at lower altitude and slower at higher altitude. [Hargreaves, 1992].

The D layer extends from approximately 50 km to 90 km from the Earth’s surface. Ionization is low in the region because of rapid recombination due to high density. This layer almost disappears at night and any remaining activity is due to cosmic radiation [Hargreaves, 1992].

Summary of Ionization Energy Sources:

- Solar Radiation: Primary source of energy for all ionosphere layers and affecting mainly mid to low latitudes
- Sun Spots: Source of geomagnetic storms causing ionosphere perturbations and high-energy radiation increasing ionization activity
- Solar Flares: Source of geomagnetic storms and ionizing particles. Solar flare ionization affects mainly high latitudes
- Solar Wind: Source of ionization at the magnetopause and at high latitudes where particles follow the Earth’s magnetic field into the atmosphere at the poles.
- Cosmic Radiation: Small source of radiation when compared to solar sources; however, in the absence of solar radiation (at night) it becomes the dominant source that maintains the ionosphere.

Electromagnetic waves travel as both waves and particles. The particles travel as part of a group of waves that make up the entire wave package. All of the waves in a group combine to create the overall group wave. The waves interact with each other constructively and destructively to produce the resulting waveform (envelope), which has a “beat” frequency resulting from that interaction (see Figure VII-1). If the contributing waves are traveling at the same phase velocity, then the group wave envelope will also travel at that speed (group velocity). In this case the group waveform envelope will be frozen relative to the individual waves. If the individual phase speeds are different, then the group velocity will also be different, and the group waveform envelope will oscillate. This occurs when the waves travel through a dispersive medium.
The following discussion delves into the process of determining phase and group velocities for the ionosphere. This section draws heavily on Hargreaves (1992), pp 14-26 and Leick (2004), pp 214-218.

The phase velocity of a wave is given by:

\[ v_p = \frac{\omega_p}{k_p} \text{ (m/s)} \]  
Eq. 1

Where \( \omega_p \) (rad/s) is the wave’s angular frequency (=2\( \pi f \)) and \( k_p \) is the wave number.

Phase velocity is also given by:

\[ v_p = \frac{c}{n_p} \text{ (m/s)} \]  
Eq. 2

Where \( c \) is the speed of light and \( n_p \) is the refractive index of the phase in the medium.

The group velocity is given by

\[ v_g = \frac{d\omega}{dk} \text{ (m/s)} \]  
Eq. 3

and
\[ v_g = \frac{c}{n_g} \quad \text{(m/s)} \quad \text{Eq. 4} \]

The group refractive index:
\[ n_g = c \frac{dk}{d\omega} \quad \text{(unitless)} \quad \text{Eq. 5} \]

Using another form of \( k \):
\[ k_p = \frac{\omega_p}{v_p} = \frac{\omega_p n_p}{c} \quad \text{(m}^{-1}) \quad \text{Eq. 6} \]

The group refractive index can be expressed in terms of the phase frequency:
\[ n_g = c \frac{d(\omega_p n_p)}{d\omega} = \frac{d(\omega_p n_p)}{d\omega} = \frac{d(f_p n_p)}{df} \quad \text{Eq. 7} \]

If the group and phase refractive indices are the same, then the velocities are the same and the medium is considered to be non-dispersive at that frequency range. If the indices are different, then the medium is dispersive, as is the case with the ionosphere.

The key to determining the wave propagation speed in the ionosphere is the determination of the refractive index. Generally, the refractive index is expressed as a complex number \( n = \mu - j\chi \), however, when collisions are infrequent (little absorption) only the real portion applies \([\text{Hargreaves, 1992, p. 26}]. \) The Appleton Equation can be used to determine the refractive index of a medium:
\[ n^2 = 1 - \frac{X}{1 - jZ - \left[ \frac{Y_f^2}{2(1 - X - jZ)} \right] \pm \left[ \frac{Y_f^4}{4(1 - X - jZ)^2 + Y_L^2} \right]^{\frac{1}{2}}} \quad \text{Eq. 8} \]
Where:

\[ X = \omega_n^2 / \omega^2 \] Angular plasma frequency \((\omega_n = 2\pi f_N)\) component

\[ Y = \omega_g / \omega \] Angular electron gyro frequency \((\omega_g)\) component

\[ Y_l = \omega_l / \omega \] Longitudinal component of \(\omega_B\)

\[ Y_t = \omega_t / \omega \] Transverse component of \(\omega_B\)

\[ Z = v / \omega \] Electron collision frequency \((v)\) component

[Taken from Hargreaves, 1992, p. 25]

The plasma frequency \(f_N\) is “the natural electron oscillation frequency for electron perturbations within the plasma” [Hargreaves, 1992, p. 17]. It is a function of electron density and permittivity and can be approximated by:

\[ f_N = (80.5N)^{1/2} \text{ (Hz)} \] Eq. 9

Where \(N\) is the electron density per m\(^3\). In the ionosphere it is this plasma frequency that has the greatest influence over the propagation of the GPS signals.

The gyro frequency of the ionosphere is a function of the magnetic field and the electron particle mass:

\[ \omega_g = Be / m \text{ (rad/s)} \] Eq. 10

Where:

- \(B\) is the magnetic flux strength, in Tesla (kg/(A s\(^2\)))
- \(e\) \(1.6 \times 10^{-19}\) C is the electron particle charge, in coulomb (Amp S)
- \(m\) \(9.1 \times 10^{-31}\) kg is the mass of an electron

If the magnetic flux \((B)\) were estimated to be \(5 \times 10^{-5}\) (kg/A s\(^2\)) \{it is at a maximum of \(3.5 \times 10^{-5}\) (kg/A s\(^2\)) on the Earth at the magnetic equator\}, the angular gyro
frequency would be $8.8 \times 10^6$. In this case, $Y$ of equation 8 becomes $9.3 \times 10^{-4}$ for the GPS frequencies ($-9.4 \times 10^9$ rad/s) and can be ignored as an initial approximation.

The electron collision frequency ($\nu$), which is an estimate of absorption, is:

$$\nu = \sqrt{\frac{3kT}{m}}$$  \hspace{1cm} \text{Eq. 11}$$

Where:

- $k = 1.381 \times 10^{-23}$ (J/°K) (Boltzmann’s constant)
- $T$ Temperature in Kelvin
- $m = 9.1 \times 10^{-31}$ kg is the mass of an electron

For a temperature of 1500 °K (nominal maximum temperature in outer ionosphere), $\nu = 2.6 \times 10^5$, and $Z$ of equation 8 becomes $2.8 \times 10^{-5}$ for the GPS frequencies and can be ignored as an initial approximation.

Ignoring magnetic effects and absorption, when dealing with the propagation of GPS signals through the ionosphere, equation 8 (*Appleton Equation*) can be greatly simplified to:

$$n_p^2 = 1 - \frac{f_p^2}{f_r^2}$$  \hspace{1cm} \text{Eq. 12}$$

To determine the group refractive index ($n_g$) of equation 7, equation 12 can be rearranged to find $f_p n_p$:

$$n_p^2 = \frac{1}{f_p^2} \left( f_p^2 - f_r^2 \right)$$

$$n_p = \frac{1}{f_p} \left( f_p^2 - f_r^2 \right)^{1/2}$$

$$n_p f_p = \left( f_p^2 - f_r^2 \right)^{1/2}$$
Differentiating and squaring:
\[
\frac{d(n_p f_p)}{df_p} = \frac{f_p}{\left(f_p^2 - f_N^2\right)^{\frac{1}{2}}} = \frac{1}{\left(1 - \frac{f_N^2}{f_p^2}\right)^{\frac{1}{2}}} = n_g
\]

\[
n_g^2 = \frac{1}{1 - \frac{f_N^2}{f_p^2}}
\]

Considering the electron density reaches a maximum of approximately 10^6/m^3 at an approximate altitude of 300 km, \(f_N \sim 9\) KHz. For the GPS L1 frequency of 1.5 GHz (~L1):
\[
\frac{f_N^2}{f_p^2} = \frac{(9 \times 10^3)^2}{(1.5 \times 10^9)^2} = 3.6 \times 10^{-11}
\]

Given that \(\frac{f_N^2}{f_p^2}\) is extremely small compared to 1:
\[
\frac{1}{1 - \frac{f_N^2}{f_p^2}} \text{ can be approximated by } 1 + \frac{f_N^2}{f_p^2} = n_g^2
\]

By the same principle the phase velocity can be approximated by:
\[

v_p = \frac{c}{n_p} = \frac{c}{\left(1 - \frac{f_N^2}{f_p^2}\right)^{\frac{1}{2}}} = c\left(1 + \frac{f_N^2}{f_p^2}\right)^{\frac{1}{2}}
\]

Eq. 13

And the group velocity can be approximated by:
\[ v_g = \frac{c}{n_g} = \frac{c}{\left(1 + \frac{f_N^2}{f_p^2}\right)^{1/2}} = c\left(1 - \frac{f_N^2}{f_p^2}\right)^{1/2} \]  

Equation 14

Equation 12 indicates that the refractive index for the ionosphere is less than one, meaning that the velocity of propagation is greater than the speed of light. This does not go against the theory of relativity because no information or matter is transferred by the individual waves. Information is conveyed only by the group wave, and the group index of refraction is greater than one. Equations 13 and 14 show that the phase velocity increases and the group velocity slows, by an equivalent amount. Thus the GPS signal time perturbations of carrier advance and code delay.
APPENDIX B

LAMBDA

LAMBDA (least-squares ambiguity decorrelation adjustment method) is an integer least-squares estimator used to determine the most probable integer representation of a set of real values. In GPS processing, LAMBDA uses real-value ambiguity estimates, and their corresponding variance covariance (VCV) matrix, to establish integer ambiguity estimates.

The inputs to LAMBDA are simply the real (float) ambiguity values (in cycles) computed from a float solution, and the corresponding VCV matrix. From this information the LAMBDA process decorrelates the covariance matrix and the real ambiguity values, then estimates the integer values. The integer ambiguities are then used in another GPS process to estimate an "ambiguity fixed" solution. An overview of the processing steps is as follows [Joosten (1996) and de Jong (2003)]:

1. Estimate real-value ambiguities ($\tilde{N}$) and corresponding VCV matrix ($Q_\tilde{N}$) using GPS position solution algorithms.
2. Separate the integer and decimal portions of the real ambiguities and send the decimal portion through the LAMBDA process.
3. Decorrelate the float ambiguities ($\tilde{N}'$) and the VCV matrix ($Q_\tilde{N}'$) using a transformation matrix ($Z$). Gauss (LDL) decomposition is used to create a transformation matrix that makes $Q_\tilde{N}'$ as diagonal as possible (off diagonals going to zero), while preserving the search space volume (determinants are the same $|Q_\tilde{N}'| = |Q_\tilde{N}'| = \pm 1$).
4. Compute a suitable multiplier ($\chi^2$) to the size of the search region such that it is large enough to contain the correct integer ambiguity and small enough for fast solution computations using $D$ and $L$ from above and the decorrelated real-value ambiguities ($\tilde{N}'$).
5. Perform the actual search using the decorrelated real-value ambiguities ($\tilde{N}'$), L, D and size of the search region to get the integer ambiguity estimates ($N'$).
6. Transform the resulting integer ambiguity estimates ($N'$) by the inverse of the transformation matrix ($Z'$) to get the final integer estimates ($N$).
7. Recombine the integer ambiguity estimates with the original integer portion of the ambiguity from step 2.
8. The fixed integer ambiguity values are then input into another GPS processing algorithm to determine the “fixed” position solution.

The following discussion will delve into the LAMBDA process in greater detail. A sample data set will be used to help describe the processes involved. The sample dataset is from a 26 km static baseline in southeast Louisiana (CORS NOLA to DSTR) observed between 0500 and 0800 UTC on Friday, June 29 (DOY 180), 2006. Data were collected at 1 Hz. A specific sample was taken from epoch 367560 seconds-of-week (102.1000 hours-of-week, 6.1000 hours-of-day, 21960 seconds-of-day), using eight satellites in the solution.

B.1 Real-Value Ambiguity Estimates

The real-value ambiguity estimates were determined from the USM OTF float, four-observation solution. This solution used CA and P2 code (pseudo-range) observations with L1 and L2 carrier observation in a single-baseline, double differenced, sequential-least-squares process to estimate the three-dimensional position vector of the unknown point and the real-value L1 and L2 double differenced ambiguities.

As part of the sequential-least-squares process, the VCV matrix of unknowns from the solution was passed to the next epoch for use in the subsequent solution. The ambiguity portion of that matrix was also passed to the LAMBDA process. In matrix notation:

\[
Y = \begin{bmatrix} \hat{X} \\ \hat{N} \end{bmatrix}
\]

\[
Q = \begin{bmatrix} Q_{\hat{X}\hat{X}} & Q_{\hat{X}\hat{N}} \\ Q_{\hat{N}\hat{X}} & Q_{\hat{N}\hat{N}} \end{bmatrix}
\]

Where:

- \( Y \) is the vector of estimates of unknowns of length \( m \)
- \( \hat{X} \) is the vector of position estimates of length 3
\( \hat{N} \) is the vector of double-differenced real-ambiguity estimates of length \( n \)

\( n \) is the number of double differenced ambiguities = \( 2 \times \text{(numbers of satellites-1)} \)

\( m \) is the total number of unknowns in the solution = \( 3 + n \)

\( Q \) is the \( m \times m \) VCV matrix of the estimates of unknowns

\( Q_{\hat{x}} \) is the \( 3 \times 3 \) VCV matrix of position estimates

\( Q_{\hat{\xi}} \) is the \( n \times n \) VCV matrix of double-differenced ambiguity estimates

\( Q_{\hat{\delta}\hat{\xi}} \) is the \( 3 \times n \) ambiguity and position covariance matrix

\( Q_{\hat{\delta}\hat{\xi}} \) is the \( n \times n \) position and ambiguity covariance matrix, and is the transpose of \( Q_{\hat{\delta}\hat{\xi}} \)

The vector of double-difference ambiguities \( \hat{N} \) and the associated VCV matrix \( Q_{\hat{N}} \) were passed to LAMBDA for the ambiguity integer estimation process.

**B.2 Decorrelation**

The first step in the LAMBDA process is the decorrelation of the VCV matrix and real ambiguities. The subsequent search processes will perform correctly without prior decorrelation; however, the processes will take much longer. Essentially, the decorrelation process reduces the search size, thereby reducing the computation time.

The aim of the decorrelation process is to create a transformation matrix such that:

1. The elements of the transformation matrix are integer
2. The volume is preserved
3. The correlation is reduced after transformation (Hofmann-Wellenhof, 2001; Teunissen, 1995).

The first step in the decorrelation process is to create the transformation matrix such that:

\[
\hat{N}' = Z^T \hat{N} \quad \text{(real-value ambiguity transformation)}
\]

\[
N' = Z^T N \quad \text{(integer ambiguity transformation)}
\]

\[
Q'_{\hat{\xi}} = Z^T Q_{\hat{\xi}} Z \quad \text{(VCV transformation)}
\]
The $Z$ transformation matrix is determined through the $LDL$ Gauss decomposition of $Q_\tilde{N}$ such that

$$Q_\tilde{N} = L^TDL$$

$$L = \begin{bmatrix}
1 & 0 & \ldots & 0 \\
I_{2,1} & \ldots & \ldots & \ldots \\
\ldots & \ldots & 1 & 0 \\
I_{n,1} & \ldots & I_{n,n-1} & 1
\end{bmatrix}$$

$$l_{i,j} = \frac{\sigma_{\tilde{N},i}}{\sigma_{\tilde{N},j}}$$

Where $i = 2 \ldots n$, and $j = 1 \ldots n-1$

$$D = Q_\tilde{N} = \begin{bmatrix}
\sigma_{\tilde{N},1}^2 & 0 & \ldots & 0 \\
0 & \sigma_{\tilde{N},2}^2 & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots \\
0 & 0 & \ldots & \sigma_{\tilde{N},n}^2
\end{bmatrix}$$

= conditional VCV

Where: $L$ is a lower triangular matrix, with diagonals set to 1, and $D$ is a diagonal matrix [de Jong, 2003].

Creation of the final $Z$ transformation matrix is an iterative process that involves the creation of a series of intermediate transformation matrices ($Z_t$), which are created from integer versions of the $L$ matrix, where $l_{i,j}$ are converted to integers. During the decorrelation process, a $Z_t$ matrix is determined from the current VCV. Transforming the original VCV using the $Z_t$ matrix creates a new VCV matrix. A new $Z_t$ matrix is determined from the current VCV, which is then used to transform the current VCV, and so on until $Z_t$ becomes a unit matrix. The final transformation matrix ($Z$) is created by combining all of the previous $Z_t$ matrices [de Jong, 2003].

The following discussion on the concepts of the decorrelation process uses two ambiguity estimations as an example (two-dimensions). The same concepts apply to the actual multi-dimensional case. The VCV matrix gives an estimate of the precision (variance) of the ambiguity value (matrix diagonals) as well as an estimate of the
correlation between ambiguities (matrix off-diagonals). For a pair of real-value ambiguities the VCV takes the form of:

\[ Q_{\tilde{N}} = \begin{bmatrix} \sigma_{\tilde{N}_i}^2 & \sigma_{\tilde{N}_{i1}} \\ \sigma_{\tilde{N}_{i1}} & \sigma_{\tilde{N}_{11}}^2 \end{bmatrix} \]

Where

- \( \sigma_{\tilde{N}_i}^2 \) represents the variance for float ambiguity \( i \)
- \( \sigma_{\tilde{N}_{ij}} \) represents the correlation between float ambiguities \( i \) and \( j \)
- \( \sigma_{\tilde{N}_{ij}} = \sigma_{\tilde{N}_{ji}} \), the matrix is symmetric

The variances and covariances can be combined to give an estimate of system uncertainty; which, in the two-dimensional case, is an ellipse centered on the two ambiguity values. The ellipse describes an area within which the true ambiguity values lie, at a certain confidence level (~68% for the standard two-dimensional case). The size and orientation of the uncertainty ellipse are determined from:

\[
a^2 = \frac{\left( \sigma_{\tilde{N}_i}^2 + \sigma_{\tilde{N}_1}^2 + \sqrt{\left( \sigma_{\tilde{N}_i}^2 - \sigma_{\tilde{N}_1}^2 \right)^2 + 4\sigma_{\tilde{N}_{i1}}^2} \right)}{2}
\]

\[
b^2 = \frac{\left( \sigma_{\tilde{N}_i}^2 + \sigma_{\tilde{N}_1}^2 - \sqrt{\left( \sigma_{\tilde{N}_i}^2 - \sigma_{\tilde{N}_1}^2 \right)^2 + 4\sigma_{\tilde{N}_{i1}}^2} \right)}{2}
\]

\[
\tan 2\varphi = \frac{2\sigma_{\tilde{N}_{i1}}}{\sigma_{\tilde{N}_i}^2 - \sigma_{\tilde{N}_1}^2}
\]

Where:

- \( a \) is the semi-major axis
- \( b \) is the semi-minor axis
- \( \varphi \) is the rotation angle

[Hofmann-Wellenhof, 2001].
Determination of the uncertainty ellipse can also be thought of as a transformation of the VCV to a diagonal matrix, where the off-diagonals go to zero (no correlation). The "variances" (diagonals) of the transformed matrix are the eigenvalues ($\lambda_i$) of the original matrix.

\[
Q'_N = \begin{bmatrix}
\lambda_1 & 0 \\
0 & \lambda_2
\end{bmatrix} \quad \text{Where} \quad \lambda_1 = a^2 \quad \text{and} \quad \lambda_2 = b^2
\]

With the covariances at zero, the transformed VCV is uncorrelated. This process is known as eigenvalue decomposition and would achieve the desired decorrelation; unfortunately, the transformation matrix used to create the decorrelated matrix must also be used to transform the integer ambiguities in the final step of the LAMBDA process. In order to ensure that the final transformation of ambiguities preserves the integer nature of those ambiguities, the transformation matrix must also be made up of integers, which is why the LAMBDA process uses a modified Gauss LDL decomposition to create the integer transformation matrix ($Z$).

The following example, taken from de Jong, 2003, is used to describe the decorrelation process.

With a variance covariance matrix of:

\[
Q_N = \begin{bmatrix}
4.9718 & 3.8733 \\
3.8733 & 3.0188
\end{bmatrix}
\]

The correlation coefficient ($r$), which is an indicator of the degree to which the two ambiguities are correlated, can be determined from:

\[
r_{12} = \frac{\sigma_N \sigma_N}{\sigma_N \sigma_N} = 99.98\% \text{ correlation for the example}
\]

The final transformation matrix is created from a series of intermediary transformations used to iteratively decorrelate the ambiguities. The transformation of one ambiguity is conditional on the other ambiguities. For a pair of ambiguities the relationship is:
\[ Z^T_1 = \begin{bmatrix} 1 & 0 \\ \alpha_1 & 1 \end{bmatrix} \quad \alpha_i = -\text{INT}\left[ \frac{\sigma_{N_{i2}}}{\sigma_{N_i}^2} \right] \]

or

\[ Z^T_2 = \begin{bmatrix} 1 & \alpha_2 \\ 0 & 1 \end{bmatrix} \quad \alpha_2 = -\text{INT}\left[ \frac{\sigma_{N_{i2}}}{\sigma_{N_2}^2} \right] \]

Note: INT refers to rounding to the nearest integer.

In the first case the transformation is conditional on \( \hat{N}_1 \) (it is preserved), and in the second case the transformation is conditional on \( \hat{N}_2 \). These relationships will be used alternately through the iterations of the decorrelation process.

For the two-dimensional example the modified (integer) LDL decomposition of the two-dimensional example is performed in three iterations. The first transformation matrix is computed from the original VCV:

\[ Z^T_2 = \begin{bmatrix} 1 & \alpha_2 \\ 0 & 1 \end{bmatrix} \]

\[ \alpha_2 = -\text{INT}[3.8734 - 3.0188] = -\text{INT}[3.8734 - 3.0188] = -1 \]

Note: The variance of \( \hat{N}_2 \) is smaller; therefore it was chosen to be preserved in the first iteration.

\[ Z^T_2 = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \]

Which holds \( \hat{N}_2 \) and adjusts \( \hat{N}_1 \).

A new VCV is computed from the transformation matrix:

\[ Q_{Z_i} = Z_i^T Q_Z Z_i = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4.9718 & 3.8733 \\ 3.8733 & 3.0188 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.2440 & 0.8545 \\ 0.8545 & 3.0188 \end{bmatrix} \]

The second transformation matrix is computed from the new VCV, using the other \( Z^T \):
\[
Z_1^T = \begin{bmatrix} 1 & 0 \\ \alpha_1 & 1 \end{bmatrix}
\]

\[
\alpha_1 = -\text{INT} \left[ \frac{\sigma_{\hat{N}_1}}{\sigma_{\hat{N}_2}} \right] = -\text{INT} \left[ \begin{bmatrix} 0.8545 \\ 0.2440 \end{bmatrix} \right] = -4
\]

\[
Z_1^T = \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix}
\]

Which holds \( \hat{N}_1 \) and adjusts \( \hat{N}_2 \).

A new VCV is computed from the new transformation matrix:

\[
Q_{z_1} = Z_1^T Q_{z_1} Z_2 = \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 0.2440 & 0.8545 \\ 0.8545 & 3.0188 \end{bmatrix} \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.2440 & -0.1215 \\ -0.1215 & 0.0868 \end{bmatrix}
\]

The next transformation matrix is computed from the new VCV and going back to \( \alpha_2 \), similar to the first step.

\[
Z_1^T = \begin{bmatrix} 1 & \alpha_2 \\ 0 & 1 \end{bmatrix} \quad \alpha_2 = -\text{INT} \left[ \frac{\sigma_{\hat{N}_1}}{\sigma_{\hat{N}_2}} \right] = -\text{INT} \left[ \begin{bmatrix} 0.1215 \\ 0.0868 \end{bmatrix} \right] = 1
\]

\[
Q_{z_2} = Z_1^T Q_{z_2} Z_3 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.2440 & -0.1215 \\ -0.1215 & 0.0868 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.0878 & -0.0347 \\ -0.0347 & 0.0868 \end{bmatrix}
\]

Which holds \( \hat{N}_2 \) and adjusts \( \hat{N}_1 \).

In this example the transformation \( Z_1 \) transforms \( \hat{N}_1 \), with respect to \( \hat{N}_2 \), which remain the same. In the second iteration \( (Z_2) \), \( \hat{N}_2 \) is transformed with respect to \( \hat{N}_1 \), which remains the same, and so on. Combining \( Z_1, Z_2 \) and \( Z_3 \) gives the transformation for the entire two-dimensional system. [Hofmann-Wellenhof, 2001; de Jon, 2003].

\[
Z^T = Z_3^T Z_2^T Z_1^T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ -4 & 5 \end{bmatrix}
\]
The final transformed VCV is

$$Q'_N = \begin{bmatrix} 0.0878 & -0.0347 \\ -0.0347 & 0.0868 \end{bmatrix}$$

The correlation coefficient for the decorrelated VCV is 39.75%.

Plots of the uncertainty ellipses for the pre and post decorrelation are shown in Figure VII-2, which assumes that the ambiguity estimates are "0". Each of the iterations can be thought of as a single step in the reduction of the search grid. The first iteration leaves the bounds of $\hat{N}_2$ and squeezes the bounds of $\hat{N}_1$. The second iteration leaves the bounds of $\hat{N}_1$ and squeezes the bounds of $\hat{N}_2$, and so on, reducing the final search area from the region bounding the red solid ellipse, to the region bounding the blue dashed ellipse of Figure VII-2.

![Figure VII-2: Pre and Post decorrelation uncertainty ellipses.](image)

If decorrelation had not taken place, the search area would encompass all integer pairs in the plot ($N_1$ from -3 to 3, and $N_2$ from -2 to 2), for a total of 35 possibilities.
After decorrelation, the search region reduces to $-1$ to $1$ in both directions, for a total of $9$ possibilities.

The following $n$-dimensional example was taken from epoch 367560 of the sample data set. Of the eight satellites used in the sample solution, one was used as a reference, leaving seven double difference sets. There were two ambiguities per double difference set ($L_1$ and $L_2$) for a total of fourteen double difference ambiguities ($n$).

The float ambiguities ($\hat{N}$) for $\hat{L}_1$ and $\hat{L}_2$ double differenced estimates are shown in Table VII-1.

| $\hat{N}$ in cycles |\hat{L}_1 | 40156.68  \\
| | $\hat{L}_2$ | 31290.71  \\
| | $\hat{L}_1$ | -33866.92  \\
| | $\hat{L}_2$ | -26389.98  \\
| | $\hat{L}_3$ | 72998.70  \\
| | $\hat{L}_4$ | 56882.79  \\
| | $\hat{L}_5$ | 79858.77  \\
| | $\hat{L}_6$ | 62226.82  \\
| | $\hat{L}_7$ | 80023.76  \\
| | $\hat{L}_8$ | 62355.79  \\
| | $\hat{L}_9$ | 190601.42  \\
| | $\hat{L}_10$ | 148520.58  \\
| | $\hat{L}_11$ | 9020.55  \\
| | $\hat{L}_12$ | 7029.53  |

Table VII-1: Estimate float ambiguities, in cycles ($L_1$ and $L_2$).

The covariance matrix of the float ambiguities ($Q_{\hat{N}}$), is given by
Where

\[ \sigma_{N_i}^2 \] represents the variance for float ambiguity \( i \)

\[ \sigma_{N_j \pm 1} \] represents the correlation between float ambiguities \( i \) and \( i+1 \)

\( i \) goes from 1 to \( n-1 \).

The covariance matrix for the sample set is shown in Table VII-2:

| \( \begin{array}{cccccccccccc} 
0.42 & 0.34 & 0.06 & 0.02 & 0.29 & 0.27 & 0.20 & 0.17 & -0.05 & -0.08 & -0.21 & -0.25 & 0.46 & 0.43 \\
0.34 & 0.42 & 0.02 & 0.06 & 0.27 & 0.29 & 0.17 & 0.20 & -0.08 & -0.05 & -0.25 & -0.21 & 0.43 & 0.46 \\
0.06 & 0.02 & 1.50 & 1.12 & -0.27 & -0.30 & -0.42 & -0.45 & -0.64 & -0.67 & -0.80 & -0.83 & 0.61 & 0.58 \\
0.02 & 0.06 & 1.12 & 1.50 & -0.30 & -0.27 & -0.45 & -0.42 & -0.67 & -0.64 & -0.83 & -0.80 & 0.58 & 0.61 \\
0.29 & 0.27 & -0.27 & -0.30 & 0.46 & 0.38 & 0.34 & 0.32 & 0.28 & 0.25 & 0.39 & 0.36 & 0.23 & 0.20 \\
0.27 & 0.29 & -0.30 & -0.27 & 0.38 & 0.46 & 0.32 & 0.34 & 0.25 & 0.28 & 0.36 & 0.39 & 0.20 & 0.23 \\
0.20 & 0.17 & -0.42 & -0.45 & 0.34 & 0.32 & 0.37 & 0.31 & 0.35 & 0.32 & 0.43 & 0.40 & 0.03 & 0.00 \\
0.17 & 0.20 & -0.45 & -0.42 & 0.32 & 0.34 & 0.31 & 0.37 & 0.32 & 0.35 & 0.40 & 0.43 & 0.00 & 0.03 \\
-0.05 & -0.08 & -0.64 & -0.67 & 0.28 & 0.25 & 0.35 & 0.32 & 0.68 & 0.62 & 1.10 & 1.07 & -0.36 & -0.39 \\
-0.08 & -0.05 & -0.64 & -0.67 & 0.28 & 0.25 & 0.32 & 0.35 & 0.62 & 0.68 & 1.07 & 1.10 & -0.39 & -0.36 \\
-0.21 & -0.25 & -0.80 & -0.83 & 0.39 & 0.36 & 0.43 & 0.40 & 1.10 & 1.07 & 2.37 & 2.19 & -0.59 & -0.62 \\
-0.25 & -0.21 & -0.83 & -0.80 & 0.36 & 0.39 & 0.40 & 0.43 & 1.07 & 1.10 & 2.19 & 2.37 & -0.62 & -0.59 \\
0.46 & 0.43 & 0.61 & 0.58 & 0.23 & 0.20 & 0.03 & 0.00 & -0.36 & -0.39 & -0.59 & -0.62 & 1.20 & 0.84 \\
0.43 & 0.46 & 0.58 & 0.61 & 0.20 & 0.23 & 0.00 & 0.03 & -0.39 & -0.36 & -0.62 & -0.59 & 0.64 & 1.20 \\
\end{array} \right|

Table VII-2: \( Q_X \) from ambiguity-fixed solution, for epoch 367560, divided by mean variance.

The ambiguity system correlation can be estimated from the variance covariance \( Q_X \) by averaging the correlation coefficients for each ambiguity pair. The correlation coefficient \( r \) for each pair can be computed from:

\[ r_{ij} = \frac{\sigma_{N_{ij}}}{\sigma_{N_i} \sigma_{N_j}} \]

For all combinations of \( i \) and \( j = 1, 2, \ldots n-1 \), and there are

\[ nr = \sum_{i=1}^{n-1} n - i \]

combinations of ambiguity pairs. The correlation coefficient \( r \) goes from
0 to 1 and indicates the correlation between ambiguities as a percentage. For the sample set, the fourteen ambiguities lead to 91 combinations with an estimated system correlation of 46.95%. After decorrelation (shown in Table VII-3), the system correlation goes down to 16.43%. Figure VII-3 shows a bar graph of all 91 correlation coefficients before decorrelation and Figure VII-4 shows them after decorrelation.

| 0.69 | 0.29 | 0.16 | 0.15 | 0.14 | 0.00 | 0.09 | 0.06 | -0.03 | 0.05 | 0.03 | 0.00 | 0.00 | 0.00 |
| 0.29 | 0.69 | 0.15 | 0.16 | 0.13 | 0.00 | 0.04 | 0.01 | -0.08 | -0.05 | 0.03 | 0.00 | 0.00 | 0.00 |
| 0.16 | 0.15 | 0.61 | 0.15 | 0.09 | 0.00 | -0.07 | 0.13 | 0.02 | 0.00 | 0.02 | 0.06 | 0.03 | -0.03 |
| 0.15 | 0.16 | 0.15 | 0.61 | 0.03 | 0.00 | -0.01 | 0.07 | 0.05 | 0.00 | 0.08 | -0.06 | -0.03 | 0.03 |
| 0.14 | 0.13 | 0.09 | 0.03 | 0.46 | -0.15 | 0.06 | 0.06 | -0.05 | 0.00 | -0.06 | 0.06 | 0.00 | 0.00 |
| 0.00 | 0.00 | 0.00 | 0.00 | -0.15 | 0.34 | 0.00 | 0.03 | 0.03 | 0.05 | -0.03 | 0.05 | -0.05 | -0.05 |
| 0.09 | 0.04 | -0.07 | -0.01 | 0.06 | 0.00 | 0.29 | 0.11 | 0.00 | 0.05 | 0.02 | -0.06 | 0.00 | 0.00 |
| 0.06 | 0.01 | 0.13 | 0.07 | 0.06 | 0.03 | 0.11 | 0.28 | 0.05 | 0.08 | 0.03 | 0.03 | 0.00 | -0.06 |
| -0.03 | -0.08 | 0.02 | 0.05 | -0.05 | 0.03 | 0.00 | 0.05 | 0.08 | 0.15 | -0.03 | 0.05 | -0.05 | -0.05 |
| -0.05 | -0.05 | 0.00 | 0.00 | 0.00 | 0.05 | 0.05 | 0.08 | 0.08 | 0.15 | -0.03 | 0.05 | -0.05 | -0.05 |
| 0.03 | 0.03 | 0.02 | 0.08 | -0.06 | -0.03 | 0.02 | 0.03 | -0.03 | -0.03 | 0.15 | -0.08 | 0.03 | 0.03 |
| 0.00 | 0.00 | 0.06 | -0.06 | 0.06 | 0.05 | -0.06 | 0.03 | -0.03 | 0.05 | -0.08 | 0.17 | -0.06 | -0.06 |
| 0.00 | 0.00 | 0.03 | -0.03 | 0.00 | -0.05 | 0.00 | 0.00 | -0.03 | -0.05 | 0.03 | -0.06 | 0.11 | 0.05 |
| 0.00 | 0.00 | -0.03 | 0.03 | 0.00 | -0.05 | 0.00 | -0.06 | -0.05 | -0.05 | 0.03 | -0.06 | 0.05 | 0.11 |

Table VII-3: $Q'_N$ from ambiguity-fixed solution, for epoch 637560, after decorrelation.

Notice in Table VII-3 that the off diagonals are greatly reduced. Note that the decorrelation process reorders the VCV by variance, with the higher variances in the upper left, decreasing to the lower right. The final transformation of the integer ambiguities by the inverse of the decorrelation matrix ($Z^{-1}$) returns the values to the original order.
Figure VII-3: Correlation coefficients for float ambiguity pairs, prior to decorrelation.

Figure VII-4: Correlation coefficients for float ambiguities after decorrelation.
Table VII-4 and Table VII-5 show the final $L$ and $D$ matrices after decorrelation, and Table VII-6 shows the final transformation matrix $Z$.

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Table VII-4: $L$ matrix for sample.

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Table VII-5: Diagonal portion of $D$ matrix. Off diagonals are zero.

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Table VII-6: Final Z transformation matrix for the sample.
B.3 Integer Ambiguity Search

LAMBDA uses Integer Least-squares (ILS) to estimate the integer ambiguities. In the ILS process the weighted squared residual differences between candidate ambiguity integers and their corresponding real values, are minimized. The inverse of the decorrelated VCV matrix ($Q^{-1}$) is used for weighting and the decorrelated real-value ambiguities ($\hat{N}'$) are differenced from candidate integers ($N'$). The integer candidates are derived from the search volume. This process can be represented by:

$$\left( \hat{N}' - N' \right)^T Q^{-1} \left( \hat{N}' - N' \right) = \text{minimum}$$  \hspace{1cm} (1)

In short, the search process evaluates all combinations of integer ambiguity representations of the real-valued ambiguities. The combination that leads to a minimum for equation (1) represents the set of most likely candidates. The search process used in LAMBDA attempts to reduce the size of the search volume in order to reduce the number of possible candidate sets. The first step in search volume reduction is the decorrelation process discussed previously. The next step is to determine a search volume size that is large enough to make sure it includes the correct integer ambiguity set, and small enough for the search process to be conducted efficiently. A multiplier ($\chi^2$) is applied to the search volume for this purpose. In this description the term “volume” is used to describe the search space, which would indicate three-dimensions, when in fact the search space is $n$-dimensional; $n$ being the number of double difference ambiguities.

With the search space multiplier, the least-squares process can be expressed as:

$$\left( \hat{N}' - N' \right)^T Q^{-1} \left( \hat{N}' - N' \right) \leq \chi^2$$

The actual search space can be defined by:
\[
\left( \hat{N}_i - N_i \right)^2 \leq \sigma^2_{\hat{N}_i} \chi^2 \quad i = 1...n
\]

Where \( \sigma^2_{\hat{N}_i} \) is the variance of the real-value ambiguity, after decorrelation (see Figure VII-5). Integer ambiguity candidates can be found anywhere within the bounds of the search box, and not necessarily within the ellipse itself. [de Jong, 2003]

Figure VII-5: Two-dimension search space [after de Jong, 2003].

The decorrelation process reduces, but does not eliminate, the correlation between ambiguity estimates. As a result, the integer estimate of one real-value ambiguity is conditional on the other real-value ambiguity values. Also, the search is performed sequentially, looking at pairs of ambiguities. Consequently, the process of adjusting the estimated integer ambiguities is known as a sequential conditional least-squares adjustment [de Jonge & Tiberius, 1996]. The search process uses the \( L \) and \( D \) matrices from the decorrelation process as well as the transformed real-value ambiguities.

In the LAMBDA process, \( \chi^2 \) is estimated from:

\[
\chi^2 = \left( \hat{N}' - N' \right)^T L' DL^{-1} \left( \hat{N}' - N' \right)
\]
Where the integer ambiguity values \( N' \) are estimated by conditionally rounding the real-values (using \( L \)) and \( L \) and \( D \) are passed from the decorrelation process.

For the two-dimensional example:

\[
\begin{bmatrix}
1000.3 \\
1002.9
\end{bmatrix} \quad \text{(selected arbitrarily)}
\]

\[
\begin{bmatrix}
2.7 \\
3.3
\end{bmatrix} \quad \text{(after decorrelation)}
\]

\[
\begin{bmatrix}
3 \\
3
\end{bmatrix} \quad \text{(estimated for } \chi^2 \text{ determination)}
\]

\[
\begin{bmatrix}
1.0 & 0 \\
-0.3998 & 1.0
\end{bmatrix} \quad \text{(from decorrelation)}
\]

\[
\begin{bmatrix}
0.0739 & 0 \\
0 & 0.0868
\end{bmatrix} \quad \text{(from decorrelation)}
\]

\[\chi^2 = 8.03\]

The results of the search process are:

\[
\begin{bmatrix}
3 \\
3
\end{bmatrix}
\]

With the final integer ambiguities:

\[
\begin{bmatrix}
1003 \\
1005
\end{bmatrix}
\]

Table VII-7 shows the ambiguity values at several stages of the process for the GPS sample set, in decorrelation space (transformed). It shows the real-valued ambiguities, the conditional ambiguities used to compute \( \chi^2 (=2.75) \), and the final values from the search. Table VII-8 shows the ambiguity values prior to and after the LAMBDA process.
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
\(N^*\) & \(N_c^*\) (for \(\chi^2\)) & \(N^*\) (from search) \\
\hline
-0.1332 & 0 & 0 \\
0.0023 & 0 & 0 \\
0.4646 & 1 & 1 \\
0.6527 & 1 & 1 \\
-0.2272 & 0 & 0 \\
-0.1451 & 0 & 0 \\
0.0327 & 0 & 0 \\
0.5804 & 1 & 1 \\
0.0088 & 0 & 0 \\
0.0247 & 0 & 0 \\
-0.0371 & 0 & 0 \\
-0.0448 & 0 & 0 \\
-0.0704 & 0 & 0 \\
-0.0079 & 0 & 0 \\
\hline
\end{tabular}
\caption{Table VII-7: Sample data set ambiguity values during process, in decorrelated space (in cycles).}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
\(\hat{N}\) & \(\hat{N}\) in cycles & \(N\) in cycles \\
\hline
\(\hat{L}_{1_i}\) & 40156.68 & 40157 \\
\(\hat{L}_{2_i}\) & 31290.71 & 31291 \\
\(\hat{L}_{3_i}\) & -33866.92 & -33867 \\
\(\hat{L}_{2_i}\) & -26389.98 & -26390 \\
\(\hat{L}_{1_k}\) & 72998.70 & 72999 \\
\(\hat{L}_{2_k}\) & 56882.79 & 56883 \\
\(\hat{L}_{1_s}\) & 79858.77 & 79859 \\
\(\hat{L}_{2_s}\) & 62226.82 & 62227 \\
\(\hat{L}_{1_s}\) & 80023.76 & 80024 \\
\(\hat{L}_{2_s}\) & 62355.79 & 62356 \\
\(\hat{L}_{1_6}\) & 190601.42 & 190602 \\
\(\hat{L}_{2_6}\) & 148520.58 & 148521 \\
\(\hat{L}_{1_7}\) & 9020.55 & 9021 \\
\(\hat{L}_{2_7}\) & 7029.53 & 7030 \\
\hline
\end{tabular}
\caption{Table VII-8: Sample data set initial and final ambiguity values (in cycles).}
\end{table}
With this GPS example, the real-valued ambiguities produced a position result with a 3d uncertainty of 7 cm (1σ) with respect to the known position. As a result, we would expect there to be very little difference between the real-valued and integer ambiguities, which is the case here. Simple rounding would have achieved the same results. This will not always be the case.
APPENDIX C
ALGORITHMS

The following discussion looks at the algorithms used in the USM_OTF software algorithms. The following subjects are covered, in order of appearance:

2. Klobuchar ionosphere model.
3. Saastamoinen standard troposphere model.
4. Sagnac effect.
5. Least-Squares code point positioning.
7. Least-Squares code and carrier point positioning (four-observation).
8. Least-Squares code and carrier double differencing (four-observation).
9. Ionosphere-free.
10. Sequential Least-squares.
12. Zenith Ionosphere Map Interpolation.

C.1 Carrier Smoothing of Code

Smoothing of the pseudo-ranges with the carrier observations was performed with an algorithm retrieved from the Internet [van Leeuwen, 1997]. The change in the carrier observations from epoch to epoch was used to compute the change in the pseudo-range. The smoothed pseudo-range observation was derived from a weighted combination of the observed change in carrier and observed change in pseudo-range.

\[ p_{ni} = \frac{p_i}{N} + \left( p_{ni-1} + \Phi_i - \Phi_{i-1} \right) \cdot \frac{N - 1}{N} \]

Where:

\[ p_{ni} = \text{smoothed pseudo-range for current epoch (i)} \]
\[ p_i = \text{observed pseudo-range for current epoch (i)} \]
\[ p_{Ni-1} = \text{smoothed pseudo-range from previous epoch (i-1)} \]
\[ N = \text{number of observations used to this point in the filter, to a maximum value (e.g. 100).} \]
\[ \Phi_i = \text{carrier observation (in meters) for current epoch (i)} \]
\[ \Phi_{i-1} = \text{carrier observation (in meters) for previous epoch (i-1)} \]

This algorithm also uses a very rudimentary cycle slip detector. The slip detector compares the change in pseudo-range with the change in carrier observation between epochs and if that difference is greater than a specified value (e.g. 15 m). If a slip is detected the filter is reset (\(N=1\)).

In the current configuration of the software, smoothing is performed prior to the application of ionosphere corrections. Because the ionosphere has the opposite effect on pseudo-range observations than carrier observations, the observation differences will diverge, thus reducing the effectiveness of the smoothing. This effect will be amplified as the time between epochs increases. This smoothing algorithm should only be used with one-second data.

C.2 Klobuchar

The ionosphere correction model employed in this software was taken directly from the GPS-ICD-200 document [Navtech (1995), pp. 107 to 108b]. This algorithm uses four alpha (\(\alpha\)) and four beta (\(\beta\)) parameters that are broadcast in GPS navigation message and available in the header portion of the RINEX navigation file. This software currently looks for a separate ASCII text file containing the parameters. The following algorithms are reproduced from Navtech (1995), pp. 107 to 108b.

\[ L_{1_{(m)}} = F \times \left[ 5.0 \times 10^{-9} + AMP \left( 1 - \frac{x^2}{2} + \frac{x^4}{24} \right) \right] (m), \text{ for } |x| < 1.57 \]

Or
\[ L_{1_{(m)}} = F \times (5.0 \times 10^{-9})m, \text{ for } |x| \geq 1.57(m) \]

Where:
\[ AMP = \sum_{n=0}^{3} \alpha_n \Phi^2_m \text{ (sec)}, \text{ if } < 0; \ AMP = 0 \text{ (sec)}. \]

\[ x = \frac{2\pi(t - 50400)}{PER} \text{ (radians), phase} \]

\[ PER = \sum_{n=0}^{3} \beta_n \Phi^n_m \text{ for } PER \geq 72000 \text{ (sec)} \]

Or;
\[ PER = 72000 \text{ (sec)}, \text{ for } PER \geq 72000 \text{ (sec)} \]

\[ F = 1.0 + 16.0 \cdot (0.53 - E)^3, \text{ obliquity factor} \]

Four (0 through 3) \( \alpha \) and \( \beta \) parameters are transmitted in the navigation message
\[ \Phi_m = \Phi_i + 0.064 \cdot \cos(A_i - 1.617), \text{ (semi-circles)} \]
\[ \Lambda_i = \Lambda_u + \frac{\psi \cos A}{\cos \Phi_i}, \text{ (semi-circles)} \]
\[ \Phi_i = \Phi_u + \psi \cos A, \text{ (semi-circles), for } |\Phi_i| \leq 0.416 \]
Or
\[ \Phi_i = 0.416, \text{ for } \Phi_i > 0.416 \]
Or
\[ \Phi_i = -0.416, \text{ for } \Phi_i < 0.416 \]
\[ \psi = \frac{0.0137}{E + 0.11} - 0.022, \text{ (semi-circles)} \]
\[ t = 4.32 \cdot 10^4 \cdot \Lambda_i + \text{GPS time, (sec)} \]

If \( t \geq 86400, t = t - 86400 \)
If \( t < 0, t = t + 86400 \)

\( E \)  
\text{satellite elevation angle}

\( A \)  
\text{Azimuth of satellite, relative to user}

\( \Phi_u \)  
\text{user geodetic latitude (semi-circles)}

\( \Lambda_u \)  
\text{user geodetic longitude (semi-circles)}

\text{GPS time}  
\text{receiver computed system time}
\[ L2_{f(m)} = L1_{f(m)} \cdot \alpha_f \]

\[ \alpha_f = \left( \frac{fL1}{fL2} \right)^2 \]

\[ fL1 = \text{L1 frequency (1575.42 MHz)} \]

\[ fL2 = \text{L2 frequency (1227.60 MHz)} \]

\[ P1_f \text{ (or CA)} = -L1_f \]

\[ P2_f = -L2_f \]

C.3 Saastamoinen

The standard Saastamoinen Total Delay Model was used to account for the
troposphere delay model when the NOAA troposphere model was not used. The model
algorithm was developed from *GPS Theory and Practice* [Hofmann-Wellenhof et al.,
2001, p. 113]. This algorithm takes into account receiver latitude, height, and satellite
elevation.

The algorithm, as presented in Hofmann-Wellenhof et al., [2001], p. 113; is as follows:

\[ Tropo = \frac{0.002277}{\cos(z)} \cdot \left[ p + \left( \frac{1255}{T} + 0.05 \right) e - B \cdot \tan^2 z \right] + \delta R \text{, (meters)} \]

Where:

- \( z \) zenith angle of satellite (90 - elevation angle)
- \( p \) atmospheric pressure at the site (derived from receiver altitude) in
  millibars
- \( T \) Temperature in Kelvin (assumed to be 293.15°)
- \( e \) Partial water vapor pressure in millibars (assumed to be 23 mbar)
- \( B \) is a height correction term derived from a lookup table, in mbar (see Table VII-9).
- \( \delta R \) is a height and elevation correction term from a lookup table, in meters
  (see Table VII-10).
Table VII-9: Saastamoinen $B$ corrector look-up table.

<table>
<thead>
<tr>
<th>Height (km)</th>
<th>$B$ (mBar)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1.156</td>
</tr>
<tr>
<td>0.5</td>
<td>1.079</td>
</tr>
<tr>
<td>1.0</td>
<td>1.006</td>
</tr>
<tr>
<td>1.5</td>
<td>0.938</td>
</tr>
<tr>
<td>2.0</td>
<td>0.874</td>
</tr>
<tr>
<td>2.5</td>
<td>0.813</td>
</tr>
<tr>
<td>3.0</td>
<td>0.757</td>
</tr>
<tr>
<td>4.0</td>
<td>0.654</td>
</tr>
<tr>
<td>5.0</td>
<td>0.563</td>
</tr>
</tbody>
</table>

Table VII-10: Saastamoinen $\delta R$ corrector look-up table, in meters.

<table>
<thead>
<tr>
<th>Zenith angle</th>
<th>Station Height Above Sea Level (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>60-00</td>
<td>0.003</td>
</tr>
<tr>
<td>66-00</td>
<td>0.006</td>
</tr>
<tr>
<td>70-00</td>
<td>0.012</td>
</tr>
<tr>
<td>73-00</td>
<td>0.020</td>
</tr>
<tr>
<td>75-00</td>
<td>0.031</td>
</tr>
<tr>
<td>76-00</td>
<td>0.039</td>
</tr>
<tr>
<td>77-00</td>
<td>0.050</td>
</tr>
<tr>
<td>78-00</td>
<td>0.065</td>
</tr>
<tr>
<td>78-30</td>
<td>0.075</td>
</tr>
<tr>
<td>79-00</td>
<td>0.087</td>
</tr>
<tr>
<td>79-30</td>
<td>0.102</td>
</tr>
<tr>
<td>79-45</td>
<td>0.111</td>
</tr>
<tr>
<td>80-00</td>
<td>0.121</td>
</tr>
</tbody>
</table>

C.4 Sagnac Effect

The algorithm for the Sagnac Effect correction was adapted from an algorithm used in UNB's DIPOP program. First, the receiver position is corrected for earth rotation during signal transmission. Then the satellite/receiver ranges for corrected and uncorrected receiver positions are differenced to establish the Sagnac correction for each satellite.

$$X_s = X \cdot \cos(\partial \rho) - Y \cdot \sin(\partial \rho)$$

$$Y_s = Y \cdot \cos(\partial \rho) + X \cdot \sin(\partial \rho)$$

$$\partial \rho = \rho \cdot \omega / c$$

$$S_{corr} = \rho_s - \rho$$
Where:

\( X, Y \) = Receiver cartesian coordinates (m).

\( X_s, Y_s \) = Receiver cartesian coordinates corrected for Sagnac effect (m).

\( \rho \) = Computed range between satellite and receiver (m).

\( \partial \rho \) = Range correction (m).

\( \omega = 7.2921151467 \times 10^{-5} \) (rad/s), rotation rate of the earth.

\( c = 2.99792458 \times 10^8 \) (m/s), speed of electromagnetic wave propagation.

C.5 Least-squares Code Point Positioning

C.5.1 Observation Equation [after Wells et al., 1986]

\[
p = \rho + \partial \rho + c(\partial t + \partial T) + \partial_{\text{ion}} + \partial_{\text{tropo}} + \epsilon \rho
\]

\[
\rho = \sqrt{(x - X_s)^2 + (y - Y_s)^2 + (z - Z_s)^2}
\]

(Actual range between satellite and receiver)

Where:

\( p \) = observed pseudo-range

\( c \) = speed of light (299 792 458 m/s)

\( \partial \rho \) = error in satellite position

\( \partial t \) = satellite clock error

\( \partial T \) = receiver clock errors

\( \partial_{\text{ion}} \) = delay due to passage through the ionosphere

\( \partial_{\text{tropo}} \) = delay due to passage through the troposphere

\( \epsilon \rho \) = receiver noise and multipath

\( x, y, z \) = satellite coordinates

\( X_s, Y_s, Z_s \) = receiver coordinates

C.5.2 Least-squares

\[
\partial X = -(A^T C_\rho^{-1} A)^{-1} A^T C_\rho^{-1} w
\]

Where:

\( \partial X \) = update to receiver coordinates (4 x 4 matrix)

\( C_\rho^{-1} \) = weight matrix, which can be considered to be unity (n x n matrix)
\[ w = (p - \rho) \text{ misclosure vector (n x 1 matrix)} \]

\[
A = \begin{bmatrix}
\frac{\partial p_1}{\partial X_r} & \frac{\partial p_1}{\partial Y_r} & \frac{\partial p_1}{\partial Z_r} & \frac{\partial p_1}{\partial (\partial T)} \\
... & ... & ... & ...
\end{bmatrix}
\]
\[ \text{(n x 4 matrix)}, \]

where \( n \) = number of pseudo-range observations

The \( i^{th} \) row of the A matrix is derived from:

\[
\frac{\partial p_i}{\partial X_r} = \frac{(X_r - x')}{\rho_i}, \quad \frac{\partial p_i}{\partial Y_r} = \frac{(Y_r - y')}{\rho_i}, \quad \frac{\partial p_i}{\partial Z_r} = \frac{(Z_r - z')}{\rho_i}, \quad \frac{\partial p_i}{\partial (\partial T)} = 1
\]

C.6 Least-squares Code Double Differencing

C.6.1 Observation Equation

\[
\nabla \Delta p_r = (p_r' - p_r^0) + (\rho_b' - \rho_b^0) - (p_{b'} - p_b^0)
\]

This rearranges to:

\[
\nabla \Delta p_r = (p_r' - p_r^0) + E
\]

\[
\rho_r' = \sqrt{(x' - X_r)^2 + (y' - Y_r)^2 + (z' - Z_r)^2}
\]

Where:

\( E \) combines all unaccounted for errors at both base and remote receivers.

\( n \) = number of pseudo-range observations

\( m \) = number of double differences (n-1)

\( i = 1 \) to \( m \),

0 refers to the reference satellite

b refers to the base station

C.6.2 Least-squares

\[
\partial X = -(A^T C_\rho^{-1} A)^{-1} A^T C_\rho^{-1} w
\]
Where:

\[ \partial X = \begin{array}{l}
\text{update to receiver coordinates (3 x 3 matrix)} \\
w = \nabla \Delta \rho_i - (\rho_i^o - \rho_i^o), \text{ misclosure vector (m x 1 matrix)} \\
C_{\rho}^{-1} = \text{ fully populated weight matrix (m x m matrix)} \\
\end{array} \]

\[
\frac{1}{2\sigma^2(m+1)} \begin{bmatrix}
 1 & -1 & -1 & \cdots \\
-1 & 1 & -1 & \cdots \\
-1 & \cdots & 1 & -1 \\
\cdots & \cdots & \cdots & \cdots \\
\end{bmatrix}, [\text{Hofmann-Wellenhof et al., 2001, p. 195}]
\]

Where the observation standard deviation (\(\sigma\)) was set to 1.

\[ A = \begin{array}{l}
\text{first design matrix (m x 3 matrix)} \\
\end{array} \]

\[
\frac{\partial (\nabla \Delta \rho_i)}{\partial X}, \frac{\partial (\nabla \Delta \rho_i)}{\partial Y}, \frac{\partial (\nabla \Delta \rho_i)}{\partial Z}, \ldots, \frac{\partial (\nabla \Delta \rho_i^m)}{\partial X}, \frac{\partial (\nabla \Delta \rho_i^m)}{\partial Y}, \frac{\partial (\nabla \Delta \rho_i^m)}{\partial Z}, \ldots, \]

[Dodd, 1994, p. 48]

The \(" i^\text{th}\) row of the A matrix is derived from:

\[
\frac{\partial (\nabla \Delta \rho_i)}{\partial X} = -\frac{(x_i - X_r)}{\rho_i^o} + \frac{(x_i^o - X_r)}{\rho_i^o} \\
\frac{\partial (\nabla \Delta \rho_i)}{\partial Y} = -\frac{(x_i - Y_r)}{\rho_i^o} + \frac{(x_i^o - Y_r)}{\rho_i^o} \\
\frac{\partial (\nabla \Delta \rho_i)}{\partial Z} = -\frac{(x_i - Z_r)}{\rho_i^o} + \frac{(x_i^o - Z_r)}{\rho_i^o} \\
\]

C.7 Least-squares Code and Carrier Point Positioning

C.7.1 Observation Equations

\[ p = \rho + \partial \rho + c(\partial t + \partial T) + \partial_{\text{ion}} + \partial_{\text{ropo}} + \varepsilon \rho \]  \hspace{1cm} \text{(code)}

\[ \phi = \rho - N \lambda + \partial \rho + c(\partial t + \partial T) - \partial_{\text{ion}} + \partial_{\text{ropo}} + \varepsilon \rho \]  \hspace{1cm} \text{(carrier in meters)}
\[ \rho = \sqrt{(x - X_r)^2 + (y - Y_r)^2 + (z - Z_r)^2} \] (actual range between satellite and receiver)

Where:

- \( \rho \) = observed pseudo-range
- \( \phi \) = observed carrier, from start of lock
- \( N \) = ambiguity at start of lock
- \( \lambda \) = carrier wave length
- \( c \) = speed of light (299 792 458 m/s)
- \( \partial p \) = error in satellite position
- \( \partial t \) = satellite clock error
- \( \partial T \) = receiver clock errors
- \( \partial_{\text{ion}} \) = delay due to passage through the ionosphere
- \( \partial_{\text{tropo}} \) = delay due to passage through the troposphere
- \( \varepsilon \) = receiver noise and multipath
- \( x, y, z \) = satellite coordinates
- \( X_n, Y_n, Z_r \) = receiver coordinates

C.7.2 Least-squares

Number of unknowns (n) = 4 + number of sats * 2.

Number of observations (m) = number of sats * 4

\[ \partial X = -(A^T C_\rho^{-1} A)^{-1} A^T C_\rho^{-1} w \]

Where:

\[ \partial X = \text{update to receiver coordinates (n x n matrix)} \]

\[ C_\rho^{-1} (m x m) = \]

\[ \begin{bmatrix}
1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 100 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 100 & \cdots & 0 & 0 & 0 & 0 \\
\end{bmatrix} \]

Variance/covariance of observations Code = 1.0 m, carrier = 0.01 m

\[ w = (\rho - \rho_\text{est} \text{ and } N_\text{est} - N_\text{pre_est}) \text{ misclosure vector (m x 1 matrix)} \]
\[
\begin{pmatrix}
-p_1^1 - \rho_1^1 \\
\phi_1^1 + N_1^1 \cdot \lambda_1 - \rho_1^1 \\
p_2^1 - \rho_2^1 \\
\phi_2^1 + N_2^1 \cdot \lambda_2 - \rho_2^1 \\
\vdots \\
\vdots \\
p_{i_{\text{Sat}}} - \rho_{i_{\text{Sat}}} \\
\phi_{i_{\text{Sat}}} + N_{i_{\text{Sat}}} \cdot \lambda_1 - \rho_{i_{\text{Sat}}} \\
p_{i_{\text{Sat}}} - \rho_{i_{\text{Sat}}} \\
\phi_{i_{\text{Sat}}} + N_{i_{\text{Sat}}} \cdot \lambda_2 - \rho_{i_{\text{Sat}}}
\end{pmatrix}
\]

\[w = \]
$A$ (m x n matrix),

$$
\begin{bmatrix}
\frac{\partial p_1^1}{\partial X_r}, & \frac{\partial p_1^1}{\partial Y_r}, & \frac{\partial p_1^1}{\partial Z_r}, & \frac{\partial p_1^1}{\partial (\partial T)_r}, & \frac{\partial p_1^1}{\partial N_{1s}^1}, & \frac{\partial p_1^1}{\partial N_{2s}^1}, & \cdots, & \frac{\partial p_1^1}{\partial N_{ns}^1}, & \cdots,
\frac{\partial p_1^n}{\partial X_r}, & \frac{\partial p_1^n}{\partial Y_r}, & \frac{\partial p_1^n}{\partial Z_r}, & \frac{\partial p_1^n}{\partial (\partial T)_r}, & \frac{\partial p_1^n}{\partial N_{1s}^n}, & \frac{\partial p_1^n}{\partial N_{2s}^n}, & \cdots, & \frac{\partial p_1^n}{\partial N_{ns}^n}, & \cdots.
\end{bmatrix}
$$

\[
\begin{bmatrix}
\frac{\partial p_2^1}{\partial X_r}, & \frac{\partial p_2^1}{\partial Y_r}, & \frac{\partial p_2^1}{\partial Z_r}, & \frac{\partial p_2^1}{\partial (\partial T)_r}, & \frac{\partial p_2^1}{\partial N_{1s}^1}, & \frac{\partial p_2^1}{\partial N_{2s}^1}, & \cdots, & \frac{\partial p_2^1}{\partial N_{ns}^1}, & \cdots,
\frac{\partial p_2^n}{\partial X_r}, & \frac{\partial p_2^n}{\partial Y_r}, & \frac{\partial p_2^n}{\partial Z_r}, & \frac{\partial p_2^n}{\partial (\partial T)_r}, & \frac{\partial p_2^n}{\partial N_{1s}^n}, & \frac{\partial p_2^n}{\partial N_{2s}^n}, & \cdots, & \frac{\partial p_2^n}{\partial N_{ns}^n}, & \cdots.
\end{bmatrix}
\]

\[
\begin{bmatrix}
X_r - x^1, & Y_r - y^1, & Z_r - z^1, & 1, & 0, & 0, & 0, & 0, & \cdots, & 0, & 0.
\end{bmatrix}
\]

\[
\begin{bmatrix}
X_r - x^1, & Y_r - y^1, & Z_r - z^1, & 1 - \lambda_1, & 0, & 0, & 0, & 0, & \cdots, & 0, & 0.
\end{bmatrix}
\]

\[
\begin{bmatrix}
X_r - x^1, & Y_r - y^1, & Z_r - z^1, & 1, & 0, & 0, & 0, & 0, & \cdots, & 0, & 0.
\end{bmatrix}
\]

\[
\begin{bmatrix}
X_r - x^1, & Y_r - y^1, & Z_r - z^1, & 1, & 0, & -\lambda_2, & 0, & 0, & \cdots, & 0, & 0.
\end{bmatrix}
\]

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C.8 Least-squares Code and Carrier Double Differencing

C.8.1 Observation Equations

Code: $\nabla \Delta \rho^i_r = (\rho^i_r - \rho^0_r) + (\rho^i_b - \rho^0_b) - (\rho^i_b - \rho^0_b) + E$

This rearranges to:

$$\nabla \Delta \rho^i_r = (\rho^i_r - \rho^0_r) + E$$

$$\rho^i_r = \sqrt{(x^i - X_r)^2 + (y^i - Y_r)^2 + (z^i - Z_r)^2}$$

Carrier: $\nabla \Delta \phi^i_r = (\phi^i_r - \phi^0_r) + (\phi^i_b - \phi^0_b) - \lambda[(N^i_r - N^0_r) - (N^i_b - N^0_b)] + E$

This rearranges to:

$$\nabla \Delta \phi^i_r = (\phi^i_r - \phi^0_r) - \lambda[(N^i_r - N^0_r) - (N^i_b - N^0_b)] + E$$

$$\nabla \Delta \phi^i_r = (\phi^i_r - \phi^0_r) - \lambda N^i_r + \lambda N^0_r + \lambda N^i_b - \lambda N^0_b + E$$

Where:

- $\rho$ = observed pseudo-range
- $\rho$ = actual range
- $\phi$ = observed carrier, from start of lock
- $N$ = ambiguity at start of lock
- $\lambda$ = carrier wave length
- $E$ = receiver noise and multipath and unmodeled atmospheric uncertainties
- $x, y, z$ = satellite coordinates
- $X, Y, Z$ = receiver coordinates
- Subscript $r$ = remote receiver
- Subscript $b$ = base receiver
- Superscript $i$ = satellite

C.8.2 Least-squares

Number of unknowns (n) = 3 + (number of sats - 1) * 2.

Number of observations (m) = number of (sats - 1) * 4

$n_{DD} =$ number of double differences (sats - 1)

$$\hat{x} = - (A^T C^{-1} A)^{-1} A^T C^{-1} w$$
Where:
\[ \partial X = \text{update to receiver coordinates (n x n matrix)} \]
\[ C^{-1}_c = \text{fully populated weight matrix (m x m matrix)} \]
\[ \frac{1}{2 \sigma^2 (m + 1)} \begin{bmatrix} m & -1 & -1 & \ldots \\ -1 & m & -1 & \ldots \\ -1 & \ldots & -1 & m \end{bmatrix} \]

[Hofmann-Wellenhof et al., 2001, p. 195]

Variance of observations \((\sigma^2)\) code = 1.0 m, carrier = 0.01 m
\[ w = (p - \rho \text{ and } N_{\text{est}} - N_{\text{pre est}}) \text{ misclosure vector (m x 1 matrix)} \]
\[ w = \begin{bmatrix} \nabla \Delta \rho_{1} - (\rho_{1} - \rho_{0}) \\ \nabla \Delta \phi_{1} + \nabla \Delta N_{1} \cdot \lambda_{1} - (\rho_{1} - \rho_{0}) \\ \nabla \Delta \rho_{2} - (\rho_{2} - \rho_{0}) \\ \nabla \Delta \phi_{2} + \nabla \Delta N_{2} \cdot \lambda_{2} - (\rho_{2} - \rho_{0}) \\ \vdots \\ \nabla \Delta \rho_{n_{DD}} - (\rho_{n_{DD}} - \rho_{1_{refSat}}) \\ \nabla \Delta \phi_{n_{DD}} + \nabla \Delta N_{n_{DD}} \cdot \lambda_{1} - (\rho_{n_{DD}} - \rho_{1_{refSat}}) \\ \nabla \Delta \rho_{n_{DD}} - (\rho_{n_{DD}} - \rho_{2_{refSat}}) \\ \nabla \Delta \phi_{n_{DD}} + \nabla \Delta N_{n_{DD}} \cdot \lambda_{2} - (\rho_{n_{DD}} - \rho_{2_{refSat}}) \end{bmatrix} \]

\[ p_{1} = \text{Ca code} \]
\[ p_{2} = \text{P2 code} \]
\[ \phi_{1} = \text{L1 carrier observation (from lock)} \]
\[ \phi_{2} = \text{L2 carrier observation (from lock)} \]
\[ N_{1} = \text{Initial integer count for L1} \]
\[ N_{2} = \text{Initial integer count for L2} \]
\[ \nabla \Delta N = (N_{i} - N_{r}) - (N_{i}^0 - N_{r}^0) \]

\[ A \text{ (m x n matrix)}, \]

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The \( i \)th row and first three columns of the A matrix is derived from:

\[
\begin{align*}
\frac{\partial (\nabla \Delta p_i)}{\partial X_r} &= \frac{\partial (\nabla \Delta \phi_i)}{\partial X_r} = \frac{- (x' - X_r)}{\rho_r^i} + \frac{(x^0 - X_r)}{\rho^0} \\
\frac{\partial (\nabla \Delta p_i)}{\partial Y_r} &= \frac{\partial (\nabla \Delta \phi_i)}{\partial Y_r} = \frac{- (y' - Y_r)}{\rho_r^i} + \frac{(y^0 - Y_r)}{\rho^0} \\
\frac{\partial (\nabla \Delta p_i)}{\partial Z_r} &= \frac{\partial (\nabla \Delta \phi_i)}{\partial Z_r} = \frac{- (z' - Z_r)}{\rho_r^i} + \frac{(z^0 - Z_r)}{\rho^0}
\end{align*}
\]
C.9 Ionosphere-Free Combination

The effects of the ionosphere can be almost completely removed by combining L1 and L2 carrier observations into an "ionosphere-free" solution. We go from a four-observation solution to a two-observation solution. The ionosphere-free solution is as follows:

\[
\phi_{IF} = \frac{\phi_2 - \alpha_f \phi_1}{1 - \alpha_f}
\]

\[
P_{IF} = \frac{P_2 - \alpha_f P_1}{1 - \alpha_f}
\]

\[
\alpha_f = \left(\frac{fL_1}{fL_2}\right)^2
\]

Where:

\[
P_{IF} = \text{new ionosphere-free pseudo-range observation}
\]
$P_i$ = observed pseudo-range on L1 (CA or P1)

$P_2$ = observed pseudo-range on L2 (P2)

$\Phi_{IF}$ = new ionosphere-free carrier observation

$\Phi_1$ = observed carrier (with estimated ambiguity) on L1

$\Phi_2$ = observed carrier (with estimated ambiguity) on L2

$f_{L1}$ = L1 frequency (1575.42 MHz)

$f_{L2}$ = L2 frequency (1227.60 MHz)

C.9.1 Observation Equations

The observation equations are as follows

**Code**

\[
\nabla \Delta \rho_i = \left( \rho_i - \rho_i^0 \right) + E
\]

\[
\rho_i = \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2}
\]

**Carrier**

\[
\nabla \Delta \phi_i = \left( \phi_i - \phi_i^0 \right) - N_i + N_i^0 - N_i^b + E
\]

The observation equations are the same as in the code and carrier double difference case, except that there are only two observables, one for the ionosphere-free code and one for the ionosphere-free carrier. Ambiguities are no longer integers and are estimated in meters.

C.9.2 Least-squares

Number of unknowns (n) = 3 + (number of satellites-1). X, Y, Z, LIF ambiguities

Number of observations (m) = number of (sats-1)*2
nDD = number of double differences (sats-1)

\[
\partial X = - (A^T C_\rho^{-1} A)^{-1} A^T C_\rho^{-1} w
\]

Where:

$\partial X$ = update to receiver coordinates (n x n matrix)

$C_\rho^{-1}$ = fully populated weight matrix (m x m matrix)
\[
\frac{1}{2\sigma^2(m+1)}\begin{bmatrix}
  m & -1 & -1 & \ldots \\
  -1 & m & -1 & \ldots \\
  -1 & -1 & \ldots & -1 \\
  \ldots & \ldots & \ldots & m
\end{bmatrix}, \text{[Hofmann-Wellenhof et al., 2001, p. 195]}
\]

Variance of observations ($\sigma^2$) code $= 1.0$ m, carrier $= 0.01$ m

\[
w = (p - \rho \text{ and } N_{\text{est}} - N_{\text{pre_est}}) \text{ misclosure vector (m x 1 matrix)}
\]

\[
w = \begin{bmatrix}
\nabla \Delta p_{IF}^1 - (\rho_{IF}^1 - \rho_{IF}^0) \\
\nabla \Delta \phi_{IF}^1 + \nabla \Delta N_{IF}^1 - (\rho_{IF}^1 - \rho_{IF}^0) \\
\nabla \Delta p_{IF}^{n\text{DO}} - (\rho_{IF}^{n\text{DO}} - \rho_{IF}^{\text{refSar}}) \\
\nabla \Delta \phi_{IF}^{n\text{DO}} + \nabla \Delta N_{IF}^{n\text{DO}} - (\rho_{IF}^{n\text{DO}} - \rho_{IF}^{\text{refSar}})
\end{bmatrix}
\]

$P_{IF}$ = Ionosphere-free code (in meters)

$\phi_{IF}$ = Ionosphere-free carrier (in meters)

$N_{IF}$ = Initial ionosphere-free ambiguity (float – in meters)

\[
\nabla \Delta N = \left( N_{IF}' - N_{IF}^0 \right) - \left( N_{IF}' - N_{IF}^0 \right)
\]

$A$ (m x n matrix),

\[
\begin{bmatrix}
\frac{\partial \nabla \Delta p_{IF}^1}{\partial X_r} & \frac{\partial \nabla \Delta p_{IF}^1}{\partial Y_r} & \frac{\partial \nabla \Delta p_{IF}^1}{\partial Z_r} & \frac{\partial \nabla \Delta p_{IF}^1}{\partial \Delta N_{IF}^1} & \ldots & \frac{\partial \nabla \Delta p_{IF}^1}{\partial \nabla \Delta N_{IF}^{n\text{DO}}} \\
\frac{\partial \nabla \Delta \phi_{IF}^1}{\partial X_r} & \frac{\partial \nabla \Delta \phi_{IF}^1}{\partial Y_r} & \frac{\partial \nabla \Delta \phi_{IF}^1}{\partial Z_r} & \frac{\partial \nabla \Delta \phi_{IF}^1}{\partial \Delta N_{IF}^1} & \ldots & \frac{\partial \nabla \Delta \phi_{IF}^1}{\partial \nabla \Delta N_{IF}^{n\text{DO}}} \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
\frac{\partial \nabla \Delta p_{IF}^{n\text{DO}}}{\partial X_r} & \frac{\partial \nabla \Delta p_{IF}^{n\text{DO}}}{\partial Y_r} & \frac{\partial \nabla \Delta p_{IF}^{n\text{DO}}}{\partial Z_r} & \frac{\partial \nabla \Delta p_{IF}^{n\text{DO}}}{\partial \Delta N_{IF}^1} & \ldots & \frac{\partial \nabla \Delta p_{IF}^{n\text{DO}}}{\partial \nabla \Delta N_{IF}^{n\text{DO}}} \\
\frac{\partial \nabla \Delta \phi_{IF}^{n\text{DO}}}{\partial X_r} & \frac{\partial \nabla \Delta \phi_{IF}^{n\text{DO}}}{\partial Y_r} & \frac{\partial \nabla \Delta \phi_{IF}^{n\text{DO}}}{\partial Z_r} & \frac{\partial \nabla \Delta \phi_{IF}^{n\text{DO}}}{\partial \Delta N_{IF}^1} & \ldots & \frac{\partial \nabla \Delta \phi_{IF}^{n\text{DO}}}{\partial \nabla \Delta N_{IF}^{n\text{DO}}}
\end{bmatrix}
\]
The "i"th row and first three columns of the A matrix is derived from:

\[
\begin{align*}
\frac{\partial (\nabla \Delta p_{if}^i)}{\partial x_r} &= \frac{\partial (\nabla \Delta \phi_{if}^i)}{\partial x_r} = -\left(x^i - x_r\right) + \left(x^o - x_r\right) \\
\frac{\partial (\nabla \Delta p_{if}^i)}{\partial y_r} &= \frac{\partial (\nabla \Delta \phi_{if}^i)}{\partial y_r} = -\left(y^i - y_r\right) + \left(y^o - y_r\right) \\
\frac{\partial (\nabla \Delta p_{if}^i)}{\partial z_r} &= \frac{\partial (\nabla \Delta \phi_{if}^i)}{\partial z_r} = -\left(z^i - z_r\right) + \left(z^o - z_r\right)
\end{align*}
\]

\[
\begin{bmatrix}
\frac{\partial \nabla \Delta p_{if}^i}{\partial x_r} & \frac{\partial \nabla \Delta p_{if}^i}{\partial y_r} & \frac{\partial \nabla \Delta p_{if}^i}{\partial z_r} & 0 & 0 & 0 \\
\frac{\partial \nabla \Delta p_{if}^i}{\partial x_r'} & \frac{\partial \nabla \Delta p_{if}^i}{\partial y_r'} & \frac{\partial \nabla \Delta p_{if}^i}{\partial z_r'} & -1 & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
\frac{\partial \nabla \Delta p_{if}^\text{DD}}{\partial x_r} & \frac{\partial \nabla \Delta p_{if}^\text{DD}}{\partial y_r} & \frac{\partial \nabla \Delta p_{if}^\text{DD}}{\partial z_r} & 0 & 0 & 0 \\
\frac{\partial \nabla \Delta p_{if}^\text{DD}}{\partial x_r'} & \frac{\partial \nabla \Delta p_{if}^\text{DD}}{\partial y_r'} & \frac{\partial \nabla \Delta p_{if}^\text{DD}}{\partial z_r'} & 0 & 0 & -1
\end{bmatrix}
\]

C.10 Sequential Least-Squares

The method of sequential least-squares (SLS) was used to propagate position and ambiguity uncertainty from one epoch to the next. The variance-covariance matrix of unknowns from one epoch is inverted and added to the weight matrix of unknowns in the subsequent matrix. As the confidence in position and ambiguity estimates increase (the uncertainty decreases) and the weight on the corresponding unknowns will also increase. When a new satellite is acquired, the weight on its ambiguity estimate is low relative, relative to that of the others. The algorithms used in USM_OTF were adapted from the precise point positioning algorithms presented by Kouba & Heroux (2000).
The standard least-squares algorithm is given by:
\[ \partial \mathbf{X} = -(A^T C_{\rho}^{-1} A)^{-1} A^T C_{\rho}^{-1} w \]

The "N" matrix (weight matrix of unknowns) is given as:
\[ N = A^T C_{\rho}^{-1} A \]

The inverse \((N')\) is the variance-covariance of unknowns, and it is this matrix that is transferred to the subsequent epoch. System noise is added to \(N'\) to account for changes in the unknowns. In all applications for the dissertation, the position noise was set to 1.0 m/s² to simulate antenna motion. The system noise for the ambiguities was set to zero.

The weight matrix \(N\) for the sequential least-squares process is computed from:
\[ N_i = A_i^T C_{\rho}^{-1} A_i + N_{i-1} + \text{Noise} * \Delta t \]

Where \(\Delta t\) is the time between epochs and \(i\) is the current epoch.

### C.11 Zenith Propagation Delay Term

A zenith propagation delay (ZPD) term was included as an additional unknown in modified versions of the four-observation and ionosphere-free solutions to remove some of the residual troposphere errors. Only the ionosphere-free version will be discussed here, because the same principal applies to both applications.

#### C.11.1 Observation Equations

The observation equations are as follows

**Code**
\[ \nabla \Delta \rho_i = (\rho_i - \rho_i^0) + \nabla \Delta E \]
\[ \rho_i = \sqrt{(x_i - X_r)^2 + (y_i - Y_r)^2 + (z_i - Z_r)^2} \]

**Carrier**
\[ \nabla \Delta \phi_i = (\phi_i - \phi_i^0) - N_i + N_i^0 + N_i^0 - N_i^0 + \nabla \Delta E \]

In this case "\(\nabla \Delta E\)" represents the double differenced residual troposphere delay.

In the process, a slant scale factor was computed for each in-view satellite, at both the base station and remote, using the wet component from the Niell Mapping function \((WMF)\). The scale factor was then double differenced to determine a combined scale
factor for the double differenced observation. The double differenced scale factor was then used to map the ZPD to the slant and finally applied to double differenced observations. In mathematical terms:

\[ E = ZPD \times WMF \]

\[ \nabla \Delta E = (E'_r - E'_h) - (E'_h - E'_n) \]

\[ \nabla \Delta E = E'_r - E'_h + E'_h \]

\[ \nabla \Delta E = ZPD \times WMF'_r - ZPD \times WMF'_h - ZPD \times WMF'_n + ZPD \times WMF'_p \]

\[ \nabla \Delta E = ZPD(WMF'_r - WMF'_h - WMF'_n + WMF'_p) \]

\[ \nabla \Delta E = ZPD \times \nabla \Delta WMF \]

C.11.2 Least-squares

Number of unknowns (n) = 4 + (number of satellites - 1).

\( X, Y, Z, L \) ambiguities

Number of observations (m) = number of (sats-1)\*2

nDD = number of double differences (sats-1)

\[ \partial X = -(A^T C^{-1} A)^{-1} A^T C^{-1} w \]

Where:

\[ \partial X \] update to receiver coordinates (n x n matrix)

\[ C^{-1} \] fully populated weight matrix (m x m matrix)

\[ \frac{1}{2\sigma^2(m+1)} \begin{bmatrix} m & -1 & -1 & \ldots \\ -1 & m & -1 & \ldots \\ -1 & \ldots & \ldots & -1 \\ \ldots & \ldots & -1 & m \end{bmatrix}, [Hofmann-Wellenhof et al., 2001, p. 195] \]

Variance of observations (\( \sigma^2 \)) code = 1.0 m, carrier = 0.01 m

\[ w \] (\( \rho - \rho \) and \( N_{est} - N_{pre est} \)) misclosure vector (m x 1 matrix)
The "ith" row and first three columns of the A matrix is derived from:

\[
\begin{align*}
\frac{\partial (\nabla \Delta p^i_{IF})}{\partial x_r} &= \frac{\partial (\nabla \Delta \phi^i_{IF})}{\partial x_r} = -\left( x^i - x_r \right) + \frac{\left( x^0 - x_r \right)}{\rho^i} \\
\frac{\partial (\nabla \Delta p^i_{IF})}{\partial y_r} &= \frac{\partial (\nabla \Delta \phi^i_{IF})}{\partial y_r} = -\left( y^i - y_r \right) + \frac{\left( y^0 - y_r \right)}{\rho^i} \\
\frac{\partial (\nabla \Delta p^i_{IF})}{\partial z_r} &= \frac{\partial (\nabla \Delta \phi^i_{IF})}{\partial z_r} = -\left( z^i - z_r \right) + \frac{\left( z^0 - z_r \right)}{\rho^i}
\end{align*}
\]
C.12 Zenith Ionosphere Map Interpolation

Several of the studies conducted for this dissertation used zenith ionosphere maps. These maps contained a two-dimensional grid of zenith ionosphere corrector values for an ionosphere shell at a given height (300 km, 350 km or 450 km for this work). In order to interpolate these maps, a signal pierce point between the user and satellite was computed. To translate the zenith corrector to a slant range, the obliquity factor was determined. The following algorithms, taken from Lin (2001), were used to determine pierce point locations and then determine the obliquity factor.

C.12.1 Pierce points

\[ \phi_{pp} = \sin^{-1}(\sin\phi_a \cdot \cos\psi_{pp} + \cos\phi_a \cdot \sin\psi_{pp} \cdot \cos A) \]

\[ \psi_{pp} = \frac{\pi}{2} - E - \sin^{-1}\left(\frac{R_e}{R_e + h} \cdot \cos E\right) \]

\[ \lambda_{pp} = \lambda_u + \sin^{-1}\left(\frac{\sin\psi_{pp} \cdot \sin A}{\cos\phi_{pp}}\right) \]

Where:

\[ \phi_{pp} = \text{Pierce point latitude} \]

\[ \lambda_{pp} = \text{Pierce point longitude} \]
\( \psi_{pp} \) = "Angle subtended at the center of the earth between the user position and the earth projection of the pierce point."

\( \phi \) = User latitude

\( \lambda \) = User longitude

\( E \) = Elevation angle from user to satellite

\( A \) = Azimuth of satellite from user

\( R_e \) = Mean Earth radius (6370973.278 m)

\( h \) = height of shell in meters (300000, 350000, or 450000)

C.12.2 Obliquity factor

\[
O_E = \left[ 1 - \left( \frac{R_e}{R_e + h} \cos E \right)^2 \right]^{-0.5}
\]

Zenith correctors were interpolated from the pierce point locations and then mapped to the slant using the obliquity factor.

Slant Range = Zenith * \( O_E \)
APPENDIX D

UNCERTAINTY AND HISTOGRAMS FOR 2006 LOUISIANA DATA

The following appendix contains tables and histogram plots from DOY 180 (June 29) of the 2006 Louisiana data discussed in 0. The tables (Table VII-11 and Table VII-12) contain the 95% (2σ and OS) up RMS uncertainty in cm. Table VII-11 contains the results from using the conventional methods (Saastamoinen troposphere, no MDP or ZPD). Table VII-12 contains the results from using the optimum methods (NOAA troposphere, with MDP and ZPD).

<table>
<thead>
<tr>
<th></th>
<th>2σ (cm)</th>
<th></th>
<th>95 OS (cm)</th>
<th></th>
</tr>
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<tr>
<td></td>
<td>N</td>
<td>E</td>
<td>U</td>
<td>N</td>
</tr>
<tr>
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<td>5.8</td>
<td>13.5</td>
<td>4.8</td>
</tr>
<tr>
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<td>10.3</td>
<td>18.4</td>
<td>6.6</td>
</tr>
<tr>
<td>UFX</td>
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<td>8.0</td>
<td>13.3</td>
<td>4.9</td>
</tr>
<tr>
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<td>6.6</td>
<td>21.4</td>
<td>10.6</td>
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<tr>
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<td>14.5</td>
<td>19.1</td>
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<tr>
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<td>17.2</td>
<td>15.1</td>
<td>24.9</td>
<td>22.2</td>
</tr>
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<td>LMCN-IF</td>
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<td>10.1</td>
<td>37.4</td>
<td>8.9</td>
</tr>
<tr>
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<td>51.6</td>
<td>14.8</td>
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<tr>
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<td>12.0</td>
<td>48.9</td>
<td>15.6</td>
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<td>63.1</td>
<td>21.0</td>
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<td>17.8</td>
</tr>
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<td>25.7</td>
<td>59.7</td>
<td>21.9</td>
</tr>
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<td>9.7</td>
</tr>
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<td>74.0</td>
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<td>35.4</td>
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<td>32.3</td>
<td>112.5</td>
<td>30.6</td>
</tr>
</tbody>
</table>

Table VII-11: Conventional up 2σ and 95% OS RMS for DOY 180 (June 29), 2006.
The following plots show the uncertainty histograms for the conventional and optimum solutions, from the Louisiana study baselines discussed in 0. Plots show histograms from the ionosphere-free (IF), 4-observation USTEC float (U FL) and 4-observation USTEC fixed (U FX) solutions. Red vertical lines show the 95% ordered statistic limits and the blue lines show the 2σ limits.
Figure VII-6: Uncertainty histograms for DSTR to NOLA.

- Red line represents 95% OS
- Blue line represents 2σ
Figure VII-7: Uncertainty histograms for DSTR to HAMM.

- Red line represents 95% OS
- Blue line represents 2σ
Figure VII-8: Uncertainty histograms for LMCN to NOLA.

- Red line represents 95% OS
- Blue line represents 2σ
Figure VII-9: Uncertainty histograms for LMCN to HAMM.

- Red line represents 95% OS
- Blue line represents 2σ
Figure VII-10: Uncertainty histograms for SIHS to HAMM.

- Red line represents 95% OS
- Blue line represents 2σ
Figure VII-11: Uncertainty histograms for LMCN to SIHS.

- Red line represents 95% OS
- Blue line represents 2σ
REFERENCES


Fuller-Rowell, T. (2006a). Personal communication on April 26, 2006 at the Space Environment Week conference in Boulder CO.


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